Implementation of The Chaotic Mobile Robot for The Complex Missions

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Abstract:
Mobile robotics, after decades of continuous development, keeps up as an intensive research issue because of its ever-increasing application to different domains and its economical and technological relevance. Interesting applications can be seen in robots performing floor-cleaning tasks, executing industrial transportation, exploring volcanoes, scanning areas to find explosive devices, and so on. a chaotic signal for an autonomous mobile robot is to increase and to take advantage of coverage areas resulting from its travelling paths. The chaotic behavior of the mobile robot is achieved by adding nonlinear equations into the robot kinematic equations, like Arnold, Lorenz, and Chua's equations, that are well known equations for had a chaotic behavior. The performance of the three guiding signals for robotics system is evaluated in the sense of the wide area coverage, the evenness index, and the total trajectory distance.

Keywords: *chaos, chaotic motion, chaotic mobile robot, Chua's circuit, Arnold and Lorenz equations*

1. Introduction

The chaos characterizes one of mysterious rich behaviors of nonlinear dynamical systems. Many research efforts have been paid to establish the mathematical theory behind chaos. Applications of chaos are also being studied and include, for example, controlling chaos and chaotic neural networks. This paper follows a method to impart chaotic behavior to a mobile robot. This is achieved by designing a controller which ensures chaotic motion [1, 2].

Further investigations on chaotic trajectories of the same type of the robot using other equations were carried out in [3–11]. The main objective in exploiting chaotic signals for an autonomous mobile robot is to increase and to take advantage of coverage areas resulting from its travelling paths. Large coverage areas are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purposes. The aim of this paper is to implement the chaotic behavior of the mobile robot and to evaluate the performance of the Arnold, Lorenz, and the Chua's circuit equations, as a controller of the mobile robot, from point of view of performance index **k**, which reflects how high the coverage area, and the evenness index **E**, which reflects the degree of variation in covering the areas between species. The paper structure is as follows: The next section presents the mobile robot model. The chaotic mobile controllers are illustrated in section 3. Section 4 is dedicated to the evaluation criteria to be applied. Section 5 is reserved to the simulation results. Finally section 6 concludes this paper.

2. The Mobile Robot Model

The mobile robot used is shown in Fig. 1. Let the linear velocity of the robot **v** [m/s], and the the angular velocity **w** [rad/s], be the inputs to the system, and the state equation of the mobile robot is written as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} \tag{1}$$

Where **x** [m], and ***y*** [m] is the position of the mobile robot, is the angle of the robot.

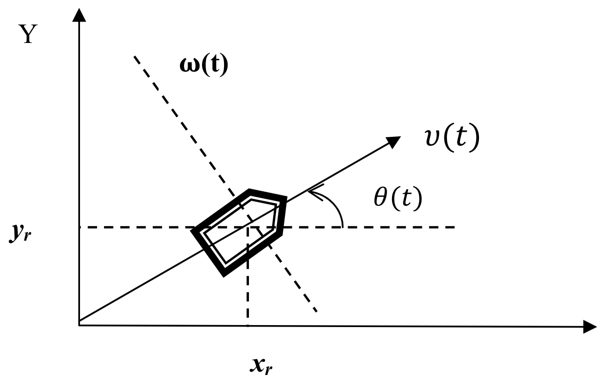

Fig. 1. Geometry of the robot motion on the Cartesian plane

3. The Chaotic Mobile Robot Controller

In order to generate chaotic motions of the mobile robot, this is achieved by designing a controller which ensures chaotic motion. The type of chaotic patterns employed to generate the robot trajectory are the Arnold, the Lorenz, and the Chua's circuit equations.

