ACCELEROMETER-BASED MEASUREMENTS OF AXIAL TILT

Submitted 27th June 2011; accepted 1st September 2011

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Abstract:

The paper deals with a specific type of tilt measurements, where an axial tilt is to be determined. The measurements are realized by means of accelerometers – MEMS devices most preferably. Various mathematical relations between the axial tilt and the Cartesian components of the gravitational acceleration are presented. Each relation is described in detail, especially in the terms of the resultant uncertainty of the measurement, as well as the requirements regarding the employed accelerometers. Results of experimental studies realized by means of commercial MEMS accelerometers are presented and discussed, especially with regard to the measurement accuracy that has been evaluated for each mathematical relation. Scope of application of each relation is proposed.

Keywords: *MEMS, accelerometer, tilt, measurements*

1. Introduction

In the case of tilt measurements, usually two component angles are determined: pitch and roll [1]. However, there are some cases when such approach is not convenient. An example can be here e.g. directional drilling [2], where the tilt angle between the rotation axis of a drill bit and the gravitational acceleration must be carefully observed and kept at constant value. Another instance may be monitoring an object against losing its stability, as far as its vertical position is concerned. In the mentioned cases both pitch and roll occur, as if it were a dual axis tilt measurement. However, in fact it is a single axis measurement, where orientation of the rotation axis is not important.

So, an unconventional approach should be employed here. Its idea is illustrated in Fig. 1, where the considered tilt angle φ is presented against the Cartesian components of the gravitational acceleration. As measurements of tilt realized by means of accelerometers are very advantageous in many aspects, the further considerations are limited to this measurement method.

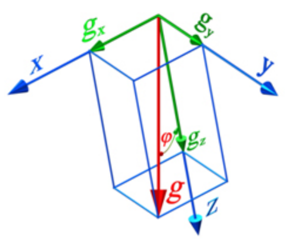

Fig. 1. Tilt angle and Cartesian components of the gravitational acceleration

Designations used in Fig. 1, have the following meaning:

g – gravitational acceleration,

g_x, g_y, g_z – components of the gravitational acceleration,

φ – tilt angle.

Let us assume that we use a triaxial accelerometer whose sensitive axes are x, y, z, and we want to determine the tilt of axis z (note that even though rotation about this axis results in a change of g_x and g_y, it should not result in a change of angle φ being determined).

2. Computations of the tilt angle and the resultant uncertainty

Whereas the pitch is primarily related to component g_x and the roll to g_y, angle φ is primarily related to g_z:

$$\phi_1 = \arccos\frac{g_z}{g} \tag{1}$$

Making use of the fact that:

$$g = \sqrt{g_x^2 + g_y^2 + g_z^2} \tag{2}$$

further formulas for determining angle φ_n can be derived (the subscript n=1…5 has been introduced in order to distinguish between particular formulas later in the text):

$$\phi_2 = \arcsin\frac{\sqrt{g_x^2 + g_y^2}}{g} \tag{3}$$

$$\phi_3 = \arctan\frac{\sqrt{g_x^2 + g_y^2}}{g_z} \tag{4}$$

Even though the nominal values of angles φ_n (n = 1…3) are the same, the uncertainty of their determination differs significantly. As suggested by the International Organization for Standardization [3], the uncertainty can be calculated according to a general formula:

$$u_c(\phi_n) = \sqrt{\left(\frac{\partial\phi_n}{\partial g_x}u(g_x)\right)^2 + \left(\frac{\partial\phi_n}{\partial g_y}u(g_y)\right)^2 + \left(\frac{\partial\phi_n}{\partial g_z}u(g_z)\right)^2 + \left(\frac{\partial\phi_n}{\partial g}u(g)\right)^2} \tag{5}$$

