Chaotic Mobile Robot Workspace Coverage Enhancement

Submitted 24th May 2011; accepted 10th January 2012

Ashraf Anwar Fahmy

Abstract:
The chaotic mobile robot implies a mobile robot with a controller that ensures chaotic motions. Chaotic motion is characterized by the topological transitivity and the sensitive dependence on initial conditions. Due to the topological transitivity, the chaotic mobile robot is guaranteed to scan the whole connected workspace. The Chua's circuit, which is low cost and easy to construct for trajectory generators, exhibits a rich variety of bifurcation and chaotic behaviors. This particular circuit is analyzed. It is shown how to generate a sequence of chaotic behaviors by varying the value of a linear resistor of the Chua's circuit. According to the simulation, with best adjusting parameters, and mapping the appropriate chaotic variables to robot's kinematic variables, the chaotic behavior is enhanced in the sense of wide area coverage and evenness index.

Keywords:*chaos, mobile robot, chaotic motion, chaotic mobile robot, Chua's circuit*

1. Introduction

Chaos is a typical behavior of nonlinear dynamical systems, and has been studied deeply in different fields such as mathematics, physics, engineering, economics, and sociology. In robotics, the first chaotic mobile robot that can navigate following a chaotic pattern was proposed by Nakamura and Sekikuchi [1, 2], where the Anorld's equation was used to generate the desired motions. Further investigations on chaotic trajectories of the same type of the robot using other equations were carried out in [3-12]. The main objective in exploiting chaotic signals for an autonomous mobile robot is to increase and to take advantage of coverage areas resulting from its travelling paths. Large coverage areas are desirable for many applications such as robots designed for scanning of unknown workspaces with borders and barriers of unknown shape, as in patrol or cleaning purposes. The Chua's pattern is the most interesting one among other candidate's patterns due to its largest coverage areas, low cost, and ease for implementation [12]. The aim of this paper is to seek the best combinations of Chua's circuit parameters and the appropriate mapping of the state variables to robot's kinematic variables in order to fulfill the requirements of high coverage coefficient and high evenness index of a specified area crossed far and wide erratically by the mobile robot.

The paper structure is as follows: The next section presents the analysis of the Chua's circuit. The mobile robot model is illustrated in section 3. Section 4 is dedicated to describe the methodology which includes the integration of Chua's circuit equations to mobile robot model, and the performance criteria to be applied. Section 5 is reserved to the simulation results. Finally section 6 concludes this paper.

2. Chua's Circuit Analysis

The circuit components are two capacitors, one inductance and two resistors – one linear and another one nonlinear. The non-linear resistor is known as Chua's diode and it's actually made from several components (as seen in the dashed rectangle of Fig.1). The diodes are nonlinear components, and the opamp together with R1, R2 and R3 represent a negative resistor [13].

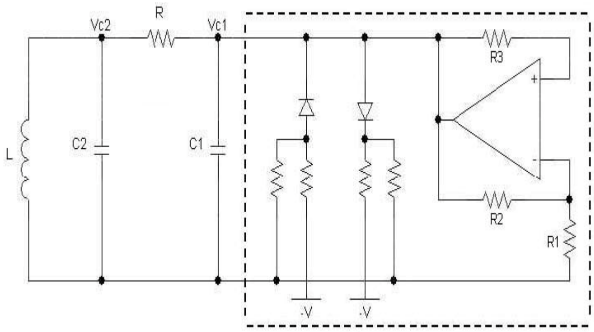

Fig. 1. Chua's circuit

The equivalent input resistor is actually a negative resistor, accordingly; the equivalent Chua's circuit is as shown in Fig. 2.