3.1. The Arnold equation

The equation of the Arnold is written as follows:

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \end{bmatrix} \begin{bmatrix} \mathbf{A}\sin\mathbf{x}_3 + \mathbf{C}\cos\mathbf{x}_2 \\ \mathbf{B}\sin\mathbf{x}_1 + \mathbf{A}\cos\mathbf{x}_3 \\ \mathbf{C}\sin\mathbf{x}_2 + \mathbf{B}\cos\mathbf{x}_1 \end{bmatrix} \tag{2}$$

Where A, B, and C are constants. It is known that the Arnold equation shows periodic motion when one of the constants, for example C, is 0 or small and shows chaotic motion when C is large [10]. The chaotic pattern of the

Arnold equation, for the following parameters: A = 0.27, B = 0.135, C = 0.135 and initial conditions: $x_{10} = 4$, $x_{20} = 3.5$, $x_{30} = 0$, is shown in Fig. 2

After integration the Arnold equation (2) into the controller of the mobile robot equation (1), the state equation of the mobile robot becomes:

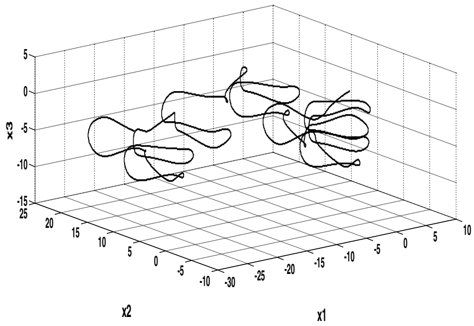

Fig. 2. Arnold chaotic pattern in 3-D space

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \\ \dot{x} \\ \dot{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} A\,sin\mathbf{x}_3 + C\,cos\mathbf{x}_2 \\ B\sin\mathbf{x}_1 + \mathbf{A}\,\mathbf{cos}\mathbf{x}_3 \\ C\,sin\mathbf{x}_2 + B\,cos\mathbf{x}_1 \\ v\,cos\,\mathbf{x}_3 \\ v\sin\mathbf{x}_3 \end{bmatrix} \quad (3)$$

The integrated system of the Arnold equation with the mobile robot equation with appropriate adjusting parameters and initial conditions guaranteed that a chaotic orbit of the Arnold equation behaves chaotically. The resultant workspace coverage trajectory of the mobile robot at iteration n = 10000 is shown in Fig. 3

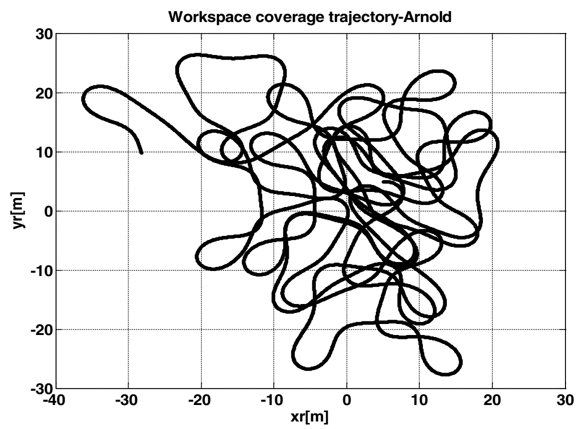

Fig. 3. Workspace coverage trajectory of the chaotic mobile robot, Arnold

3.2. The Lorenz equation

The Lorenz attractor is generated by the differential equation given by:

$$\dot{\mathbf{X}} = -10\,\mathbf{X} + 10\mathbf{Y}$$

$$\dot{\mathbf{Y}} = 28\,\mathbf{X} - \mathbf{Y} - \mathbf{X}\mathbf{Z} \quad (4)$$

$$\dot{\mathbf{Z}} = -\frac{8}{3}\mathbf{Z} + \mathbf{X}\mathbf{Y}$$

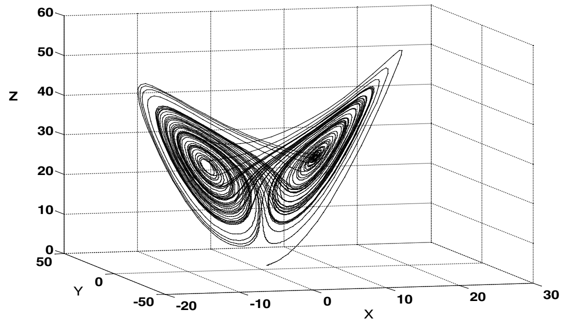

Fig. 4. The Lorenz attractor in 3D space

The parametric values in the differential equation (4) are needed in order to generate a chaotic behavior. The Lorenz attractor is shown in Fig. 4

