Pole-placement Adaptive Control for a Plant with Unknown Structure and Parameters – a Simulation Study

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Abstract:

Adaptive pole-placement control for the plant with unknown orders and coefficients of its model is presented in the paper, in an on-line approach. In order to adapt to the plant, the considered controller changes its structure and parameters, along with the identification process.

In order to combine structural and parametric identification, the approach presented in [5] has been used, with the simulation runs performed for continuous plant and a discrete-time controller and identification algorithms.

Keywords: adaptive control, identification

1. Introduction

Full knowledge of the plant is required to design the classical controller with parameters computed on the basis of well-known tuning rules, such as of Ziegler-Nichols for PID controllers. A good choice of controller parameters assures achieving expected performance of the control system. Because the tuning is performed for a specific plant which structure (e.g., order) or parameters may change with time, the computed controller parameters may turn out to be inappropriate. In such a case, one uses adaptive control, tuning the controller to improve the performance by using information about current polynomial orders and estimates of plant parameters.

The paper presents the topic of adaptive control with estimation of parameters and orders of the plant polynomials in the reference signal tracking task in a fully discretetime control system. The control algorithm combined with estimation yields time- and structure-varying controller with parameters tuned on-line to the current structure and properties of the plant.

2. General model of the plant

Let $G(q^{-1})$ and $H(q^{-1})$ be certain transfer functions and q^{-1} be a one-sample shift operator, $q^{-1}y_t = y_{t-1}$. The general structure of the model [3, 4]

$$y_t = G(q^{-1})u_t + H(q^{-1})\xi_t = = \frac{B(q^{-1})}{A(q^{-1})}u_{t-d} + \frac{C(q^{-1})}{A(q^{-1})}\xi_t, \qquad (1)$$

where $G(q^{-1})$ is a transfer function of control circuit, and $H(q^{-1})$ of disturbance circuit, can be put in the form

$$A(q^{-1})y_t = B(q^{-1})u_{t-d} + C(q^{-1})\xi_t, \qquad (2)$$

where d is a dead-time and polynomials from (2) are given as:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{nA} q^{-nA}, \quad (3)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_{nB} q^{-nB}, \quad (4)$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \dots + c_{nC} q^{-nC}.$$
 (5)

The basic model considered in the paper is the autoregressive model with moving-average with auxogenous input (ARMAX) as in (2). A special form of ARMAX will be of interest here, namely CARMA model.

3. Pole-placement controller

The considered controller is to assure appropriate placement of poles of discrete-time system (with discrete-time controller and discrete-time model of the plant). Such an adaptive controller can be put in RST form with control signal

$$u_t = \left(1 - \hat{R}(q^{-1})\right) u_t - \hat{S}(q^{-1})y_t + \hat{T}(q^{-1})r_t \,, \quad (6)$$

where r_t is the reference signal tracked by y_t .

Having omitted estimate symbols, the controller polynomials:

$$R(q^{-1}) = 1 + r_1 q^{-1} + \dots + r_{nB+d-1} q^{-nB-d+1}, (7)$$

$$S(q^{-1}) = s_0 + s_1 q^{-1} + \dots + s_{nA-1} q^{-nA+1}. (8)$$

Using the knowledge about plant polynomials $A(q^{-1})$ and $B(q^{-1})$, known delay d and choosing closed-loop characteristic polynomial

$$A_M(q^{-1}) = 1 + a_{M,1}q^{-1} + \dots + a_{M,nA_M}q^{-nA_M}$$
(9)

one can introduce Diophantine equation

$$A(q^{-1})R(q^{-1}) + q^{-d}B(q^{-1})S(q^{-1}) = A_M(q^{-1}),$$
(10)

with

$$T(q^{-1}) = \frac{A_M(1)}{B(1)}.$$
 (11)

By solving this equation one obtains controller parameters, what in turn allows the current control sample to be computed. Having substituted control law to the plant equation and using the Diophantine equation one can obtain closed-loop transfer function

$$\frac{y_t}{r_t} = \frac{q^{-d}B(q^{-1})T(q^{-1})}{A_M(q^{-1})},$$

from which it can be seen that closed-loop poles are in prescribed locations.

