SIMULATIONS OF THE ACTIVE CAB SUSPENSION

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Abstract

In the context of growing ergonomic concerns and pressing competition on the market, designing machines and vehicles offering a better operator comfort has become a major trend in development of heavy-duty machines and vehicles. During the ride over the rough terrain, the cab is subjected to excitations in the form of low frequency and high-amplitude vibration. This study investigates the vibration reduction strategy whereby the machine structure should incorporate an active suspension of the cab. An actuating mechanism is incorporated, connected to the machine frame and the cab, and placed in between. The main system component is a mobile platform to which the cab is attached. Respective drives set in motion the passive links in the actuating mechanism. The drives are equipped with cylinders capturing the instantaneous velocities derived in the control sub-system. The machine frame, subjected to kinematic excitations, performs a movement in space, which has to be measured with a set of sensors to support the control process. Basing on the measured movements of the machine frame, the control sub-system calculates the real-time values of the anticipated load and the required drive velocities. This study focuses on the development of a mechanical model of the actuating mechanism operating in several degree of freedom options. Solving the direct and inverse problems involving the position and velocity of the mechanism allows the Jacobean matrix to be applied in Newton-Euler's equations. The purpose of the active suspension system is to stabilise the cab in the vertical position and to reduce its lateral vibrations and seat vibrations in the vertical. This study summarises the results of simulations performed to evaluate the system's performance and its power demand.

Keywords: *heavy duty machines, active vibration control, platform mechanism*

1. Introduction

The work of operators in heavy machinery requires constant attention to gather information about the machine's surroundings, its current status and the operations performed. Operators have to analyse the received information on the continuous basis and make decisions accordingly, to have them implemented via the control system and to perform the scheduled tasks in the optimal manner. The more powerful the machine, the more serious the consequence of errors committed by operators. The typical frequency range of vibration of machines and their equipment is determined based on testing done on heavy machines and is found to be 0.5-80 Hz [1].

Machine vibrations are induced by the drives' action, movements of the equipment, variable loading and machine ride. The ride of heavy machines, tractors, forestry vehicles over a rough terrain lead to cyclic tilting of the machines, which can be regarded as low-frequency (up to several Hz) and high-amplitude (about 10 degrees) vibration of the machine. The angular motions of the frame are transmitted onto the cab, and the higher the cab position, the larger the amplitude range of linear vibration of the point SIP (about 70 cm). Vibrations negatively impact on the machine structure, control processes, performance quality and the operator's comfort. Growing ergonomic concerns and competition on the market have prompted the design of machines ensuring the better comfort for the operator. Cab suspensions are now incorporated in the machine structure as a new solution [5, 6].

To improve the operator's comfort, an active suspension of a cab can be incorporated in the machine structure (Fig. 1), to reduce the cab's vibration. The active suspension system comprises several sub-systems:

1. The actuator mechanism, connected to the machine frame and the cab is placed in between. The main element (link) of the mechanism is a mobile platform to which the cab is attached. The platform is suspended or supported on the frame and depending on the mechanism's mobility, it can move with respect to the frame in the selected DOFs.
2. The drives set in motion the passive links in the active suspension mechanism. On account of the stiffness requirements and availability of the given type of energy, and to ensure fast response the control signals the hydraulic drives are going to be used. The drives are provided with actuators to capture the instantaneous velocities, derived in the control sub-system.
3. Measuring sub-system. Displacement and velocity are chosen as control quantities for the active suspension system. Directly measured data yield the error signal to be used in the control process. The machine frame subjected to kinematic excitations executes a spatial movement, measured with a set of sensors.
4. Control sub-system. Errors of drive positions and their derivatives are going to be used in the feedback control of the active suspension system. Basing on the frame motion measurements, the control sub-system performs the real time calculation of the anticipated loads and the required drive velocities.

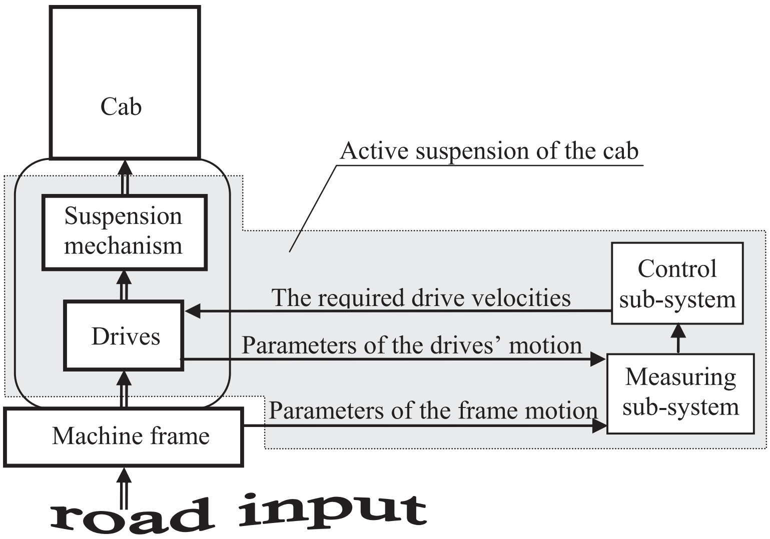

Fig. 1. Block diagram of the active suspension of a cab

2. Kinematic model of the active suspension mechanism

The active suspension mechanism, shown schematically in Fig. 2 has been engineered specifically for the purpose of modelling and simulations and its design involves a certain trade-off between functionality and simplicity. The presented active suspension mechanism is capable of reducing the amplitudes of the cab's linear vibrations in the direction y_r and its angular vibrations around the axes x_r, y_r. The active suspension mechanism comprises just three passive links, set in motion by two linear drives. The separate seat suspension mechanism reduces the vibrations along the axis z_r [4]. The main function of the active suspension system is to stabilise the cab such that the correct control of the drives 1 and 4 should enable its vertical movement in the direction of the gravity force.

To determine the influence of the active suspension system on the cab motion, the kinematic model is developed based on vector calculus. Versors used to define the positions of the active suspension mechanism links are shown in Fig. 2a.