In the same way that was made with the Arnold equation, we coupled the Lorenz equation with the mobile robot equation, and the integrated system will be:

$$\dot{X} = -10\,X + 10Y$$

$$\dot{\mathbf{Y}} = 28\,\mathbf{X} - \mathbf{Y} - \mathbf{X}\mathbf{Z}$$

$$\dot{\mathbf{Z}} = -\frac{8}{3}\mathbf{Z} + \mathbf{X}\mathbf{Y}$$

$$\dot{\mathbf{x}} = \mathbf{v}\cos\mathbf{Z}$$

$$\dot{\mathbf{y}} = \mathbf{v}\sin\mathbf{Z}$$

The resultant workspace coverage trajectory of the mobile robot at initial conditions: $X_0 = 1$, $Y_0 = 0$, $Z_0 = 1$, $x_0 = 1$, $y_0 = 0$ at iteration n = 10000 is shown in Fig. 5

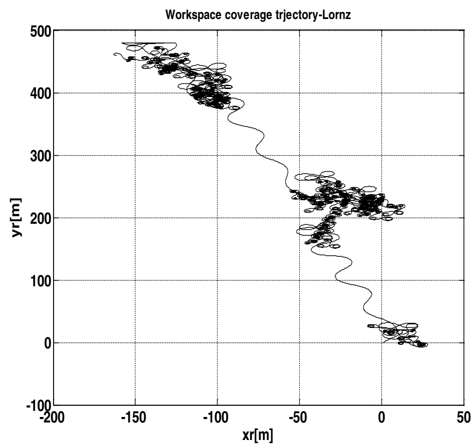

Fig. 5. Workspace coverage trajectory of the chaotic mobile robot, Lorenz

3.3. The Chua's circuit

The chaotic controller used herein as a trajectory generator is Chua's circuit which is low cost and easy to construct for trajectory generators. The general equations of Chua's circuit are:

$$\dot{X}_1 = -\left(X_2 - X_1 - f\left(X_1\right)\right)$$

$$\dot{X}_2 = X_1 - X_2 - X_3$$

$$\dot{X}_3 = -\beta\,X_2$$

Where: $F(X_1) = bX_1 + \frac{1}{2}(a-b)[|X_1+1|-|X_1-1|]$

$\alpha = 9, \beta = 100/7$, a = -5/7, b = -8/7.

These parameters generate double scroll attractor shown in Fig. 6.

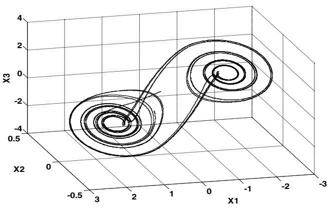

Fig. 6. Chua's attractor in the 3D space

The integrated system of the Chua's circuit equation as a controller of the mobile robot will be as follows:

$$\dot{X}_1 = -\left(X_2 - X_1 - f\left(X_1\right)\right)$$

$$\dot{X}_2 = X_1 - X_2 - X_3$$

$$\dot{X}_3 = -\beta\, X_2$$

$$\dot{x} = v\cos X_2$$

$$\dot{y} = v\sin X_2$$

The resultant workspace coverage trajectory of the mobile robot at iteration n = 10000 is shown in Fig. 7.

3.4. Workspace coverage constrain

The workspace coverage trajectory of the three mentioned controllers, Arnold, Lorenz, and Chua's equations are studied and analyzed. We can deduce from Fig. 5, Fig. 6, and Fig. 7 that each controller has a specific workspace coverage trajectory where the chaotic mobile robot moves in. We candidate three areas: (20 x 20, 40 x 40, and 60 x 60) for each controller as follows in Table 1.

