Postural Equilibrium Criteria Concerning Feet Properties for Biped Robots

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Abstract:

This article presents a study on the postural equilibrium conditions for biped robots. Criteria for dynamic walking, such as ZMP and CoP are introduced and their similarities discussed. We also introduce the effects of a compliant foot and take them into consideration during the evaluation of the criteria.

A model of a planar biped is used to imitate the movements of a human subject as recorded by the VICON motion capture system. In order to estimate the criteria, body segments accelerations and ground reaction forces are needed. ZMP and CoP are analyzed during both single and double support phases for the model's motion.

A linear shift function is used to transport the load of the biped between the supporting legs during the double support phase.

We compare simulated CoP trajectories obtained using a rigid foot and a compliant, deformable spring-damper system located between the ankle joint and the sole of the foot. It is seen that the foot's deformation smoothes the CoP trajectory and improves the biped's stability.

Keywords: *biped robots, motion analysis*

1. Introduction

The role of the ankle and toe joints during bipedal locomotion has been discussed for a long time. It has been observed that they are able to store energy at heel strike and release it during push-off with a spring like action [2, 4, 10]. This energy release decreases the total energy consumption for the gait cycle [2]. It is only natural that this advantage is sought to be replicated in both walking robots and prosthetic devices [1, 2, 6, 11].

Two main approaches exist. The first actively reproduces the motion of the ankle, and in some cases of the toes. The second stores energy dissipated at heel-strike, by means of a variety of springs, releasing it at the appropriate time 2.

This second, compliant approach can reduce the overall weight of the foot. It is also able to closely mimic human gait in regards to the distribution of ground reaction forces at key events of the gait cycle [2, 6].

The work presented here focuses on biped locomotion and the calculation of equilibrium criteria ZMP and CoP. These criteria are widely used in the field of walking robots [3, 14, 17].

Following sections will describe the method used for the evaluation of these criteria, and a brief discussion over the obtained results.

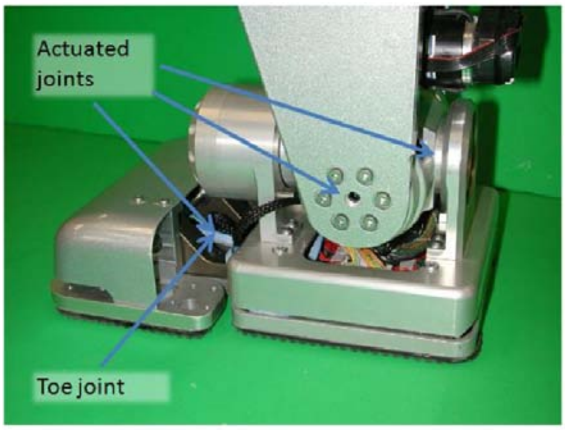
1.1. Robotic Feet

The study of feet in humanoid robotics has an ample field of application. In 2002 the humanoid robot H6 and its successor H7 were fitted with an actuated toe-jointed foot (see Fig. 1a) to increase their range of motion [11]. These articulated feet made H6 capable of kneeling while maintaining contact of the soles with the ground, walk faster and climb up higher steps. Speed was increased from 160 mm·s^{-1} to 270 mm·s^{-1}, while gait cycle characteristics remained unchanged. Also, the increased height of affordable obstacles (such as stairs) required a smaller torque at the ankle when compared to the un-jointed foot.

On the other hand, passive toe-joints have been proposed [1, 6]. They rely on the use of springs-damper systems to support the motion and improve the energy consumption. Compliant limbs have been shown to adjust better to difficult terrains [9].

Robot WABIAN-2R [6] was fitted with a foot capable of recreating the role of a human foot's longitudinal arch

a)



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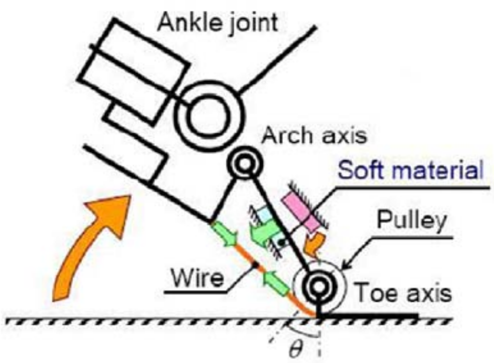


Fig. 1. Robotic feet. a) shows the actuated ankle of the H7 robot [12]; while b) presents an schematic diagram of WABIAN-2R's compliant foot [6]

(see Fig. 1b). WABIAN-2R's foot also has two toes and is divided along its sagital plane; this creates a transversal arch. It has a soft, elastic material limiting the motion of the toe joint and features a steel wire which recreates the function of the plantar aponeurosis ligament[1]. This wire is run from the heel to the toes as shown in Fig. 1b. The pull force created by the wire increases the push-off forces and gives the gait a natural look.