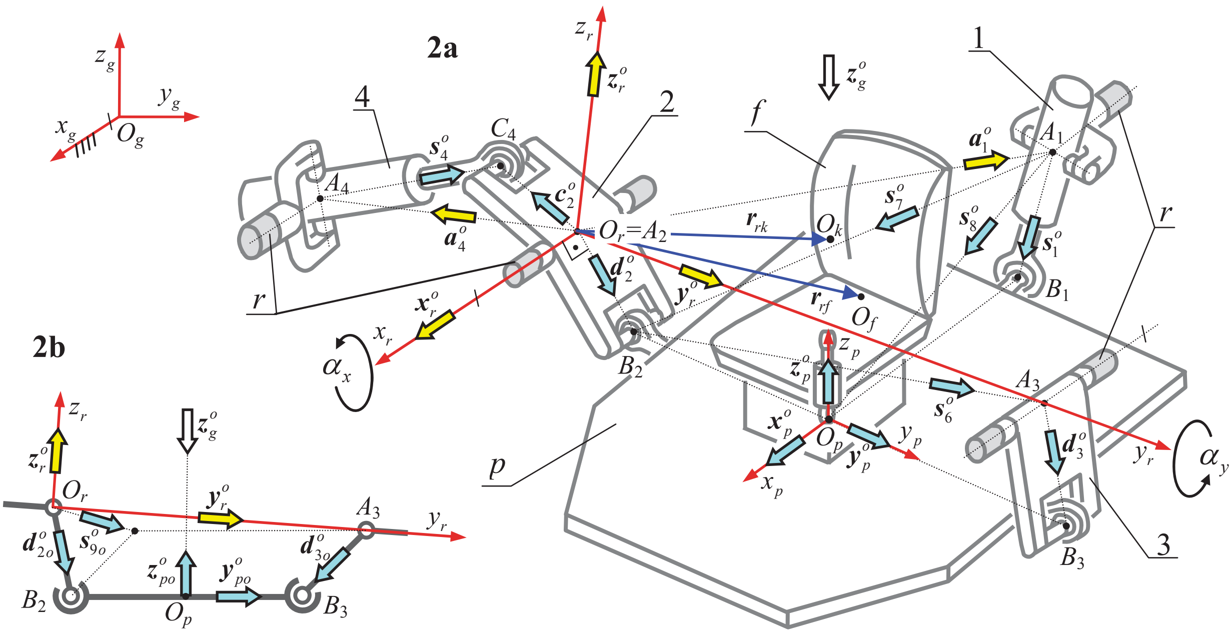

Fig. 2a, b. Vectors and versors in the kinematic model of cab suspensions

2.1. Direct and inverse kinematics problems of link position

Solving the direct problem consists in finding the cab orientation and position of its centre of gravity Q_k and of the point Q_f - the centre of gravity of the seat-operator system with respect to the reference system $\{O_r x_r y_r z_r\}$ associated with the machine frame. The cab, platform and seat orientation and $\boldsymbol{r}_{rk}$ - the radius vector of the of the cab's c.o.g with respect to the reference system associated with the frame depend on variable lengths of actuators s_1, s_4 and the known constant dimensions of links in the active suspension mechanism. The radius vector of the c.o.g of the seat-operator system; $\boldsymbol{r}_{rf}$ is controlled by the lengths of three actuators s_1, s_4, s_5 and the fixed dimensions of links in the active suspension mechanism. The solution of the direct problem involving the link position is explicit and consists in determining versors on the basis of two already known or already established ones. Basing on already known versors: $\boldsymbol{a}_4^o$, $\boldsymbol{x}_r^o$, $\boldsymbol{y}_r^o$, $\boldsymbol{a}_1^o$, expressed in the system associated with the frame, we are able to determine other versors: $\boldsymbol{c}_2^o$, $\boldsymbol{s}_4^o$, $\boldsymbol{d}_2^o$, $\boldsymbol{s}_6^o$, $\boldsymbol{d}_3^o$, $\boldsymbol{y}_p^o$, $\boldsymbol{s}_7^o$, $\boldsymbol{s}_8^o$, $\boldsymbol{s}_1^o$, $\boldsymbol{x}_p^o$, $\boldsymbol{z}_p^o$ (Fig. 2a). The cab orientation is determined by directional versors in the reference system associated with the platform $\boldsymbol{x}_p^o$, $\boldsymbol{y}_p^o$, $\boldsymbol{z}_p^o$, which is used in the direction matrix:

$$ {}_p^r\boldsymbol{R} = \begin{bmatrix} \boldsymbol{x}_p^o \cdot \boldsymbol{x}_r^o & \boldsymbol{y}_p^o \cdot \boldsymbol{x}_r^o & \boldsymbol{z}_p^o \cdot \boldsymbol{x}_r^o \\ \boldsymbol{x}_p^o \cdot \boldsymbol{y}_r^o & \boldsymbol{y}_p^o \cdot \boldsymbol{y}_r^o & \boldsymbol{z}_p^o \cdot \boldsymbol{y}_r^o \\ \boldsymbol{x}_p^o \cdot \boldsymbol{z}_r^o & \boldsymbol{y}_p^o \cdot \boldsymbol{z}_r^o & \boldsymbol{z}_p^o \cdot \boldsymbol{z}_r^o \end{bmatrix}. \tag{1} $$

The cab and seat positions determined:

$$ \boldsymbol{r}_{rk} = d_2 \boldsymbol{d}_2^o + b_2 \boldsymbol{y}_p^o + {}_p^r\boldsymbol{R}\, {}^p\boldsymbol{r}_{pk}, \tag{2} $$

$$ \boldsymbol{r}_{rf} = d_2 \boldsymbol{d}_2^o + b_2 \boldsymbol{y}_p^o + {}_p^r\boldsymbol{R}\, {}^p\boldsymbol{r}_{pf}, \tag{3} $$

where: $d_2 = |A_2 B_2|$, $b_2 = |O_p B_2|$, ${}^p\boldsymbol{r}_{pk} = \overrightarrow{O_p O_k}$ - the radius vector of the point O_k expressed in the reference system associated with the platform, ${}^{p}\boldsymbol{r}_{pf}=\overrightarrow{O_pO_f}$ - the radius vector of the point O_f reference system associated with the platform.

The inverse problem handled in the coordinate system associated with the machine frame involves the orientation of the platform p. The platform should be stabilised in the vertical whilst the active suspension system is in use. The platform position is related to the gravity force versor $\boldsymbol{z}_g^o$ (Fig. 2a,b), which can be expressed in the coordinate system associated with the frame according to the formula:

$$\boldsymbol{z}_g^o={}_g^r\boldsymbol{R}\,{}^g\boldsymbol{z}_g^o, \tag{4}$$

where: ${}_g^r\boldsymbol{R}=\begin{bmatrix}\cos\alpha_y & 0 & \sin\alpha_y\\ \sin\alpha_x\sin\alpha_y & \cos\alpha_x & -\sin\alpha_x\cos\alpha_y\\ -\cos\alpha_x\sin\alpha_y & \sin\alpha_x & \cos\alpha_x\cos\alpha_y\end{bmatrix}$, α_x, α_y - the frame deflection angles (Fig. 2a) measured in the measuring sub-system, ${}^g\boldsymbol{z}_g^o=[0,0,-1]^T$.