The general solution of the Diophantine equation for given nA and nB is

$$\underline{w}_{RS} = \boldsymbol{W}_{AB}^{-1} \underline{w}_{A_MA},$$

where:

$$\boldsymbol{W}_{AB} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ a_{1} & 1 & \vdots & \vdots & \ddots & \vdots \\ a_{2} & a_{1} & & 0 & \cdots & 0 \\ \vdots & a_{2} & \ddots & 0 & b_{0} & 0 & \cdots & 0 \\ \vdots & a_{2} & \ddots & 1 & b_{1} & b_{0} & \vdots \\ a_{nA} & \ddots & a_{1} & \vdots & b_{1} & \ddots & 0 \\ 0 & a_{nA} & a_{2} & b_{nB} & \vdots & \ddots & b_{0} \\ 0 & 0 & & 0 & b_{nB} & b_{1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nA} & 0 & \cdots & b_{nB} \end{bmatrix} \in \mathscr{R}^{(nR+nS+1)\times(nR+nS+1)}, \qquad (12)$$

$$\underline{w}_{RS} = \begin{bmatrix} r_{1}, \cdots, r_{nR}, s_{0}, \cdots, s_{nS} \end{bmatrix}^{T}, \qquad (13)$$

$$\underline{w}_{AMA} = \begin{bmatrix} a_{M,1} - a_{1}, a_{M,2} - a_{2}, \cdots, 0, \cdots, 0 \end{bmatrix}^{T}. \qquad (14)$$

The parameters of $A(q^{-1})$ are put in the first nR columns of \boldsymbol{W}_{AB} , and the zero matrix in right upper corner has the $(d-1) \times (nS+1)$ dimension.

4. Formulation of the problem

The main problem is given the triple $(n\hat{A}, n\hat{B}, n\hat{C})$ of model orders and the current estimate (21), to find the improved model for which no improvement in polynomial orders are unnecessary. Subsequently, for such a model the adaptive pole-placement discrete-time controller of the form (6) has to assure tracking properties for the discretetime model of the plant (2).

5. Estimation of orders and parameters for linear plants in ARMAX form [5]

5.1. Preliminaries

The algorithm has been designed for ARMAX-type plants but by omitting the information about $C(q^{-1})$ polynomial one can use the information concerning $A(q^{-1})$ and $B(q^{-1})$.

The method will be cited in brief, the complete algorithm can be found in [5]. It requires the following assumptions:

- polynomials $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$ are co-prime and their leading coefficients are non-zero,
- positive-real condition is satisfied,
- true polynomial orders (nA, nB, nC) belong to known and finite set

$$\begin{split} \mathcal{M} &= \left\{ (\hat{nA}, \, \hat{nB}, \, \hat{nC}) : 0 \leq \hat{nA} \leq nA^o, \\ 0 \leq \hat{nB} \leq nB^o, \, 0 \leq \hat{nC} \leq nC^o \right\} \,, \end{split}$$

where nA^{o} , nB^{o} , nC^{o} are maximal assumed orders.

5.2. Recursive estimation method

Let the regression vector

$$\underline{\varphi}_{t}^{o} = [y_{t-1}, y_{t-2}, \dots, y_{t-nA^{o}}, \\
 u_{t-d}, u_{t-d-1}, \dots, u_{t-d-nB^{o}}, \\
 \varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-nC^{o}}]^{T}$$
(15)

correspond to the information gathered for maximal polynomial orders and ε_t is computed as in RELS algorithm based on equations [4]:

$$\varepsilon_t = y_t - \hat{\underline{\theta}}_{t-1}^{o^{-T}} \underline{\underline{\varphi}}_t^o, t \ge 0, \ \varepsilon_t = 0 \ (t < 0) \ (16)$$

$$\frac{\hat{\theta}^o}{t} = \hat{\theta}^o_{t-1} + \underline{k}^o_t \varepsilon_t, \qquad (17)$$

$$\underline{k}_{t}^{o} = \frac{\boldsymbol{F}_{t-1}\underline{\boldsymbol{\Psi}}_{t}}{1 + \underline{\boldsymbol{\varphi}}_{t}^{oT}\boldsymbol{P}_{t-1}^{o}\underline{\boldsymbol{\varphi}}_{t}^{o}}, \qquad (18)$$

$$\boldsymbol{P}_{t}^{o} = \boldsymbol{P}_{t-1}^{o} - \underline{k}_{t}^{o} \underline{\boldsymbol{\varphi}}_{t}^{oT} \boldsymbol{P}_{t-1}^{o} .$$
⁽¹⁹⁾