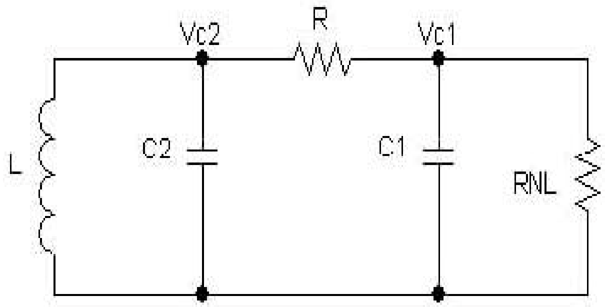

Fig. 2. Equivalent Chua's circuit

Using kirchoff laws:

$$\frac{dV_{c1}}{dt} = -\frac{V_{c1}}{RC1} + \frac{V_{c2}}{RC1} - \frac{i_{NL}}{C1} \tag{1}$$

$$\frac{dV_{c2}}{dt} = \frac{V_{c1}}{RC2} - \frac{V_{c2}}{RC2} + \frac{i_L}{C2} \tag{2}$$

$$\frac{di_L}{dt} = -\frac{V_{c2}}{L} \tag{3}$$

Knowing the Chua's diode I-V piecewise-linear characteristic given by Fig. 3, the non-linear part of the circuit can be solved by the following system of equations:

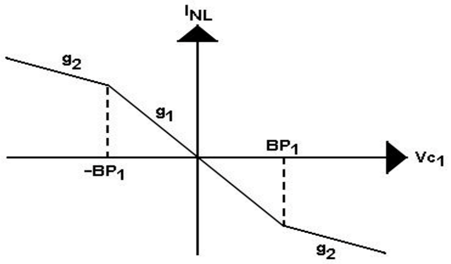

Fig. 3. Chua's diode I-V characteristic

$$I_{NL} = \begin{cases} -g_2 V_{c1} + (g_1 - g_2)BP_1, & V_{c1} < -BP_1 \\ -g_1 V_{c1} & -BP_1 \le V_{c1} \le BP \\ -g_2 V_{c1} + (g_2 - g_1)BP_1, & V_{c1} < -BP_1 \end{cases} \tag{4}$$

This set of equations can be put in the form of state-space:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_L \end{bmatrix} = A \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_L \end{bmatrix} + \mathrm{b} \tag{5}$$

From (1) to (4), the state space equations of the Chua's circuit can be written as in (6-8):

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC1} + \frac{g_2}{C1} & \frac{1}{RC1} & 0 \\ \frac{1}{RC2} & \frac{-1}{RC2} & \frac{1}{C2} \\ 0 & \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{(g_2 - g_1}{C1} \\ 0 \\ 0 \end{bmatrix}$$

$$V_{C1<-BP_1} \tag{6}$$

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC1} + \frac{g_1}{C1} & \frac{1}{RC1} & 0 \\ \frac{1}{RC2} & \frac{-1}{RC2} & \frac{1}{C2} \\ 0 & \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{(g_2 - g_1}{C1} \\ 0 \\ 0 \end{bmatrix}$$

$$-BP_1 \le V_{C1} \le Bp_1 \tag{7}$$

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC1} + \frac{g_2}{C1} & \frac{1}{RC1} & 0 \\ \frac{1}{RC2} & \frac{-1}{RC2} & \frac{1}{C2} \\ 0 & \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{(\mathrm{g}_1 - \mathrm{g}_2}{\mathrm{C1}} \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{V}_{\mathrm{C1>BP_1}} \tag{8}$$

This set of ordinary differential equations (ODE) can be solved using a numerical method such as Forward Euler (FE).

The FE method states that [13]:

$$X_{n+1} = X_n + h\ f(x_n, t_n) \tag{9}$$

Where Xn+1 is the solution of the ODE at time tn+1, Xn is the current value, and h is the step-size.

Using (9) to solve (5) we obtain:

$$X_{n+1} = X_n + h(Ax_n + b) \tag{10}$$

3. The Mobile Robot Model

The mobile robot considered in this work is of two degrees-of-freedom, including two active, parallel and independent wheels, a third passive wheel with exclusively equilibrium functions, and proximity sensors capable of obstacles detection. The active wheels are independently controlled on velocity and rotation sense. The sensors provide short-range distances to obstacles. For instance, these sensors can be infrared devices commonly used in mobile robots, with adequate accuracy. Additionally, the robot is supposed to be equipped with specific sensors for detection and recognition of searched objects.