In order to solve the inverse problem it is required that the lengths of the actuators s_{1o}, s_{4o} should be established, corresponding to the predetermined and expected platform position with respect to the system associated with the frame and expressed by versor coordinates: $\boldsymbol{x}_{po}^o$, $\boldsymbol{y}_{po}^o$ and $\boldsymbol{z}_{po}^o$. Actually, the cab will reach the position nearing the expected one. The anticipated values (indicated with a subscript "$_o$") obtained from solving the inverse problem will be used to derive the error signal required for the control process. The direction of the cab's vertical axis versor should be opposite to that of the gravity force versor $\boldsymbol{z}_{po}^o=-\boldsymbol{z}_g^o$. The solution of the inverse problem involving the link position is explicit and consists in determining versors on the basis of two already known or already established ones. Basing on already known versors, (Fig. 2a, 2b): $\boldsymbol{z}_{po}^o$, $\boldsymbol{x}_r^o$, $\boldsymbol{y}_r^o$, $\boldsymbol{a}_1^o$, $\boldsymbol{a}_4^o$ expressed in the system associated with the frame, we are able to determine other versors: $\boldsymbol{y}_{po}^o$, $\boldsymbol{x}_{po}^o$, $\boldsymbol{s}_{9o}^o$, $\boldsymbol{d}_{2o}^o$, $\boldsymbol{c}_{2o}^o$. The expected length of the drive 4 yields:

$$s_{4o}=\sqrt{(\boldsymbol{s}_{4o}\cdot\boldsymbol{x}_r^o)^2+(\boldsymbol{s}_{4o}\cdot\boldsymbol{y}_r^o)^2+(\boldsymbol{s}_{4o}\cdot\boldsymbol{z}_r^o)^2}\,, \tag{5}$$

where: $\boldsymbol{s}_{4o}=c_2\boldsymbol{c}_{2o}^o-a_4\boldsymbol{a}_4^o$ - vector of the drive 4.

The expected length of the drive 1 yields:

$$s_{1o}=\sqrt{(\boldsymbol{s}_{1o}\cdot\boldsymbol{x}_r^o)^2+(\boldsymbol{s}_{1o}\cdot\boldsymbol{y}_r^o)^2+(\boldsymbol{s}_{1o}\cdot\boldsymbol{z}_r^o)^2}\,, \tag{6}$$

where: $\boldsymbol{s}_{1o}=d_2\boldsymbol{d}_{2o}^o+b_2\boldsymbol{y}_{po}^o-a_1\boldsymbol{a}_1^o-b_1\boldsymbol{x}_{po}^o$ - vector of the actuator 1.

2.2. Direct and inverse kinematics problems of link velocity

In order to solve the direct problem to derive velocity of the active suspension system in the coordinate system associated with the frame it is required that the following vectors be determined: $\boldsymbol{\omega}_{p,r}=\boldsymbol{\omega}_{k,r}=\boldsymbol{\omega}_{f,r}$ - identical angular velocity of the platform p, of the cab k and the operator seat f, $\boldsymbol{v}_{O_k,r}$ - linear velocity of the cab's c.o.g Q_k, $\boldsymbol{v}_{O_f,r}$ - linear velocity of the c.o.g. in the seat-operator system O_f as functions of drives velocity.

The angular velocity vector of the platform, cab and the seat is linearly related to the velocity of actuators 1 and 4:

$$\boldsymbol{\omega}_{p,r}=\boldsymbol{\omega}_{k,r}=\boldsymbol{\omega}_{f,r}=\boldsymbol{J}_\omega\,\dot{\boldsymbol{s}}\,, \tag{7}$$

where: $\boldsymbol{J}_{\omega}=\begin{bmatrix}\boldsymbol{h}_1 & \boldsymbol{h}_4 & \boldsymbol{0}\end{bmatrix}$, $\dot{\boldsymbol{s}}=\begin{bmatrix}\dot{s}_1 & \dot{s}_4 & \dot{s}_5\end{bmatrix}^T$, $\boldsymbol{h}_1=\frac{\boldsymbol{y}_p^o}{r_{py_p1}}$, $\boldsymbol{h}_4=\frac{\boldsymbol{x}_p^o}{r_{px_p}}+\frac{\boldsymbol{y}_p^o}{r_{py_p4}}+\frac{\boldsymbol{z}_p^o}{r_{pz_p}}$, $r_{py_p1}=b_1(\boldsymbol{z}_p^o\cdot\boldsymbol{s}_1^o)$,

$$r_{px_p}=\frac{b_{23}}{(\boldsymbol{z}_p^o\times\boldsymbol{x}_r^o)\cdot\left(\frac{d_3}{r_3}\boldsymbol{d}_3^o-\frac{d_2}{r_2}\boldsymbol{d}_2^o\right)},\qquad r_{py_p4}=\frac{b_1\boldsymbol{z}_p^o\cdot\boldsymbol{s}_1^o}{\left[\frac{b_1}{r_{pz_p}}\boldsymbol{y}_p^o-\frac{d_2}{r_2}(\boldsymbol{x}_r^o\times\boldsymbol{d}_2^o)-\frac{b_2}{r_{px_p}}\boldsymbol{z}_p^o+\frac{b_2}{r_{pz_p}}\boldsymbol{x}_p^o\right]\cdot\boldsymbol{s}_1^o},$$

$$r_{pz_p}=\frac{b_{23}}{(\boldsymbol{x}_p^o\times\boldsymbol{x}_r^o)\cdot\left(\frac{d_2}{r_2}\boldsymbol{d}_2^o-\frac{d_3}{r_3}\boldsymbol{d}_3^o\right)},\; r_2=c_2[(\boldsymbol{x}_r^o\times\boldsymbol{c}_2^o)\cdot\boldsymbol{s}_4^o],\; r_3=\frac{c_2d_3}{d_2}\frac{[(\boldsymbol{x}_r^o\times\boldsymbol{d}_3^o)\cdot\boldsymbol{y}_p^o][(\boldsymbol{x}_r^o\times\boldsymbol{c}_2^o)\cdot\boldsymbol{s}_4^o]}{(\boldsymbol{x}_r^o\times\boldsymbol{d}_2^o)\cdot\boldsymbol{y}_p^o}.$$

The linear velocity vector of the point O_k:

$$\boldsymbol{v}_{O_k,r}=\boldsymbol{J}_{vk}\dot{\boldsymbol{s}},\tag{8}$$

where: $\boldsymbol{J}_{vk}=\left[\boldsymbol{h}_1\times(b_2\boldsymbol{y}_p^o+\boldsymbol{r}_{pk})\quad \frac{d_2}{r_2}(\boldsymbol{x}_r^o\times\boldsymbol{d}_2^o)+\boldsymbol{h}_4\times(b_2\boldsymbol{y}_p^o+\boldsymbol{r}_{pk})\quad \boldsymbol{0}\right]$.

The linear velocity vector of the point O_f:

$$\boldsymbol{v}_{O_f,r}=\boldsymbol{J}_{vf}\dot{\boldsymbol{s}},\tag{9}$$

where: $\boldsymbol{J}_{vf}=\left[\boldsymbol{h}_1\times(b_2\boldsymbol{y}_p^o+\boldsymbol{r}_{pf})\quad \frac{d_2}{r_2}(\boldsymbol{x}_r^o\times\boldsymbol{d}_2^o)+\boldsymbol{h}_4\times(\boldsymbol{y}_p^o b_2+\boldsymbol{r}_{pf})\quad \boldsymbol{z}_p^o\right]$.