Assuming that:

$$u(g_x) = u(g_y) = u(g_z) = u(g_{x\ldots z}) >> u(g) \tag{6}$$

the uncertainty for Eq. (1), (3) and(4) will be respectively:

$$u_c(\phi_1) = \frac{u(g_{x...z})}{|g|}\frac{1}{\sin\phi_1} \qquad (7)$$

$$u_c(\phi_2) = \frac{u(g_{x...z})}{|g|}\frac{1}{\cos\phi_2} \qquad (8)$$

$$u_c(\phi_3) = \frac{u(g_{x...z})}{|g|} = const \qquad (9)$$

Courses of the Eq. (7)-(9), as well as the following Eq. (10) and (12), are presented in Fig. 2. It was assumed that the illustrated uncertainties are related to a unit relative standard uncertainty of the accelerometer:

$$\frac{u(g_{x...z})}{|g|} = 1 \qquad (10)$$

In the case of Eq. (7)-(8) (courses u_2 and u_1), their maximal values (approaching infinity) have been limited in the chart.

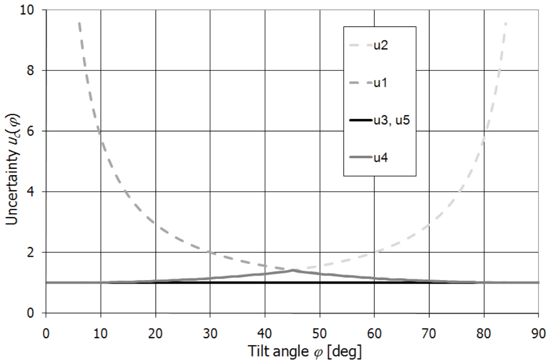

Fig. 2. *Uncertainty of tilt measurements for various mathematical formulas*

Analyzing the courses of uncertainties $u_c(\varphi_1)$ and $u_c(\varphi_2)$ presented in Fig. 2, one may easily conclude that it is worthwhile to introduce still another way of determining angle φ, defined as follows [4]:

$$\begin{Bmatrix} \phi_1, \phi_2 < 45° \Rightarrow \phi_4 = \phi_2 \\ \phi_1, \phi_2 > 45° \Rightarrow \phi_4 = \phi_1 \end{Bmatrix} \qquad (11)$$

As results from Eq. (11), uncertainty $u_c(\varphi_4)$ is a combination of $u_c(\varphi_1)$ and $u_c(\varphi_2)$, expressed in the following way:

$$\begin{Bmatrix} \phi_1, \phi_2 < 45° \Rightarrow u_c(\phi_4) = u_c(\phi_2) \\ \phi_1, \phi_2 > 45° \Rightarrow u_c(\phi_4) = u_c(\phi_1) \end{Bmatrix} \qquad (12)$$

Additionally, the tilt can be computed using a principle of a weighted average having variable weight coefficients, as follows [5]:

$$\phi_5 = \phi_1 \sin^2\phi_4 + \phi_2 \cos^2\phi_4 \qquad (13)$$

Then, the considered uncertainty approximately equals [5]:

$$u_c(\phi_5) \approx \frac{u(g_{x...z})}{|g|} \approx const \qquad (14)$$

If the uncertainties featured by particular sensitive axes of the applied accelerometer, related to measurements of the respective component accelerations, are not equal, as expressed by Eq. (6) and assumed in Fig. 2 (and thus in Eq. (11) and (13), consequently), the relevant formulas should be rearranged. Eq. (11) should feature an angle other than 45°, and the variable weight coefficients in Eq. (13) should be expressed by functions other than $\sin^2(x)$ or $\cos^2(x)$, otherwise the respective uncertainty would increase.

3. The experimental studies

The presented theoretical considerations have been fully confirmed in an experimental way. In order to carry out appropriate tests a special test station was used. It has been described by the author in [6], whereas the observed methodology of performing the experiments minutely discussed in [7]-[8].

The measurements were realized by means of a tilt sensor built of two dual-axis MEMS accelerometers ADXL 202E from Analog Devices Inc. [9], whose sensitive axes were arranged into a Cartesian coordinate system.