The robot is supposed to operate on a horizontal plane with a motion described in terms of linear velocity $v(t)$ and direction $\theta(t)$. The geometry of this motion scheme is shown in Fig. 4 [7]. The mathematical model, of this kinematic problem considers two control variables, velocity $v(t)$ and rotational velocity $\omega(t)$ and three state variables, the robot position and orientation ($x_r(t)$, $y_r(t)$, $\theta_r(t)$), is as follows:

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta & \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{11}$$

The discrete form of the equation (11):

$$\begin{aligned} x_{r(n+1)} &\approx x_{r(n)} + tv\cos\theta_{r(n)} \\ y_{r(n+1)} &\approx y_{r(n)} + tv\sin\theta_{r(n)} \\ \theta_{r(n+1)} &\approx \theta_{r(n)} + t\omega_{r(n)} \end{aligned} \tag{12}$$

Where t is sufficiently small step

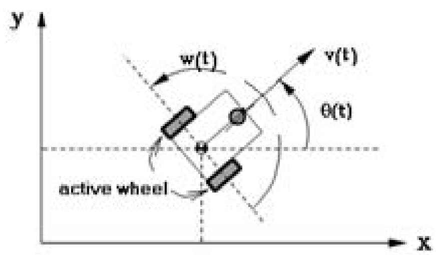

Fig. 4. Geometry of the robot motion on Cartesian plane

4. Methodology

4.1. Integrated system.

In order to integrate the Chua's circuit into the controller of the mobile robot, we name the following states

$$x_1 \equiv V_{c1},\ x_2 \equiv V_{c2},\ x_3 \equiv i_L \tag{13}$$

From equation (5) the set of equations of Chua's circuit become:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + b \tag{14}$$

Consequently, the state equation of the chaotic mobile robot after integrating the set of equations of Chua's circuit with the mathematical model of the mobile robot equation (11), we obtain the following system of equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_r \\ \dot{y}_r \end{bmatrix} = \begin{bmatrix} \mathrm{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \mathrm{b} \\ v \ \cos(\theta) \\ v \ \sin(\theta) \end{bmatrix} \tag{15}$$

The corresponding mapping parameters from 3-D chaotic circuit into 2-D one is as follows:

Table 1. Mapping chaotic variables to robot's kinematic variables

System	θ
Case 1	x_1
Case 2	x_2
Case 3	x_3

Where θ corresponds to the orientation angle of the mobile robot.

4.2. Evaluation criteria

The evaluation criteria are set according to the application purpose. Since we would like to use the robot in wandering around area in the area of no maps, the chaotic trajectory should cover the entire areas of patrolling as much as possible. The following two performances criteria are to be considered to evaluate the coverage rate of the chaotic mobile robot, namely the performance index k and the evenness index E

a) A performance index K representing a ratio of areas that the trajectory passes through or used space (A_u), over the total working area (A_t)

$$K = \frac{A_u}{A_t} \tag{16}$$

The used area A_u and the total area A_t can be calculated by the following algorithm:

1. We divide the specified area into (NxN) pixels.
2. Initially, assign the value 0 for all pixels.
3. We get the x-coordinate and y-coordinate of the pixels which passes through the trajectory of the robot.
4. We assign the value 1 for each pixel passes through the trajectory.
5. We count the number of ones (pixels which passes through the trajectory) which is A_u
6. We count the number of zeroes Z (pixels which don't passes through the trajectory).
7. The total area A_t is the sum of A_u and Z.

Similarly, let us consider a rectangular shape area, Fig. 5. The total area can be partitioned into four quarter, denoted Q=1, 2, 3, 4. The quantitative measurement of the trajectory can be evaluated by using the following equation:

$$K_Q = \frac{A_{uQ}}{A_{tQ}} \tag{17}$$

Where K_Q is the performance index of the Q^{th} quadrant, A_{uQ} is the area used by the trajectory in the Q^{th} quadrant. In our case, we have:

$$A_{tQ} = \frac{A_t}{4} \tag{18}$$

Equations (16)-(18) will be used as performance indices in section 5.

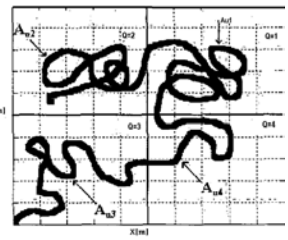

Fig. 5. Partition of the specified area

b) An evenness index E refers to how close in numbers each species in an environment are. The evenness index can be represented in our situation by [14]:

$$E = 1 - \frac{\sum_{Q=1}^{s} K_Q \ln(k_Q)}{\ln(s)} \tag{19}$$

Where s: No of species = 4 Quarters in our case.