To define the operating conditions of the drives in the active suspension mechanism, velocity vectors related to the road system are of key importance. The absolute angular velocity of the cab in the reference system $\{O_gx_gy_gz_g\}$ associated with the road becomes:

$$\boldsymbol{\omega}_{p,g}=\boldsymbol{\omega}_{k,g}=\boldsymbol{\omega}_{f,g}=\boldsymbol{\omega}_{p,r}+\boldsymbol{\omega}_{r,g}=\boldsymbol{J}_{\omega}\dot{\boldsymbol{s}}+\boldsymbol{\omega}_{r,g},\tag{10}$$

where: $\boldsymbol{\omega}_{r,g}$ - angular velocity of the frame with respect to road, based on measurement data.

Absolute linear velocities of points O_k and O_f expressed in the reference system associated with the frame are:

$$\boldsymbol{v}_{O_k,g}=\boldsymbol{v}_{O_m,g}+\boldsymbol{\omega}_{r,g}\times(\boldsymbol{r}_{mr}+\boldsymbol{r}_{rk})+\boldsymbol{v}_{O_k,r},\tag{11}$$

$$\boldsymbol{v}_{O_f,g}=\boldsymbol{v}_{O_m,g}+\boldsymbol{\omega}_{r,g}\times(\boldsymbol{r}_{mr}+\boldsymbol{r}_{rf})+\boldsymbol{v}_{O_f,r},\tag{12}$$

where: $\boldsymbol{v}_{O_m,g}$ - measured linear velocity of the control point O_m associated with the frame with respect to the road; $\boldsymbol{r}_{mr}=[r_{(mr)x},r_{(mr)y},r_{(mr)z}]^T$ - vector between the points O_m and O_r expressed in the reference system associated with the frame.

The inverse problem involves finding the drive velocities for the predetermined cab velocity with respect to the road. As the active suspension mechanism displays three degrees of freedom (DOFs), three constraints can be imposed upon the cab and seat velocities. The function of the active suspension system is to stabilise the cab in the vertical direction, hence the condition is adopted prohibiting the absolute rotating motion of the platform around its two axes $\boldsymbol{x}_{po}^o$ and $\boldsymbol{y}_{po}^o$. The third condition implicates that the absolute value of linear velocity of the point O_f in the direction of the gravity force should be zero:

$$\left.\begin{aligned}\boldsymbol{\omega}_{(p,g)o}\cdot\boldsymbol{x}_{po}^o&=0\\ \boldsymbol{\omega}_{(p,g)o}\cdot\boldsymbol{y}_{po}^o&=0\\ \boldsymbol{v}_{(O_f,g)o}\cdot\boldsymbol{z}_g^o&=0\end{aligned}\right\}.\tag{13}$$

Recalling Eq (7), (10), (9), (12) and conditions (13), we get the formulas inclusive the expected velocities of actuators 1, 4 and 5:

$$\left.\begin{array}{r}\dot{s}_{1o}\boldsymbol{h}_{1o}\cdot\boldsymbol{x}^{o}_{po}+\dot{s}_{4o}\boldsymbol{h}_{4o}\cdot\boldsymbol{x}^{o}_{po}+\boldsymbol{\omega}_{r,g}\cdot\boldsymbol{x}^{o}_{po}=0\\ \dot{s}_{1o}\boldsymbol{h}_{1o}\cdot\boldsymbol{y}^{o}_{po}+\dot{s}_{4o}\boldsymbol{h}_{4o}\cdot\boldsymbol{y}^{o}_{po}+\boldsymbol{\omega}_{r,g}\cdot\boldsymbol{y}^{o}_{po}=0\\ \dot{s}_{1o}(\boldsymbol{k}_{1O_f}\cdot\boldsymbol{z}^{o}_{g})+\dot{s}_{4o}(\boldsymbol{k}_{4O_f}\cdot\boldsymbol{z}^{o}_{g})+\dot{s}_{5o}(\boldsymbol{k}_{5O_f}\cdot\boldsymbol{z}^{o}_{g})+\boldsymbol{v}_{O_m,g}\cdot\boldsymbol{z}^{o}_{g}+[\boldsymbol{\omega}_{r,g}\times(\boldsymbol{r}_{mr}+\boldsymbol{r}_{(rf)o})]\cdot\boldsymbol{z}^{o}_{g}=0\end{array}\right\}. \qquad (14)$$

2.3. Restriction of the drive seat movement

The machine, when in service or during the ride, may change its position in the vertical direction such that in order to stabilise the seat position in this direction the operating range of the actuator 5 should be exceeded. To solve this problem, it is suggested that a penalty function (Fig. 3) should be introduced, its argument being the instantaneous length of the actuator s_5:

$$\dot{s}_{5k}=K_5\dot{s}_{5k\max}\left[\frac{0.5(s_{5\max}+s_{5\min})-s_5}{0.5(s_{5\max}-s_{5\min})}\right]^{2n-1}, \qquad (15)$$

where: K_5 - amplification factor penalty function $\dot{s}_{5k\max}$ - maximal velocity of the actuator, $n\in N$.

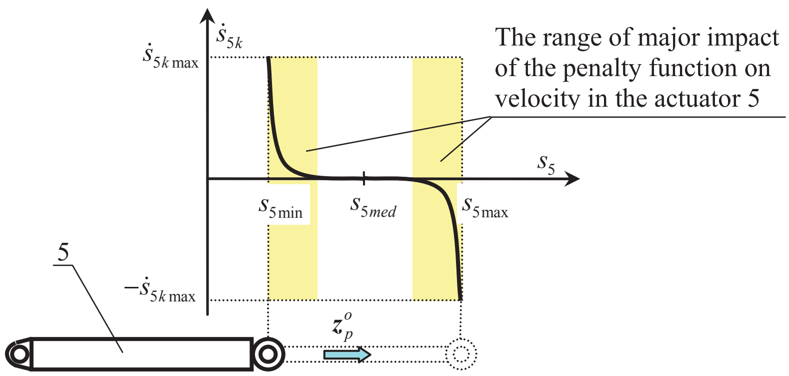

Fig. 3. Penalty function

The final expected velocity of the actuator 5 should involve a term responsible for the seat "drifting" towards the middle position:

$$\dot{s}^{*}_{5o}=\dot{s}_{5o}+\dot{s}_{5k}, \qquad (16)$$

where: $\dot{s}_{5o}$ - expected velocity in the actuator 5, derived from formula (14).

The second term in (16) represents the seat movement towards the middle position being superimposed on its relative movement. The assumed penalty function (15) guarantees that the relative velocity during the seat's return movement to the middle position should be significant at extreme points of the actuator's displacement range.