In order to implement suitable control strategies, the feet proposed by Borovac and Slavnic [1] and Nishiwaki *et al.* [11] are equipped with force sensors in the sole for real time calculation of ZMP. Hashimoto *et al.* [6] make no mention of the control strategy used in WABIAN-2R but the effectiveness of their foot in imitating that of a human was validated by comparing ground reaction forces during push-off and overall motion of the foot.

1.2. Biped model

We define a walker as an 8 element linkage (see Fig. 2). The knees are represented by revolute joints, while the joints between hip and thighs and those at the ankles are considered to be spherical. This allows motion both along the motion direction and a direction normal to it.

The foot consists of a spring-damper system, placed perpendicular to the ground. In Fig. 2 the spring-damper system is shown only as a spring, for the sake of readability. No toe joint is considered.

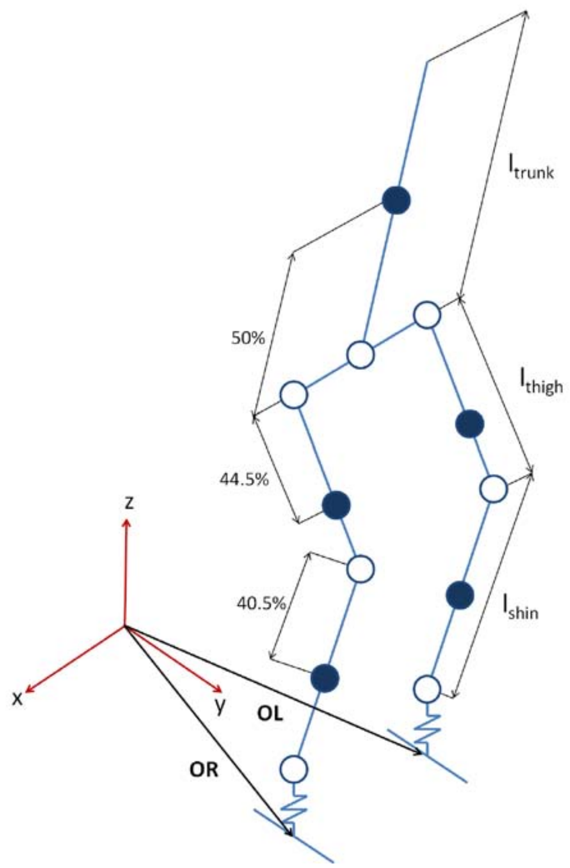


Fig. 2. Biped walker model. A spring is shown in place of a spring-damper system for the sake of readability

Since we will focus on the lower limbs, the upper part of the body is treated as single link with equivalent mass sometimes referred to as HAT (Head And Trunk).

Global reference frame was defined with the y-axis along the motion direction and the z-axis normal to the ground surface pointing upwards.

The set of parameters taken into consideration can be found in Table 1 [8]. It is important to note that the values for moment of inertia (Ji) are presented as measured with the local frame place at the center of mass of the corresponding segment. Local frames attached to the model's links are not shown as their definition is not definitive for the method followed here. For analysis, we consider the subject to fall within the values of a 5 percentile U.S. male crew member.

Limbs will be represented by point masses located at a distance measured from the joint of the link closest to the hip.

Table 1. Geometric and dynamic parameters for the 5% U.S. male crew member. [8]

	Shin	Thigh	Trunk
Length [m]	0.47	0.43	0.80
Mass [kg]	3.30	10.60	47.20
J^x [kgm^2]	4.37e-2	12.25e-2	21.53e-2
J^y [kgm^2]	4.30e-2	11.63e-2	25.56e-2
J^z [kgm^2]	0.51e-2	3.16e-2	107.31e-2

1.3. Stability criteria

1.3.1. Zero Moment Point [ZMP]

It is impossible to mention ZMP without referring to M. Vukobratović's works, such as [17, 18], and their importance in the control of biped walking robots.

ZMP is defined as the point on the ground where the tipping moment due to gravitational, inertial and reaction forces acting on the biped equals zero.