E is constrained between 0 and 1. The less variation in covering the areas between the species, the higher E is.

5. Simulation Results

The system of equations (15), introduced in section 4 is simulated through Matlab, including circuit simulation of the Chua's circuit, introduced in section 2 and mobile robot model introduced in section 3. The values used for the Chua's circuit simulation were the following [13]:

C1	C2	L	Vc1(0)	Vc2(0)	IL(0)	g1	g2	BP1	h	R
450 PF	1.5nF	1mH	10-5	0V	0A	1/1358	1/2464	0.114	10-7	1-2 KΩ

The values of all the components of the circuit are constant except the resistor R which is varied from (1000-2000) Ω to get the best value of performance index K and the evenness index E, introduced in section 4 as a performance measure of the chaotic trajectory of the Chua's circuit for the integrated system, described in section 4.

For simplicity, an area of 20m x 20m is used as a workspace coverage trajectory in computer simulation. The robot moves as if is reflected by the boundary "mirror mapping". The initial conditions were established and the program was set to run n iterations. The resultant Chua's pattern in three dimensions at iteration n=3000, is depicted in Fig. 6

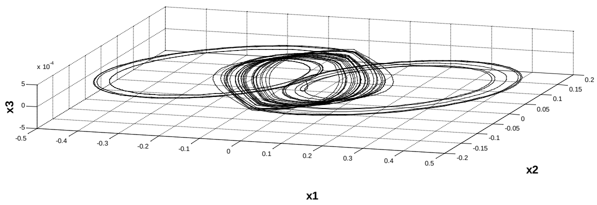

Fig. 6. Chua's pattern in three dimensions

The simulation results for case 1, case 2, and case 3, for number of iterations n= 3000 and by varying the value of the resistance R is shown in Table 2, Table 3, Table 4, respectively. From the results we can deduce the following:

a) In case 1, the performance index k and the evenness index E are maximum when R=1100.
b) In case 2, the performance index k and the evenness index E are maximum when R=1500.
c) In case 3, the performance index k and the evenness index E are maximum when R= 1625.

The resultant chaotic trajectories of the robot in case 1, case 2, and case 3 at the specified values of the resistor R are illustrated in Fig. 7, Fig. 8, and Fig. 9, respectively.

Table 2. Case 1 and run time for n=3000

Resistance R[Ω]	% of K	% of Q=1	% of Q=2	% of Q=3	% of Q=4	% of E
1000	85.82	86.06	88.15	85.58	83.52	62.19
1100	85.93	89.22	80.04	82.49	91.95	62.79
1200	85.17	92.35	82.00	78.23	88.10	61.06
1300	76.01	63.89	59.71	87.18	93.28	43.84
1400	83.07	77.70	79.40	88.54	86.64	55.91
1500	80.52	84.14	90.24	76.43	71.26	50.61
1600	82.36	88.13	80.28	76.76	84.27	54.20
1700	67.34	95.64	46.65	41.66	85.41	35.24
1800	81.71	83.32	74.64	79.80	89.08	52.88
1900	77.85	74.12	70.10	81.27	85.93	44.47
2000	67.70	95.84	58.56	44.16	72.27	31.49

Table 3. Case 2 and run time for n= 3000

Resistance R[Ω]	% of K	% of Q=1	% of Q=2	% of Q=3	% of Q=4	% of E
1000	82.10	93.69	87.85	71.07	75.80	54.74
1100	80.47	88.84	90.97	71.88	70.19	51.17
1200	85.00	86.98	80.81	82.94	89.28	60.34
1300	84.08	94.86	84.28	73.96	83.24	58.89
1400	81.76	70.89	77.63	93.31	85.21	53.74
1500	88.07	86.66	86.48	89.48	89.67	67.77
1600	83.99	82.08	77.76	85.68	90.44	58.09
1700	86.39	92.15	84.12	80.78	88.49	63.84
1800	83.90	82.40	88.34	85.29	79.56	57.69
1900	84.78	77.31	78.30	92.33	91.16	60.44
2000	74.91	90.42	64.86	60.30	84.07	40.66