2.4. Cab and seat acceleration

In order to solve the direct problem involving acceleration of the active suspension mechanism in the reference system associated with the frame, it is required that certain quantities should be determined in the function of length, velocity and acceleration of actuators 1, 4, 5. These include: $\boldsymbol{\varepsilon}_{p,r}$ - angular acceleration of the platform, cab and seat, $\boldsymbol{a}_{O_k,r}$ - linear acceleration of the cab's c.o.g, $\boldsymbol{a}_{O_f,r}$- linear acceleration of the c.o.g of the seat-operator system. Differentiating Eq.

(7), (8), (9) with respect to time yields the angular acceleration of the platform, cab and seat and linear acceleration at points O_k and O_f:

$$\boldsymbol{\varepsilon}_{p,r} = \dot{\boldsymbol{J}}_{\omega}\,\dot{\boldsymbol{s}} + \boldsymbol{J}_{\omega}\,\ddot{\boldsymbol{s}}\,, \tag{17}$$

$$\boldsymbol{a}_{O_k,r} = \dot{\boldsymbol{J}}_{vk}\,\dot{\boldsymbol{s}} + \boldsymbol{J}_{vk}\ddot{\boldsymbol{s}}\,, \tag{18}$$

$$\boldsymbol{a}_{O_f,r} = \dot{\boldsymbol{J}}_{vf}\,\dot{\boldsymbol{s}} + \boldsymbol{J}_{vf}\ddot{\boldsymbol{s}}\,, \tag{19}$$

where: $\ddot{\boldsymbol{s}} = \begin{bmatrix}\ddot{s}_1 & \ddot{s}_4 & \ddot{s}_5\end{bmatrix}^T$.

The inertia loads are determined basing on absolute acceleration values related to the inertial reference system $\{O_g x_g y_g z_g\}$. In accordance with Eq (10), the absolute angular accelerations of the platform, cab and seat expressed in the mobile reference system associated with the frame become:

$$\boldsymbol{\varepsilon}_{p,g} = \boldsymbol{\varepsilon}_{k,g} = \boldsymbol{\varepsilon}_{f,g} = \boldsymbol{\varepsilon}_{r,g} + \boldsymbol{\varepsilon}_{p,r} + \boldsymbol{\omega}_{r,g} \times \boldsymbol{\omega}_{p,r}\,, \tag{20}$$

where: $\boldsymbol{\varepsilon}_{r,g}$ - measured angular acceleration of the machine frame with respect to the inertial reference system.

Recalling Eq (11), (12), the absolute linear acceleration of the points O_r, O_k and O_f in the mobile reference system associated with the frame become:

$$\boldsymbol{a}_{O_r,g} = \boldsymbol{a}_{O_m,g} + \boldsymbol{\varepsilon}_{r,g} \times \boldsymbol{r}_{mr} + \boldsymbol{\omega}_{r,g} \times (\boldsymbol{\omega}_{r,g} \times \boldsymbol{r}_{mr})\,, \tag{21}$$

$$\boldsymbol{a}_{O_k,g} = \boldsymbol{a}_{O_r,g} + \boldsymbol{\varepsilon}_{r,g} \times \boldsymbol{r}_{rk} + 2\boldsymbol{\omega}_{r,g} \times \boldsymbol{v}_{O_k,r} + \boldsymbol{\omega}_{r,g} \times (\boldsymbol{\omega}_{r,g} \times \boldsymbol{r}_{rk}) + \boldsymbol{a}_{O_k,r}\,, \tag{22}$$

$$\boldsymbol{a}_{O_f,g} = \boldsymbol{a}_{O_k,g} + \boldsymbol{\varepsilon}_{k,g} \times \boldsymbol{r}_{kf} + 2\boldsymbol{\omega}_{k,g} \times \boldsymbol{v}_{O_f,k} + \boldsymbol{\omega}_{k,g} \times (\boldsymbol{\omega}_{k,g} \times \boldsymbol{r}_{kf}) + \boldsymbol{a}_{O_f,k}\,, \tag{23}$$

where: $\boldsymbol{a}_{O_m,g}$ - measured linear acceleration of a point O_m on the frame with respect to the inertial reference system, $\boldsymbol{r}_{kf} = \overrightarrow{O_k O_f}$, $\boldsymbol{r}_{rk} = \overrightarrow{O_r O_k}$, $\boldsymbol{r}_{mr} = \overrightarrow{O_m O_r}$.

3. Inverse problem of dynamics

The external loads acting on the active suspension mechanism involve the gravity forces, inertia forces and moments of force of the platform together with the cab, the seat and the operator. These are governed by the Newton-Euler equations, referenced in [2]:

$$\boldsymbol{P}_{bk} = -m_k \boldsymbol{a}_{O_k,g}\,, \tag{24}$$

$$\boldsymbol{M}_{bk} = -\boldsymbol{\varepsilon}_{k,g}\boldsymbol{I}_k - \tilde{\boldsymbol{\omega}}_{k,g}\boldsymbol{I}_k\boldsymbol{\omega}_{k,g}\,, \tag{25}$$

$$\boldsymbol{P}_{bf} = -m_f \boldsymbol{a}_{O_f,g}\,, \tag{26}$$

$$\boldsymbol{M}_{bf} = -\boldsymbol{\varepsilon}_{f,g}\boldsymbol{I}_f - \tilde{\boldsymbol{\omega}}_{f,g}\boldsymbol{I}_f\boldsymbol{\omega}_{f,g}\,, \tag{27}$$

where: $\tilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$, $\boldsymbol{I}_k = {}^r_p\boldsymbol{R}\,{}^k\boldsymbol{I}_k\,{}^r_p\boldsymbol{R}^T$, $\boldsymbol{I}_f = {}^r_p\boldsymbol{R}\,{}^f\boldsymbol{I}_f\,{}^r_p\boldsymbol{R}^T$, ${}^k\boldsymbol{I}_k$, ${}^f\boldsymbol{I}_f$ - mass moments of inertia of the cab and the seat with operator in their own reference systems.

The sum total of instantaneous power applied by the active suspension mechanism and power of the gravity and inertia forces are brought down to zero:

$$\dot{\boldsymbol{s}}^T\boldsymbol{F} + m_k\boldsymbol{v}_{O_k,r}^T(\boldsymbol{g} - \boldsymbol{a}_{O_k,g}) + \boldsymbol{\omega}_{k,r}^T(-\boldsymbol{I}_k\boldsymbol{\varepsilon}_{k,g} - \tilde{\boldsymbol{\omega}}_{k,g}\boldsymbol{I}_k\boldsymbol{\omega}_{k,g}) +$$

$$+ m_f\boldsymbol{v}_{O_f,r}^T(\boldsymbol{g} - \boldsymbol{a}_{O_f,g}) + \boldsymbol{\omega}_{f,r}^T(-\boldsymbol{I}_f\boldsymbol{\varepsilon}_{f,g} - \tilde{\boldsymbol{\omega}}_{f,g}\boldsymbol{I}_f\boldsymbol{\omega}_{f,g}) = 0\,, \tag{28}$$

where: $\boldsymbol{F} = [F_1, F_4, F_5]^T$ - forces developed by the drives.