For case of the planar walker, the ZMP coordinates can be determined from equations (1).

$$x_{ZMP} = \frac{\sum_i^n m_i(\ddot{z}_i+g)\cdot x_i - \sum_i^n m_i \ddot{x}_i z_i - \sum_i^n I_i^y \dot{\omega}_i^y}{\sum_i^n m_i(\ddot{z}_i+g)}$$
$$y_{ZMP} = \frac{\sum_i^n m_i(\ddot{z}_i+g)\cdot y_i - \sum_i^n m_i \ddot{y}_i z_i + \sum_i^n I_i^x \dot{\omega}_i^x}{\sum_i^n m_i(\ddot{z}_i+g)} \quad (1)$$

Where:

x_{ZMP}, y_{ZMP} – position of the ZMP along the corresponding axis;

m_i – mass of link *i*;

x_i, y_i, z_i – position of center of mass of link *i* along appropriate axis;

g – acceleration due to gravity ($g = -9.81\ m \cdot s^{-2}$);

I_i^j – moment of inertia of link *i* along axis *j* expressed on the global reference frame;

ω_i^j – angular velocity of link *i* along axis *j*.

1 The *planar aponeurosis* ligament is the main component of the plantar fascia, a group of ligaments and connective tissue located on the sole of the foot. It is capable of maintaining the shape of the longitudinal arch by pulling on its ends, resembling a taut bow.

These use the notation found in [19] (interested readers may refer to [cite{Sardain}, cite{Vukobratovic2004} for further information) and hold for both single and double support phase.

Owing to its definition, the coordinate along z-axis belongs in the ground plane.

1.3.2. Center of Pressure [CoP]

CoP is a point in the ground surface. It is located in such a way that the sum of moments caused by ground reaction forces is equal to zero.

In other words; CoP is defined as a function of the measured ground reaction forces, while ZMP is determined *via* the kinematic behavior of the walker.

It can be proved that these two points, though differently defined, are in fact equivalent [14]. As such, they can be used somewhat interchangeably.

CoP position can be found by means of (2) which are modified from the work of Sardain and Bessonnet [14] and are similar to those presented by Schepers *et al.* [15].

$$M = OR \times F_r + OL \times F_l$$
$$CoP = \frac{n \times M}{n \cdot (F_r + F_l)}$$

Where:

F_i – pressure force on the corresponding foot (l for left and r for right);

n – vector normal to the ground surface (for as model defined as in Fig. 2, $n=[0\ \ 0\ \ 1]^T$)

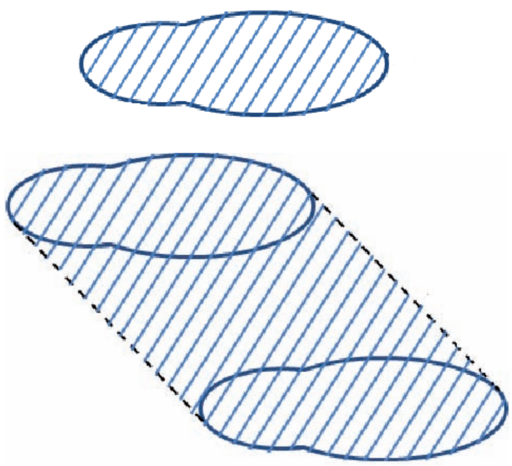
OR, OL – distance from the frame's origin to the application point of the corresponding pressure force (see Fig. 2).

Relation (2) will hold for both single and double support phases; and with an appropriate choice of vector n it does not restrict the ground to a horizontal surface.

1.3.3. Usage of stability criteria

While using a ZMP/COP based control, the corresponding point is required to stay inside the supporting polygon in order for the gait to be considered stable [3, 17]. During single support phase, this support polygon is limited by the footprint (see Fig. 3a). For the double support phase, it can be considered to span the area connecting both feet (see Fig. 3b).

a)



b)

Fig. 3. Support polygon for a) single support and b) double support phase

Knowing the shape of the polygon is useful for generating a ZMP/CoP based gait. Erbatur *et al*. [3] prescribe the desired location of ZMP point to be located in this polygon and determine the motion of the legs in order to realize it.

2. Reconstruction of the gait and equilibrium conditions.

A human subject was recorded while walking, by means of a VICON motion capture system. After processing, the angles between body segments (hip, knees and shins) are stored and used to analyze the motion. The procedures used are to be detailed in the following.

2.1. Kinematics

To reconstruct the individual's gait, we must determine the biped's pose at each time instant.

For each set of measured angles, we think of the biped's structure as a serial linkage fixed at the hip. This will allow us to determine the position of the trunk, knees and ankles in the hip's frame. Motion is then recreated by placing correctly the supporting foot/feet in the global reference frame.