Table 4. Case 3 and run time for n=3000

Resistance R[Ω]	% of K	% of Q=1	% of Q=2	% of Q=3	% of Q=4	% of E
1000	79.97	84.62	68.67	74.63	91.96	49.88
1100	77.57	89.80	66.19	65.48	88.82	45.74
1200	74.74	87.04	93.88	61.24	56.74	42.17
1300	75.12	94.46	80.09	57.79	68.16	41.59
1400	74.84	65.49	69.52	83.88	79.02	37.72
1500	80.34	83.90	84.63	76.74	76.07	49.53
1625	80.86	84.23	68.88	76.63	93.70	51.94
1700	76.35	78.32	80.63	74.92	72.16	40.76
1800	46.64	72.41	57.49	25.08	31.59	8.90
1900	76.48	85.73	73.35	67.72	79.15	41.69
2000	45.01	54.29	29.17	33.75	62.81	2.64

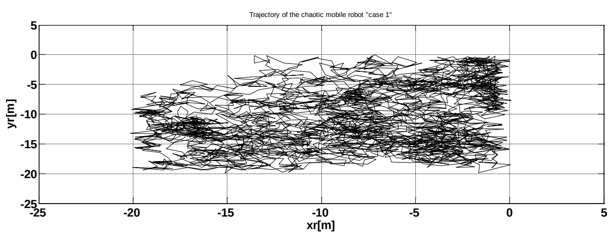

Fig. 7. The chaotic trajectory of the robot in case 1

To investigate the relation between the value of the resistance R of the Chua's circuit and the performance indices K and E, the bar plots of this relation for the case 1, case 2, and case 3, are illustrated in Fig. 10, Fig. 11, and Fig. 12, respectively.