Recalling Jacobean matrices (7), (8), (9), Eq (28) can be rewritten as:

$$\dot{\boldsymbol{s}}^T[\boldsymbol{F} + m_k\boldsymbol{J}_{vk}^T(\boldsymbol{g} - \boldsymbol{a}_{O_k,g}) + \boldsymbol{J}_{\omega}^T(-\boldsymbol{I}_k\boldsymbol{\varepsilon}_{k,g} - \tilde{\boldsymbol{\omega}}_{k,g}\boldsymbol{I}_k\boldsymbol{\omega}_{k,g}) +$$

$$+ m_f\boldsymbol{J}_{vf}^T(\boldsymbol{g} - \boldsymbol{a}_{O_f,g}) + \boldsymbol{J}_{\omega}^T(-\boldsymbol{I}_f\boldsymbol{\varepsilon}_{f,g} - \tilde{\boldsymbol{\omega}}_{f,g}\boldsymbol{I}_f\boldsymbol{\omega}_{f,g})] = 0\,. \tag{29}$$

Knowing the loads due to gravity and inertia, Eq (29) yields the forces acting in the drives:

$$\boldsymbol{F} = m_k\boldsymbol{J}_{vk}^T(\boldsymbol{a}_{O_k,g} - \boldsymbol{g}) + m_f\boldsymbol{J}_{vf}^T(\boldsymbol{a}_{O_f,g} - \boldsymbol{g}) + \boldsymbol{J}_{\omega}^T\{(\boldsymbol{I}_k + \boldsymbol{I}_f)\boldsymbol{\varepsilon}_{k,g} + [\tilde{\boldsymbol{\omega}}_{k,g}(\boldsymbol{I}_f + \boldsymbol{I}_k)\boldsymbol{\omega}_{k,g}]\}\,. \tag{30}$$

4. Simulation of the active suspension system

The operation of the active suspension system is investigated using two mutually supportive programmes. MSC visualNastran 4D is used to develop the model of the input inducing the machine motion, of the machine suspension, the active suspension mechanism for the cab and the seat. All these modelled elements are simplified (Fig. 4).

The programme enables the measurements of the actuator length, the angle of frame rolling α_x and pitching α_y, velocity and acceleration O_m and of velocity and acceleration of the cab's angular motion. These are shown in the block diagram "Measurements of the machine frame movements". During the simulation procedure, these quantities are sent to be further handled by Matlab/Simulink (Fig. 4).

The proposed control strategy to be applied to the active suspension of the cab uses the feedback control system with compensation for the measured disturbances in the form of the machine frame movements. The expected states of the cab motion, determined in the block "Preset cab motion" involve the requirement whereby the cab is to be stabilised in the vertical direction and the seat must not be displaced along the cab's vertical axis, at the same time the operating range of the actuator 5 should be duly taken into account. Once frame movements are known from measurements and assumptions as to the anticipated cab movements being taken into account, an unambiguous procedure is applied to compute drive movements in the active suspension mechanism. On the output from the block "Inverse problem of kinematics of the active suspension mechanism" we get the expected velocities and accelerations of three drives, represented by vectors $\boldsymbol{s}_o = [s_{1o} \quad s_{4o} \quad s_{5o}]^T$, $\dot{\boldsymbol{s}}_o = [\dot{s}_{1o} \quad \dot{s}_{4o} \quad \dot{s}_{5o}]^T$. Actuators should be equipped with sensors for measuring their length $\boldsymbol{s} = [s_1 \quad s_4 \quad s_5]^T$ in order to determine the control error $\boldsymbol{e} = \left[s_{1o} - s_1 \quad s_{4o} - s_4 \quad s_{5o} - s_5\right]^T$. The control error should tend to zero if the velocities implemented in actuators are in accordance with the formula:

$$\dot{\boldsymbol{s}}_w = \dot{\boldsymbol{s}}_o + \boldsymbol{K}_P\boldsymbol{e}\,, \tag{31}$$

where: $\boldsymbol{K}_P = \begin{bmatrix} k_{p1} & 0 & 0 \\ 0 & k_{p4} & 0 \\ 0 & 0 & k_{p5} \end{bmatrix}$ - gain matrix in the position path.

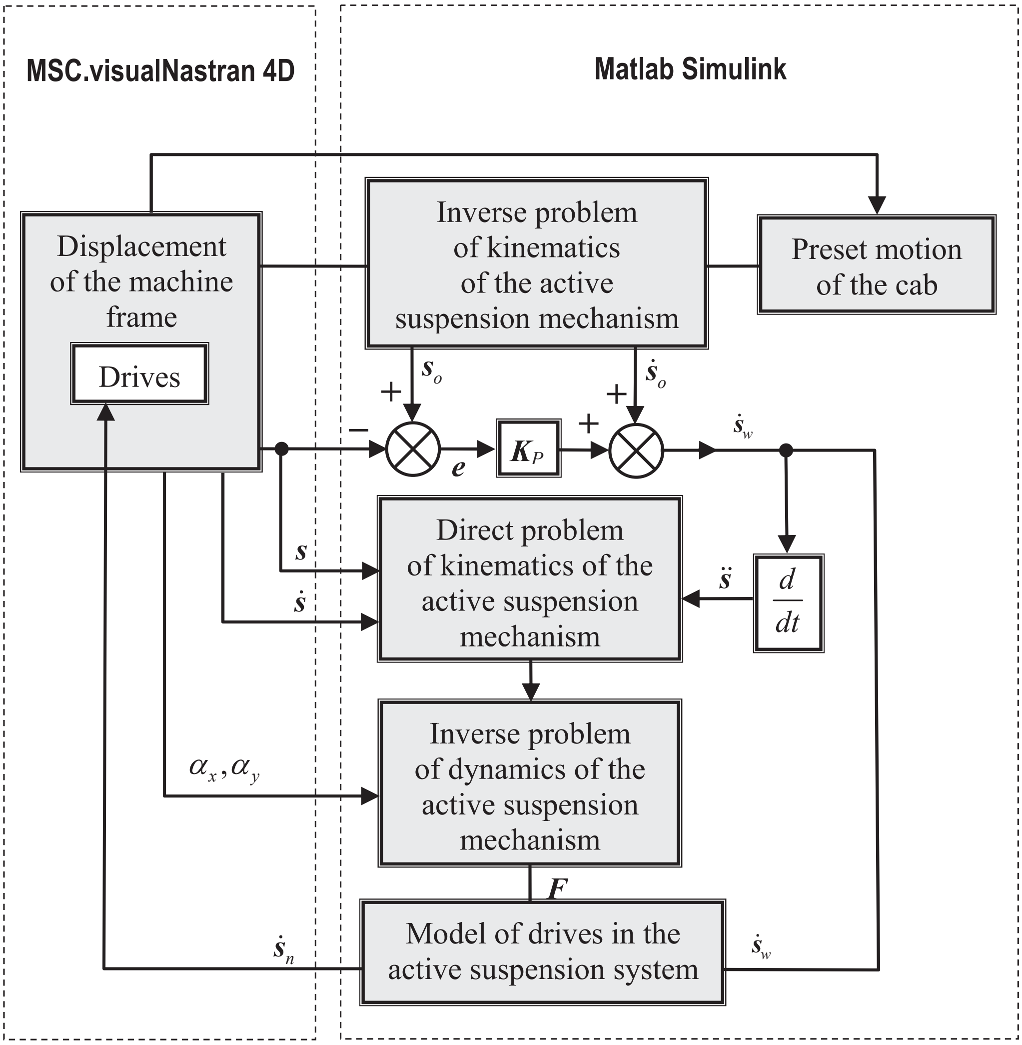

Fig. 4. Model of drives control in the active suspension system - schematic diagram