The arbitrary vector $\hat{\underline{\theta}}_0$ (for t = 0) is of size $h = nA^o + nB + nC^o + 1$ and $P_0^o = hI$. Vector of estimates

$$\hat{\underline{\theta}}_{t} = [-\hat{a}_{1,t}, -\hat{a}_{2,t}, \dots, -\hat{a}_{nA^{o},t}, \\
 \hat{b}_{0,t}, \hat{b}_{1,t}, \dots, \hat{b}_{nB^{o},t}, \\
 \hat{c}_{1,t}, \hat{c}_{2,t}, \dots, \hat{c}_{nC^{o},t}]^{T}$$
(20)

is obtained from ELSRO [4] algorithm (16)–(19) (extended least-squares in reducing orders) for overparametrised model.

The ELSRO estimate

$$\frac{\hat{\theta}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}}{\hat{\theta}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}} = [-\hat{a}_{1,t},\,-\hat{a}_{2,t},\,\ldots,\,-\hat{a}_{\hat{n}\hat{A},t},\\ \hat{b}_{0,t},\,\hat{b}_{1,t},\,\ldots,\,\hat{b}_{\hat{n}\hat{B},t},\\ \hat{c}_{1,t},\,\hat{c}_{2,t},\,\ldots,\,\hat{c}_{\hat{n}\hat{C},t}]^{T} \qquad (21)$$

of the vector of plant parameters at time t is given by

$$\frac{\hat{\theta}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}}{\left(\sum_{i=1}^{t}\underline{\varphi}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}\underline{\varphi}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})^{T}} + \frac{1}{h}I\right)^{-1}} \times \sum_{i=1}^{t}\underline{\varphi}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}y_{i},$$
(22)

and

$$\underline{\varphi}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} = [y_{t-1},\,y_{t-2},\,\ldots,\,y_{t-\hat{nA}},\\ u_{t-d},\,u_{t-d-1},\,\ldots,\,u_{t-d-\hat{nB}},\\ \varepsilon_{t-1}^{(\hat{nA},\,\hat{nB},\,\hat{nC})},\,\varepsilon_{t-2}^{(\hat{nA},\,\hat{nB},\,\hat{nC})},\\ \dots,\,\varepsilon_{t-\hat{nC}}^{(\hat{nA},\,\hat{nb},\,\hat{nC})}]^{T}.$$
(23)

The estimate (22) can be recursively computed from the algorithm:

$$\varepsilon_t^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} =$$

$$y_t - \hat{\underline{\theta}}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})^T} \underline{\varphi}_t^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}, t \ge 0, \qquad (24)$$

$$I_t(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})$$

$$\frac{\underline{k}_{t}^{(n,n),(n)},(n)}{1 + \underline{\varphi}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} \underline{P}_{t-1}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} \underline{\varphi}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})}}{1 + \underline{\varphi}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} \underline{P}_{t-1}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} \underline{\varphi}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})}}{\hat{\underline{\theta}}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})}} = \frac{\hat{\underline{\theta}}_{t-1}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} + \underline{k}_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})} \varepsilon_{t}^{(\hat{n}\hat{A},\hat{n}\hat{B},\hat{n}\hat{C})}, \quad (26)$$

$$\boldsymbol{P}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \boldsymbol{P}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} - \\ k_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} \varphi_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})^{T}} \boldsymbol{P}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}, \quad (27)$$

with the initial condition $\hat{\underline{\theta}}_{0}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{nC})}$ as a part of $\hat{\underline{\theta}}_{0}^{o}$ (see (17)) and $\boldsymbol{P}_{0}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = m\boldsymbol{I}$, where $m = \hat{n}\hat{A} + \hat{n}\hat{B} + \hat{n}\hat{B}$ $\hat{nC} + 1 \le h.$