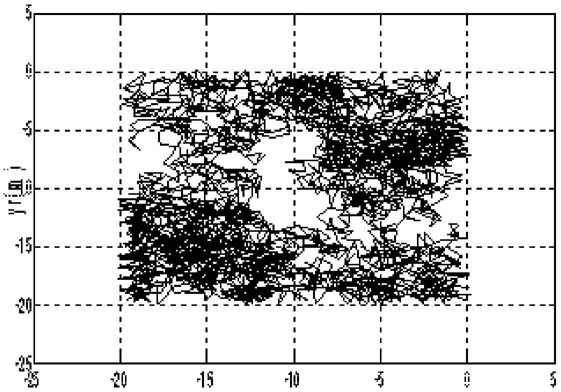

Fig. 8. The chaotic trajectory of the robot in case 2

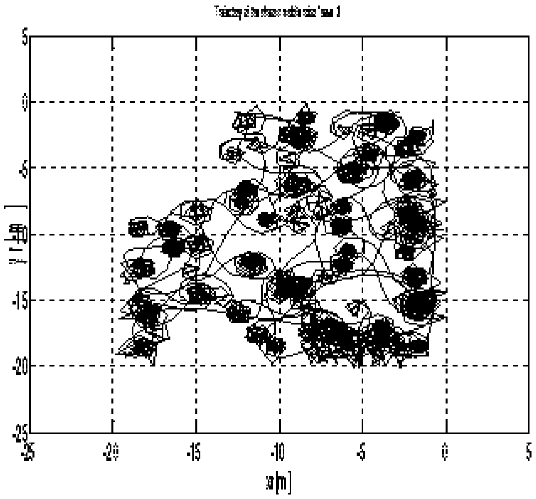

Fig. 9. The chaotic trajectory of the robot in case 3

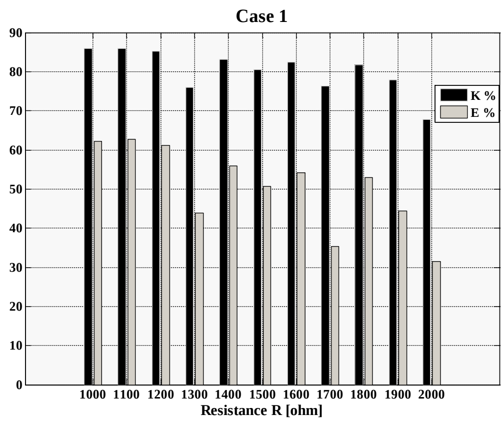

Fig. 10. Bar plot of K and E vs. R for case 1

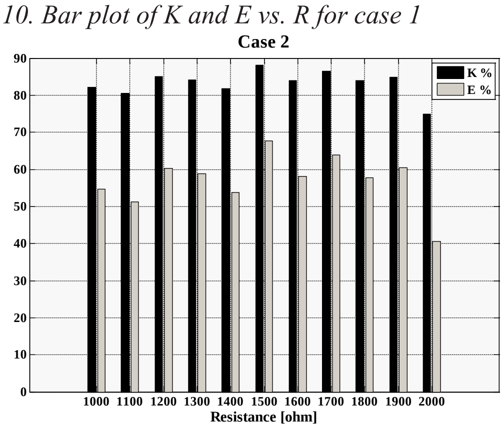

Fig. 11. Bar plot of K and E vs. R for case 2

Fig. 12. Bar plot of K and E vs. R for case 3

The relation of the performance indices K and E for the specified values of the resistance R as n changed from 1000 to 10000, for case 1, case 2, and case 3 are illustrated in Fig. 13, Fig. 14, and Fig. 15, respectively.

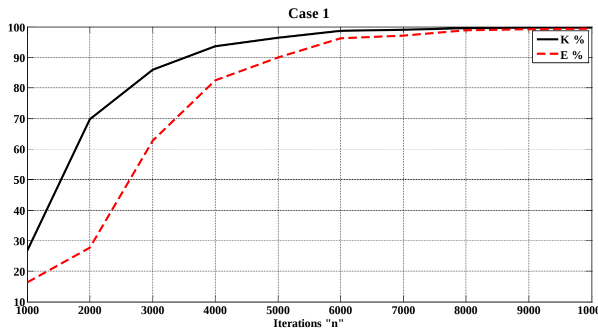

Fig. 13. The performance indices K and E vs. iterations in case1

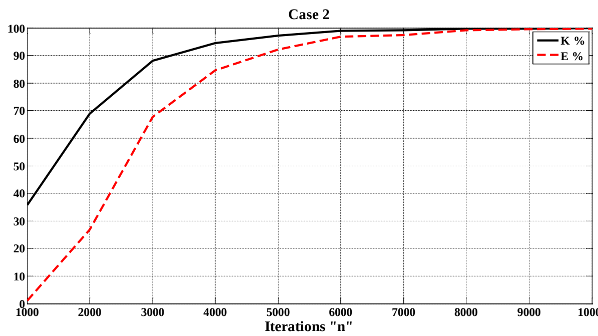

Fig. 14. The performance indices K and E vs. iterations in case 2

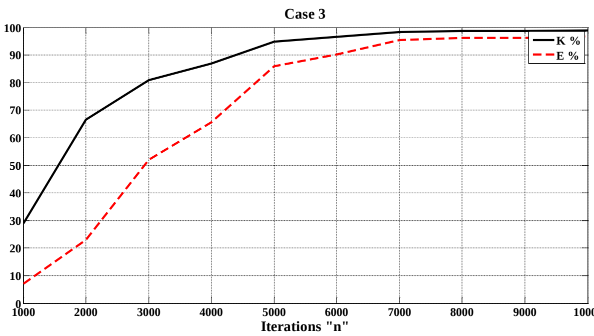

Fig.15. The performance indices K and E vs. iterations in case 3

6. Conclusion

This article introduced an enhancement to the chaotic mobile robot, using Chua's circuit as a controller. The Chua's circuit, which is low cost and easy to construct for trajectory generators, exhibits a rich variety of bifurcation and chaotic behaviors.

First, we studied the effect of the "resistor" value of the Chua's circuit on the resultant robot kinematics for enhancing the robot trajectory in the sense of wide area coverage and evenness index.

Second, we studied the effect of the setting of mapping between circuit variables and robot angle of rotation. We deduce from simulation results that, we can get the highest value of the covering coefficient and evenness index at R= 1500 Ω, conditionally that we map Vc2 of the Chua's circuit to the angle of rotation (θ), of the robot.