Computed accelerations $\ddot{s}$ and measured velocities $\dot{s}$ and displacements s of the drives become the inputs to the block "Straight problem of kinematics of the active suspension mechanism problem of kinematics of the active suspension", which calculates the anticipated movement of the active suspension mechanism of the cab and of the cab itself. Basing on anticipated cab movements, inertia interactions are found which, alongside the gravity forces, become the major loads applied to the cab. The inverse problem of the active suspension dynamics involves the calculation of the driving forces in the form of a vector $\boldsymbol{F}$, counterbalancing the external loads. The contribution of gravity forces to the load of particular drives depends on the frame tilt angles: α_x, α_y. Basing on computed loads $\boldsymbol{F}$ and the required instantaneous velocities $\dot{\boldsymbol{s}}_w$, the block "Model of active suspension drives" generates the realisable instantaneous velocities of actuators $\dot{\boldsymbol{s}}_n = \left[\dot{s}_{n1} \quad \dot{s}_{n4} \quad \dot{s}_{n5}\right]^T$. Velocity values $\dot{\boldsymbol{s}}_n$ are sent to be further processed by MSCvisualNASTRAN 4D. This work does not include the analysis of the drive model. It is assumed in simulations $\dot{\boldsymbol{s}}_n = \dot{\boldsymbol{s}}_w$.

The control of the active suspension system gives rise to certain errors ***e***, and in consequence the constraints imposed on the angular velocity of the cab and linear velocity of the cab and the seat cannot be accurately reproduced. These errors are attributable to inaccurate measurements of the frame movements, the time delay involved in implementation of the drive velocity or the drives' failure to implement the required velocity (moving beyond the limits of their typical operating range).

5. Simulation data of the active suspension system

Machine specification data used in simulations: $l_m = 2.810\,[m]$- distance between the front and rear axle of the machine frame, $w_m = 1.980\,[m]$- wheel spacing, w_r =1.4 [m] - frame width, $\boldsymbol{r}_{PO_r} = [-2.1, -0.818, 1.6]\,[m]$- position vector of the point O_r, $w_k = 1.200\,[m]$- cab width, $x_{sck} = 0.800\,[m]$, $y_{sck} = 0.600\,[m]$, $z_{sck} = 0.700\,[m]$- coordinates of the cab's c.o.g, m_k=480 [kg] - cab mass, m_f=160 [kg] - mass of the seat with an operator,

$${}^{k}J_k = \begin{bmatrix} 180 & 0 & 0 \\ 0 & 208 & 0 \\ 0 & 0 & 133 \end{bmatrix} [kg\,m^2]$$

- inertia matrix of the cab in the reference system associated with the cab,

$${}^{f}J_f = \begin{bmatrix} 23.8 & 0 & 0 \\ 0 & 24.7 & 0 \\ 0 & 0 & 13.2 \end{bmatrix} [kg\,m^2]$$

- inertia matrix of the seat and operator in the system associated with the seat.

Road profile:

$$z_{1,2,3,4} = h_g \left[1 - \cos\left(2\pi \frac{v_{Px}\left(t - \frac{l_m}{v_{Px}} - \frac{\varphi L_g}{2\pi v_{Px}}\right)}{L_g} \right) \right]$$

- the road profile function for the four wheels of the machine,

$2h_g$= 0.250 [m] - height of the unevenness range,

L_g= 2[m] - wave length of the road unevenness,

$\varphi = \pi/2\,[rad]$ - the phase shift angle between the left-and right-hand side of the machine, $v_{Px,\max} = \dfrac{L_g}{2\pi}\sqrt{\dfrac{g}{h_g}} = 2.82\,[m/s]$ - maximal speed of the machine ride computed for the free wheel in contact with the road surface.

Active suspension mechanism for the cab:

$h_m = 2.420\,[m]$ - distance of joints in the rocker arm connections A_2 and A_3 from the ground, $\delta_w = 0.05\,[m]$ - admissible distance between the cab's side wall from the joint axis A_2 or A_3, $a_3 = 1.636\,[m]$, $d = d_3 = d = 0.227\,[m]$, $b_{23} = 1.490\,[m]$, $c_2 = 0.099\,[m]$, $\square(\boldsymbol{c}_2^o, \boldsymbol{d}_2^o) = 4.561\,[rad]$, $a_4 = 0.541\,[m]$, $\boldsymbol{a}_4^o = [0.000, 0.218, -0.976]\,[m]$, $\boldsymbol{a}_1^o = [-0.722, 0.692, 0.000]\,[m]$, $a_1 = 1.182\,[m]$, $b_2 = b_3 = 0.5\,b_{23}$, $b_1 = 0.850\,[m]$, $s_{5\min} = 0.415\,[m]$, $s_{5\max} = 0.715\,[m]$, $\dot{s}_{5k\max} = 1\,[m/s]$, $K_5 = 50\,[-]$.

$$\boldsymbol{K}_P = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix} [1/s]$$

- matrix of gain in the position path.

The distance covered during the simulation - 10[m].

Plots in Fig. 6-10 show the relevant parameters in the function of the coefficient η linearly related to the machine ride velocity, whilst for $v_{Px} = v_{Px,\max}$ the value of η becomes 1.