4. Evaluation Criteria

The evaluation criteria are set according to the application purpose. Since we would like to use the robot in wandering around area in the area of no maps, the chaotic trajectory should cover the entire areas of patrolling as much as possible. The following three performances criteria are to be considered to evaluate the coverage rate of the chaotic mobile robot, namely the performance index k, the evenness index E, and the distance of the trajectory D.

a) A performance index K representing a ratio of areas that the trajectory passes through or used space (Au), over the total working area (At)

$$\mathbf{K} = \frac{\mathbf{A_u}}{\mathbf{A_t}} \tag{8}$$

The used area and the total area can be calculated by the following algorithm:

- We divide the specified area into (NxN) pixels.
- Initially, assign the value 0 for all pixels.
- We get the x-coordinate and y-coordinate of the pixels which passes through the trajectory of the robot.
- We assign the value 1 for each pixel passes through the trajectory.
- We count the number of ones (pixels which passes through the trajectory) which is.
- We count the number of zeroes Z (pixels which don't passes through the trajectory).
- The total area is the sum of and Z.

Similarly, let us consider a rectangular shape area, Fig. 8. The total area can be partitioned into four quarter, denoted Q = 1, 2, 3, 4. The quantitative measurement of the trajectory can be evaluated by using the following equation:

$$\mathbf{K_Q} = \frac{\mathbf{A_{uQ}}}{\mathbf{A_{tQ}}} \tag{9}$$

Where is the performance index of the Qth quadrant, is the area used by the trajectory in the Qth quadrant. In our case, we have

$$\mathbf{A_{tQ}} = \frac{\mathbf{A_t}}{4} \tag{10}$$

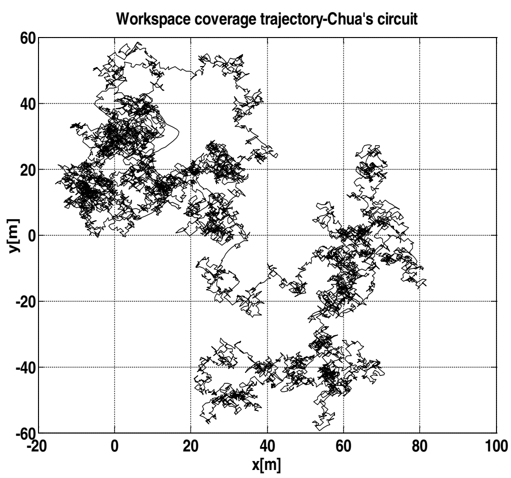

Fig. 7. Workspace coverage trajectory of the chaotic mobile robot, Chua's circuit

Table1. Workspace coverage constrain for the controllers

Controller Area	Chua's workspace constrain	Lorenz's workspace constrain	Arnold's workspace constrain
20x20	X: 0 20 Y: 0 20	X: 0 20 Y: 0 20	X: 0 20 Y: 0 20
40x40	X: 0 40 Y: 0 40	X: - 40 0 Y: 200 240	X: 0 40 Y: 0 40
60x60	X: 20 80 Y: -60 0	X: -50 10 Y: 200 260	X: -30 30 Y: -30 30

Equations (8)-(10) will be used as performance indices in section 4.

b) An evenness index E refers to how close in numbers each species in an environment are. The evenness index can be represented in our situation by [12].

$$\mathbf{E} = 1 - \frac{\sum_{Q=1}^{s} \mathbf{K_Q \ln(k_Q)}}{\mathbf{\ln(s)}} \qquad (11)$$

Where s: No of species = 4 Quarters in our case.
E is constrained between 0 and 1. The less variation in covering the areas between the species, the higher E is.

c) The total distance of the trajectory D.
The total distance of the generated trajectory of each controller should be taken in the account to measure the performance of the controller in coverage a certain area, and it can be calculated by the following formula:

$$\mathbf{D} = \sqrt{(\mathbf{x_{i+1}} - \mathbf{x_i})^2 + (\mathbf{y_{i+1}} - \mathbf{y_i})^2} \qquad (12)$$

Where: X_{i+1} and X_i are the x- coordinates at successive instants & Y_{i+1} and Y_i are the y- coordinates at successive instants.