Third, we deduced that: by increasing iterations of simulations (time), the covering coefficient and evenness index are enhanced with different rate depending on appropriate mapping, is used. In case 1 and case 2, the covering coefficient and evenness index reach above 90% in 5000 iterations (time unit), but case 3 realizes that in 6000 iterations.

We can deduce that, by setting optimal values of the parameters of the Chua's circuit and the appropriate mapping of chaotic variables to robot's kinematic variables, is suitable for generating navigating signal to implement the robot target-searching tasks due to its excellent trajectory throughout a given work-area.

Author:

Ashraf Anwar Fahmy – Assistant Professor, Department of Computer Engineering, College of Computers and Information Technology, Taif University, Taif, Saudia Arabia. His research interests are mainly in the area of tracking system, control, and robotics.

E-mails: ashraaf@tu.edu.sa; ashrafmanwar90@yahoo.com

References

[1] Y. Nakamura and A. Sekiguchi, "Chaotic mobile robot," *IEEE Transaction on Robotics and Automation*, vol. 17, no. 6, 2001, pp. 898–904.

[2] A. Sekiguchi and Y. Nakamura, "The Chaotic mobile robot", in: *Proc. IEEE/RSJ. Int. Conf. Intelligent Robots and Systems*, vol. 1, 1999, pp. 172-178.

[3] A. Jansri, K. Klomkarn, and P. Sooraksa, "Further investigation on trajectory of chaotic guiding signals for robotics system". In: *Proc. Int. Symp. Communication and Information Technology*, 2004, pp. 1166–1170.

[4] A. Jansri, K. Klomkarn, and P. Sooraksa, "On comparison of attractors for chaotic mobile robots". In: *Proc. 30th Annu. Conf. IEEE Industrial Electronics Society*, IECON, vol. 3, 2004, pp. 2536–2541.

[5] C. Chanvech, K. Klomkarn, and P. Sooraksa, "Combined chaotic attractors mobile robots". In: *Proc. SICE-ICASE Int. Joint Conf.*, 2006, pp. 3079–3082.

[6] L. S. Martins-Filho, R. F. Machado, R. Rocha, and V. S. Vale, "Commanding mobile robots with chaos". In: J. C. Adamowski, E. H. Tamai, E. Villani, and P. E. Miyagi (Eds.) *ABCM Symposium Series in Mechatronics*, vol. 1, ABCM, Rio de Janeiro, Brazil, 2004, pp. 40–46.

[7] S. Martins *et al.*," Kinematic control of mobile robots to produce chaotic trajectories". In: *ABCM Symposium Series in Mechatronics*, vol. 2, 2006, pp. 258–264.

[8] S. Martins *et al.*, „Patrol Mobile Robots and Chaotic Trajectories". In: *Mathematical problems in engineering*, vol. 2007, Article ID61543, 13 pages, 2007.

[9] J. Palacin, J. A. Salse, I. Valganon, and X. Clua, "Building a mobile robot for a floor-cleaning operation in domestic environments", *IEEE Transactions on Instrumentation and Measurement*, vol. 53, 2004, no. 5, pp. 1418–1424.

[10] M. Islam and K. Murase, "Chaotic dynamics of a behavior-based miniature mobile robot: effects of environment and control structure", *Neural Networks*, vol. 18, no. 2, 2005, pp. 123–144.

[11] U. Nehmzow, "Quantitative analysis of robot-environment interaction-towards scientific mobile robotics", *Robotics and Autonomous Systems*, vol. 44, no. 1, 2003, pp. 55–68.

[12] P. Sooraksa and K. Klomkarn, „No-CPU chaotic robots from classroom to commerce". In: *IEEE circuits and systems magazine*, 10.1109/MCAS, 2010, pp. 46–53.

[13] V. Araujo, „Circuit analysis and matlab simulation of chua oscillator", *ARISER*, vol. 5, No. 1, pp. 49–57, 2009.

[14] J. Nicolas *et al.*, „A comparative analysis of evenness index sensitivity", *Int. Review Hydrobiology*, vol. 88, no. 1, 2003, pp. 3–15.