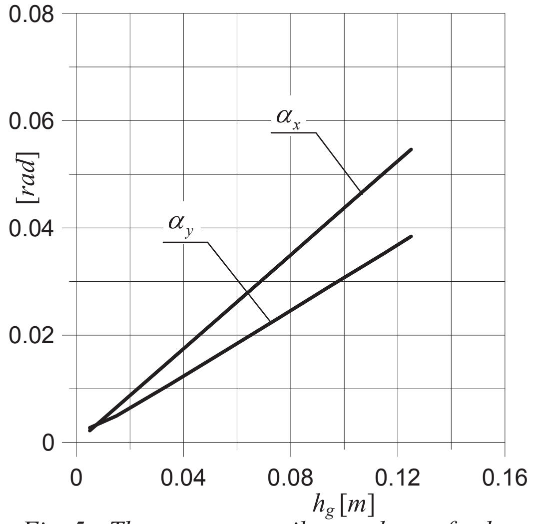

Fig. 5. The average tilt angles of the machine frame with respect to its longitudinal and lateral axes

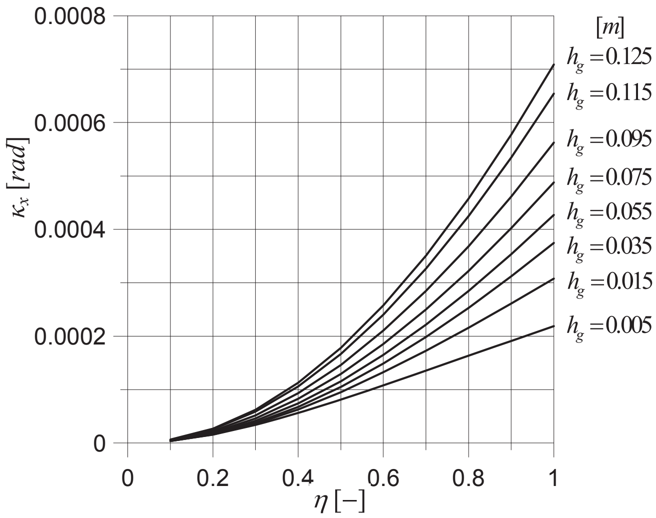

Fig. 6. The average tilt angle of the cab with respect to its longitudinal axes

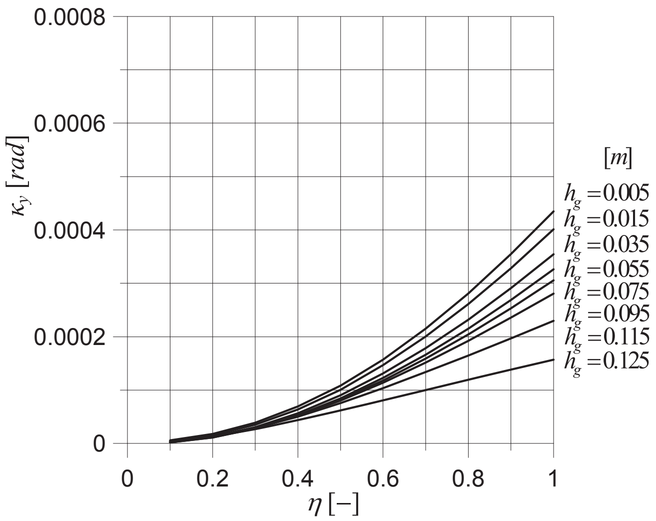

Fig. 7. The average tilt angle of the cab with respect to its lateral axes

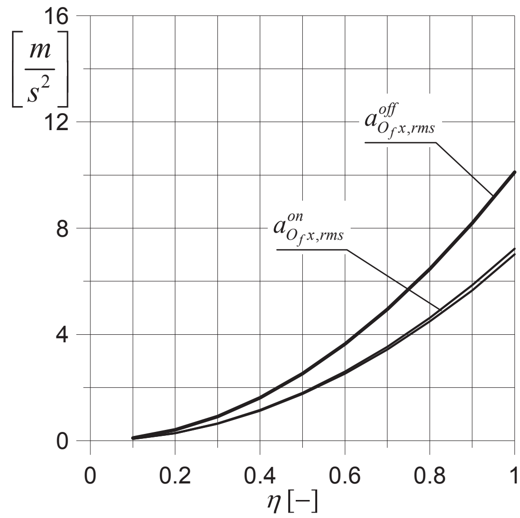

Fig. 8. The rms acceleration O_f in the direction of the machine ride (h_g=0.005-0.125 [m])

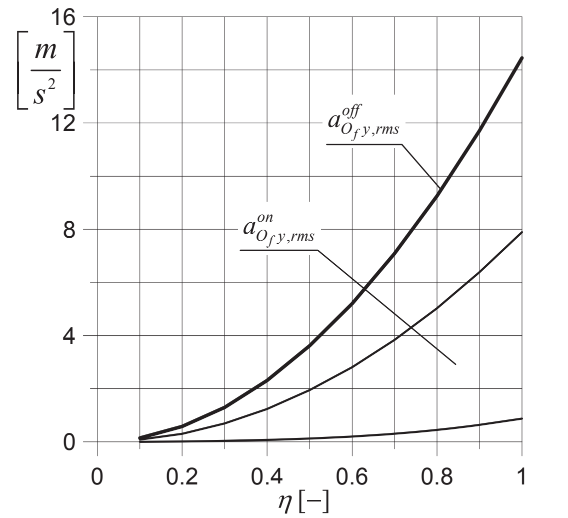

Fig. 9. The rms acceleration O_f in the lateral direction (h_g=0.005-0.125 [m])

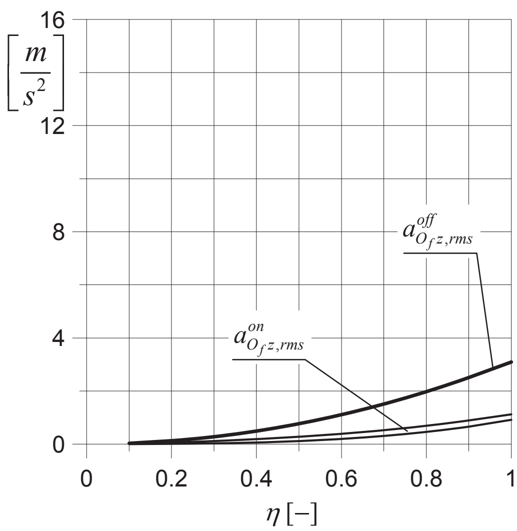

Fig. 10. The rms acceleration O_f in the vertical direction (for h_g=0.005-0.125 [m])

5. Conclusions

Simulations of the active suspension system performance have proved its adequacy in vibration reduction of angular vibrations of the cab around the longitudinal axis of the machine x_r and around the transverse axis y_r. Seat vibrations along the vertical axis z_r are successfully controlled, too. Besides, the cab and seat vibrations in the direction y_r are significantly reduced, and reduction of angular cab vibration around the axis y_r leads to reduction of linear seat vibration in the direction x_r.

The operation of the active suspension system involves the real-time measurements of mechanical quantities which can be accurately measured with state-of-the-art sensors: angular velocity and acceleration of the frame, linear acceleration of a selected point O_m on the machine frame, two angles of the frame tilting from the direction of the gravity forces and the length and velocities implemented by actuators.

Underlying the simulation procedure is that assumption that each computed drive velocity will be implemented without any time delay (provided that is allowed by collaborating programmes). Results therefore can be utilised when selecting drives which, when in extreme conditions, may not be able to perform the required movements. Besides, the overall time constant, taking into account the response time of the measurement system, the controls and drives becomes another limiting factor, particularly at higher frequencies of road input.

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