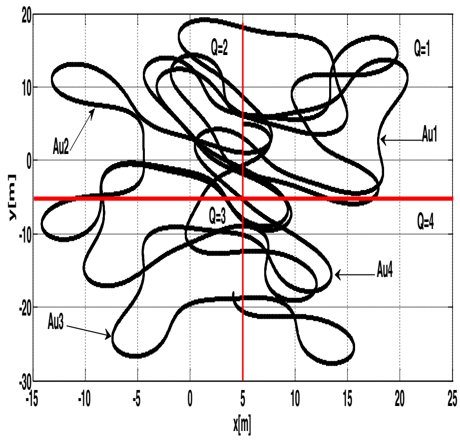

Fig. 8. Partition of the specified area

5. Simulation Results

In order to evaluate the performance of the three controllers, Arnold, Lorenz, and Chua's equations, used to generate the chaotic motion of the mobile robot, we simulate the three systems of equations (3), (5), and (7) given in section 3, in three different workspace areas (20 m x 20 m), (40 m x 40 m), and (60 m x 60 m), as specified in Table 1 illustrated in section 3. We simulate the system of equations using the parameters given in section 2 for each controller and the velocity v of the robot is 1 m/s. We use the performance index K, the evenness index E and the total distance of the chaotic mobile robot D, as the evaluation criteria to distinguish the performance between the three controllers in the three specified workspace areas.

The chaotic trajectory of the mobile robot for the three specified controllers, Arnold, Lorenz, and Chua's equations, in (20 m x 20 m) workspace at iteration n = 6000, number of pixels to cover area = N x N, N = 2000, integration step h = 0.1 and the parameters given in section 3, are shown in Fig. 9, Fig. 10, and Fig. 11 respectively. The robot moves as if is reflected by the boundary „mirror mapping".

The plot of the performance index K, the evenness index E, and the total distance of the trajectory versus iterations n of the simulation for the Arnold, Lornz, and Chua's controller in the three areas (20 m x 20 m), (40 m x 40 m), and (60 m x 60 m) are depicted in Fig. 12, Fig. 13, and Fig. 14, respectively.

6. Conclusion

In this paper, we proposed the implementation of chaotic behavior on a mobile robot, which implies a mobile robot with a controller that guarantees its chaotic motion. The Arnold, the Lorenz, and the Chua's equations, which are known to show the chaotic behavior, were adopted as the chaotic dynamics to be integrated into the mobile robot and the behaviors of these equations were evaluated from point of view of performance index k, which reflects

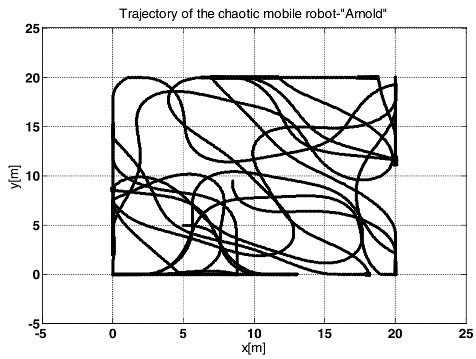

Fig. 9. The trajectory of the chaotic mobile robot controlled by the Arnold equation

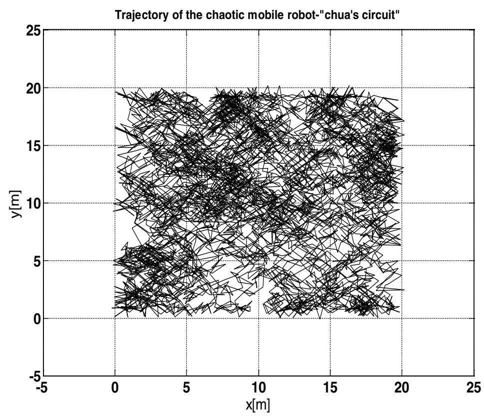

Fig.10. The trajectory of the chaotic mobile robot controlled by the Lorenz equation