Plant-model mismatching residual

$$\sigma_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \sum_{i=1}^{t} \left(y_{i} - \frac{\hat{\theta}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}^{T} \underline{\varphi}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}}{\underline{\varphi}_{i}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}} \right)^{2} = Y_{t} - 2\underline{\hat{\theta}}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}^{T} \underline{h}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} + \frac{\hat{\theta}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}^{T} N_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} \underline{\hat{\theta}}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}$$
(28)

can be recursively evaluated as

$$\begin{aligned} \sigma_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} &= \sigma_{t-1}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} + \\ \left(y_{t} - \underline{\hat{\theta}}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})}^{T} \underline{\varphi}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} \right)^{2} + \\ + \Delta \underline{\hat{\theta}}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})}^{T} \left(N_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} \Delta \underline{\hat{\theta}}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} - \\ 2\underline{h}_{t}^{(\hat{nA},\,\hat{nB},\,\hat{nC})} \right) , \end{aligned}$$
(29)

with:

$$\Delta \underline{\hat{\theta}}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \underline{\hat{\theta}}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} - \underline{\hat{\theta}}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}, (30)$$
$$\boldsymbol{N}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \boldsymbol{N}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} + \varphi_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}\varphi_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}^{T}, (31)$$

$$\boldsymbol{N}_{0}^{(\hat{n}A,\,\hat{n}B,\,\hat{n}C)} = \boldsymbol{0}, \qquad (32)$$

$$\underline{h}_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \underline{h}_{t-1}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} + \varphi_{t}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}y_{t}, \qquad (33)$$

$$\underline{h}_{0}^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})} = \underline{0}, \qquad (34)$$

$$Y_t = Y_{t-1} + y_t^2, Y_0 = 0,$$

$$\sigma_0^{(\hat{n}\hat{A}, \, \hat{n}\hat{B}, \, \hat{n}\hat{C})} = 0.$$
(35)

As a convention $N_t^{(\hat{n}\hat{A}',\,\hat{n}\hat{B}',\,\hat{n}\hat{C}')}$ and $\underline{h}_t^{(\hat{n}\hat{A}',\,\hat{n}\hat{B}',\,\hat{n}\hat{C}')}$ respectively, submatrix and subvector of are.

 $N_t^{(\hat{nA},\,\hat{nB},\,\hat{nC})}$ and $\underline{h}_t^{(\hat{nA},\,\hat{nB},\,\hat{nC})}$ and their values can be obtained directly from $N_t^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})},\,\underline{h}_t^{(\hat{n}\hat{A},\,\hat{n}\hat{B},\,\hat{n}\hat{C})}$ without additional computations.

Let for $(nA, nB, nC) \in \mathcal{M}$ the information function be given

$$S_{t}(\hat{nA}, \hat{nB}, \hat{nC}, \hat{nA}', \hat{nB}', \hat{nC}') = \sigma_{t}^{(\hat{nA}', \hat{nB}', \hat{nC}')} - \sigma_{t}^{(\hat{nA}, \hat{nB}, \hat{nC})} - \beta_{t}, \qquad (36)$$

where $\beta_n = \alpha (\log(t))^2$ and $\alpha > 0$. If $(\hat{nA}', \hat{nB}', \hat{nC}') \leq (\hat{nA}, \hat{nB}, \hat{nC})$, then the difference between residuals $\sigma_t^{(\hat{nA}', \hat{nB}', \hat{nC}')} - \sigma_t^{(\hat{nA}, \hat{nB}, \hat{nC})} \geq$ 0, that is $S_t(\hat{nA}, \hat{nB}, \hat{nC}, \hat{nA}', \hat{nB}', \hat{nC}') < 0$, thus the orders $(\hat{nA}, \hat{nB}, \hat{nC})$ should be modified to $(\hat{nA}', \hat{nB}', \hat{nC}').$

5.3. Recursive algorithm of simultaneous estimation of orders and parameters

The shortened algorithm for ARMAX model is cited from [5]:

step 0 (initialisation)

set the values of start t = 0 and stop time $t = n_k$ $(n_k > 0);$

step 1

set initial values of appropriate variables;

– step 2

stop if $t \ge t_k$, otherwise substitute t := t + 1; evaluate $\varepsilon_t^{(\hat{nA}_t,\,\hat{nB}_t,\,\hat{nC}_t)}$ and $\underline{\hat{\theta}}_t^{(\hat{nA}_t,\,\hat{nB}_t,\,\hat{nC}_t)}$ for current plant model on the basis of (24)–(27); compute $\sigma_t^{(\hat{n}A, \hat{n}B, \hat{n}C)}$ from (28) on the basis of (29)–(35), with $(nA_t, nB_t,$ nC_t) substituted for (nA, nB, nC);

- step 3

if $S_t(nA_t, nB_t, nC_t, nA_t - 1, nB_t, nC_t) < 0$, then drop the last parameter of polynomial A;

- step 4

if $S_t(\hat{nA}_t, \hat{nB}_t, \hat{nC}_t, \hat{nA}_t, \hat{nB}_t - 1, \hat{nC}_t) < 0$, then then drop the last parameter of polynomial B;

- step 5

if $S_t(\hat{nA}_t, \hat{nB}_t, \hat{nC}_t, \hat{nA}_t, \hat{nB}_t, \hat{nC}_t - 1) < 0$, then then drop the last parameter of polynomial C;

- step 6

go to step 2.