Results of the tests are illustrated in Fig. 3, which presents variations of errors corresponding to each kind of the considered formulas for determining the tilt angle.

The error e_n (n=1…5) has been defined as [3]:

$$e_n = |\theta - \phi_n| \qquad (15)$$

where θ is the real tilt angle applied by means of the test station and φ_n is the tilt angle calculated according to the respective formula, on the basis of accelerometer indications.

The graphical form of each error is consistent with its corresponding uncertainty in Fig. 2.

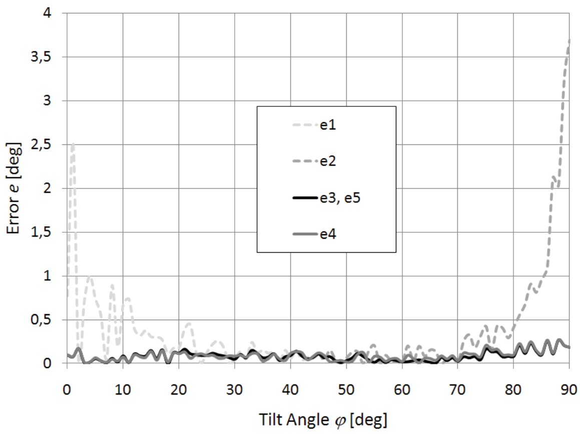

Fig. 3. The sensitivity of tilt measurements for various mathematical formulas

4. Conclusions

As results from the courses of the uncertainties illustrated in Fig. 2, as well as variations of the measurement errors in Fig. 3, the most accurate way of determining the considered axial tilt is to employ Eq. (4) or Eq. (13) – uncertainties u_3 and u_5, errors e_3 and e_5. The later is much more complicated, however while using triaxial MEMS accelerometers it may turn out that it is more advantageous, as the manufacturing technologies of MEMS devices are in fact usually semi-three-dimensional [10]. As a result, accuracy in the third axis of a triaxial accelerometer is often lower than in the other two. Then Eq. (13) may be constructed in such a way as to regard this fact.

In the case of using the most simple formulas: Eq. (7)-(8), the resultant uncertainty significantly increases for tilt angles of 80°÷**90**° and 0°÷**10**° respectively – uncertainties u_2 and u_1, what corresponds to clearly higher errors e_2 and e_1. This way of determining the tilt is acceptable rather in the case of using only the remaining part of the measuring range, or using both formulas interchangeably, as proposed by Eq. (11).

In conclusion, while comparing the respective courses of uncertainties and the corresponding errors, presented in Fig. 2 and Fig. 3, it can be stated that the experimental studies proved the reasoning presented in section 2 to be true.

5. Summary

While determining tilt of an axis with application of MEMS accelerometers, one must first decide what kind of formula will be applied for this purpose. In Tab.1 there are gathered the most important features of determining tilt according to particular kind of the mathematical formula.

Tab. 1. *Characteristics of the mathematical formulas.*

A	B	C	D
Eq. (1)	∞÷1	1	³ ±1g
Eq. (3)	1÷∞	2	³ ±1g
Eq. (4)	1	3	³ ±1g
Eq. (11)	1÷1.41	3	³ ±0,71g
Eq. (13)	1	3	³ ±1g

A – no. of the formula
B – uncertainty (defined as in Fig. 2)
C – necessary no. of sensitive axes of the applied accelerometer
D – measurement range of the applied accelerometer ($g = 9.81\ m/s^2$)

It must not be neglected that application of particular formula is connected with a different complication of the related data processing, as it results from the structure of each formula.

Application of Eq. (1) is advantageous, because a single-axis accelerometer can be applied. However, if the measured tilt angles are small, the errors have considerable values. It is just contrary in the case of Eq. (2). On the other hand, application of Eq. (3) and (5) ensures the lowest uncertainty of measurements (so, the highest accuracy), whereas application of Eq. (11) makes it possible to use an accelerometer with a smaller measuring range.

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