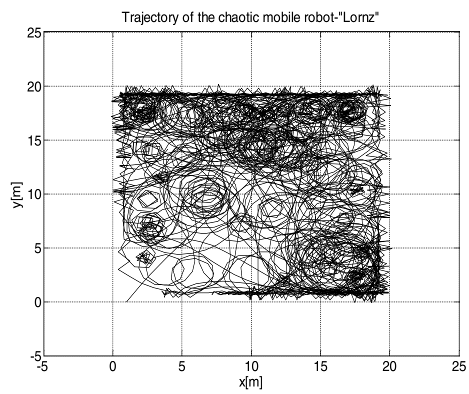

Fig.11. The trajectory of the chaotic mobile robot controlled by the Chua's circuit

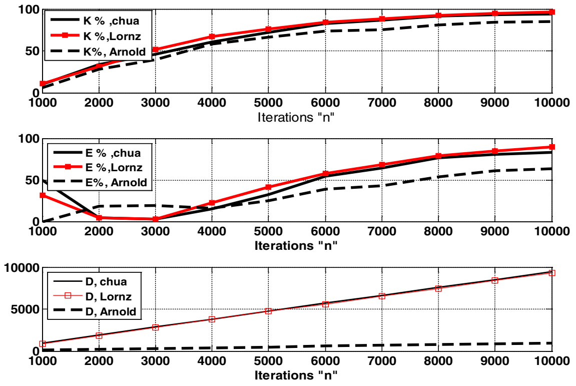

Fig. 12. Plot of K, E, and D of the controllers in (20 m x 20 m) workspace area

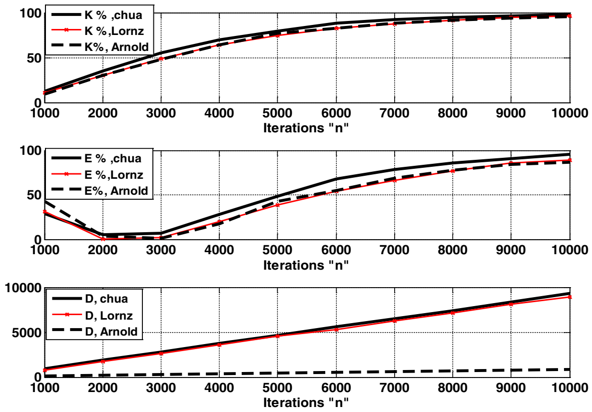

Fig. 13. Plot of K, E, and D of the controllers in (40 m x 40 m) workspace area

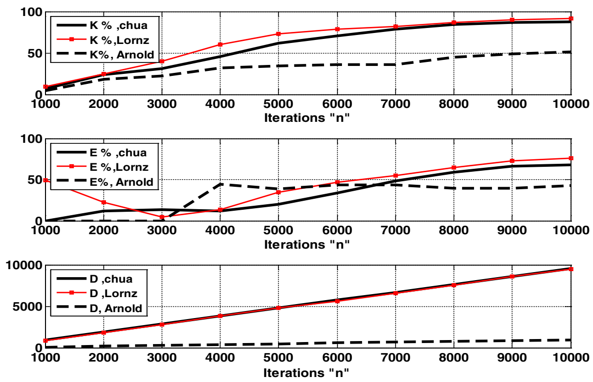

Fig. 14. Plot of K, E, and D of the controllers in (60 m x 60 m) workspace area

how high the coverage area, the evenness index E, which reflects the degree of variation in covering the areas between species, and the total distance D of the generated trajectory.

The effects of workspace size on the rate of convergence were studied. The results show that in low area coverage (20 m × 20 m), the performance of the Chua's equation as a controller is slightly better than the others, but the performance of the Lornz's equation as a controller is the best among the other in high area coverage (40 m × 40 m), and (60 m × 60 m). The performance of the Arnold equation as a controller is the worst among

the other controllers especially in high area coverage. The total distance of the trajectory is increased semi-linear with the time depending on the linear velocity of the mobile robot and the assumed obstacles and boundary area.

The effects of the shape of the workspace, and to compare the results of described simulations with other algorithms – not based on chaos motion, will be considered in the future work.

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