If the initial assumptions hold and initial orders have been set as maximal, then estimates (nA_t, nB_t, nC_t) form monotonically non-increasing trains.

6. Simulation results

6.1. Discrete-time model of the plant

The plant can be described as first-order inertia with discrete-time transfer function

$$\frac{y_t}{u_t} = \frac{16.5q^{-1}}{1 - 0.9q^{-1}},$$
(37)

from which:

$$A(q^{-1}) = 1 - 0.9q^{-1}, \quad B(q^{-1}) = 16.5$$
 (38)

and d = 1.

30

200

+

200

200

250

-+-

250

250

t

t

t

300

300

300



Fig. 1. Simulation I



t

6.2. Simulations

In order to verify the performance of simultaneous order and parameter estimation algorithm, the following simulations have been carried out in discrete-time adaptive control system by stipulating the triplets (nA, nB, nC) or plant orders:

1)
$$(3, 4, 1),$$

$$\underline{\theta}_0 = \begin{bmatrix} -0.5, -0.5, -0.5, -15, 10, -10, 5, -3, 2 \end{bmatrix}^T$$

II) (3, 2, 3),

$$\underline{\theta}_0 = [-0.05, 0.8, 0.3, -15, 10, 1.2, 0.2, 0.2, 0.2]^T$$

with attaching discrete-time transfer function in series with plant

$$\frac{0.4}{1 - 0.6q^{-1}},$$

and resulting transfer function of a additional inertiaplant connection

$$\frac{y_t}{u_t} = \frac{16.5q^{-1}}{1-0.9q^{-1}} \cdot \frac{0.4}{1-0.6q^{-1}} = \frac{6.6q^{-1}}{1-1.5q^{-1}+0.54q^{-2}}.$$
(39)

The reference has the same shape in all simulations and $a_{M1} = -0.908$.

The estimates of $C(q^{-1})$ during computations have chosen the values close to zero (or equal to zero), and as it has already been mentioned, the polynomial has been ignored here.

As it can be seen in Fig. 1 for Simulation I, the algorithm has caused overparametrisation of polynomial B with the last three parameters almost equal to zero. In fact, one can assume that the order or B is zero. The orders of plant model changed their values twice (by this changing the structure of the controller twice).

Having fixed the orders, it has taken a few sample times to find improved estimates' values in the initial stage. Large changes in the reference signal caused the control input to saturate on a certain level, leading to insufficient excitation and poor tracking.

Nevertheless, as it can be seen, the tracking is of satisfactory performance past the adaptation period (say, 50th sample) and the values of parameters do not change.

In the case of Simulation II (Fig. 2), by initially choosing high order values one could obtain acceptable tracking performance in comparable time with respect to the previous simulation, but with appropriate order of B. The structure of the adaptive controller changed three times (model of the plant has changed). As in the previous simulation, the model has been initially inaccurate, what has caused control signals to saturate.

The performance of tracking is the same and with the same prescribed dynamics as in the previous simulation – the pole placement has been performed correctly.

7. Summary

The presented algorithm, as shown in the simulations, allows one to estimate the orders of plant model polynomials in a closed-loop system.

On the basis of numerous simulations and the first simulation included in the paper, it can be said that structural identification is not flawless. A common case was that estimation stopped with orders higher than real orders, leading to overparametrised models, as in Simullation I. Nevertheless, control performance in such a case does not suffer from overparametrisation as much as in the case of underparametrisation which leads to poor tracking.

It has been verified that the controller assures good performance in a case of an unknown plant, assuring full adaptivity features. Since discrete-time system analysis allows one to draw conclusions about behaviour of sampled-data systems, one can expect that a sampled-data pole-placement control for a real plant could have similar performance.

The conclusions drawn from this paper, have been applied to a real-time control of a servo drive with minimumvariance controller (not a subject of this paper) and the same identification algorithms, leading to comparable results in real-world experiment (for reference see [2]).

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