Joint positions at each time instant are calculated and stored. In order to make the supporting foot appear stationary, it is necessary to offset the position of the model for each time instant. This offset is represented by the vector d_{shift} (see Fig. 4) and may be used to set an initial condition for the walk.

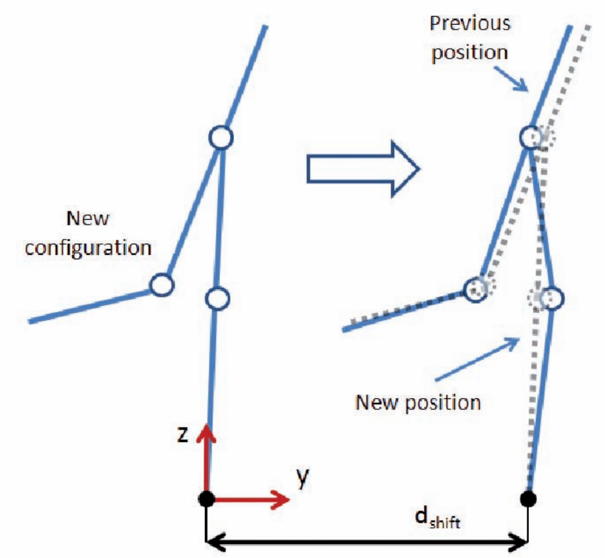


Fig. 4. Shifting of the biped to give the impression of motion

After finding the trajectory of each body segment, the location of the walker's center of mass (P_{CM}) may be calculated as given by equation (3).

$$P_{CM} = \frac{\Sigma m_i P_{CMi}}{\Sigma m_i}$$

Where:

P_{CMi} – corresponds to the global position link's i center of mass;

P_{CM} – is the position of the biped's center of mass.

In order to find the ZMP of the biped, as defined by (1), we must first find the angular velocity and acceleration of its links, as well as the linear acceleration of their centers of mass. Numerical differentiation can be used to find these values.

Determining reaction forces during single support offers no difficulty. On the contrary, during the double support phase we find the system does not have a unique solution. In order to obtain one, we assume the forces experienced are transferred between the supporting legs by means of a *shift function* [7, 16] denoted by f. In order to do this, the time instants for the beginning and ending of the single support phases must be known.

A function f_i is created with a value of one for the single support phase, and a value of zero during transfer. Transition between both states is taken to be linear and takes place during the double support phase. Different shift functions have been proposed. In her work, Ruiz Garate [16] points to the work of Lei Ren *et al.* [13], who use an exponential function for the analysis of 3D data with good results in the sagital plane. A linear function (4) as used here yields good results while remaining simple to implement.

$$f_i = \begin{cases} 0 & \textit{during swing} & t_{pi} < t <= t_{hj} \\ \dfrac{t - t_{hi}}{t_{pj} - t_{hi}} & \textit{after heel-strike of } i & t_{hi} < t <= t_{pj} \\ 1 & \textit{double support} & t_{pj} < t <= t_{hj} \\ \dfrac{t - t_{hj}}{t_{hj} - t_{pi+1}} & \textit{after heel-strike of } j & t_{hj} < t <= t_{pi+1} \end{cases} \quad (4)$$

Function f_i is defined for the left and right legs. This function changes between 0 and 1 during the double support phase. Here t_h and t_p refer to the time instant of heel-strike and push-off for the corresponding leg; subindex i refers to the first supporting leg, j to the other one during one complete gait cycle (see Fig. 5).

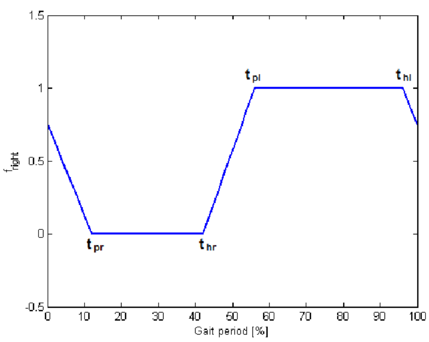


Fig. 5. Shift function defined for transfer of ground reaction forces during walk

Following this, ground reaction forces on each leg are given by relation (5).

$$F_r = -f_r \cdot F_{CM}$$
$$F_l = -f_l \cdot F_{CM} \quad (5)$$

2.2. Ground reaction force's point of application

CoP is the point where the sum of moments caused by the ground reaction force's vectors is equal to zero. With this in mind, it is necessary to locate the point of application of the reaction forces along the sole of the foot (we denote it here as 'C').

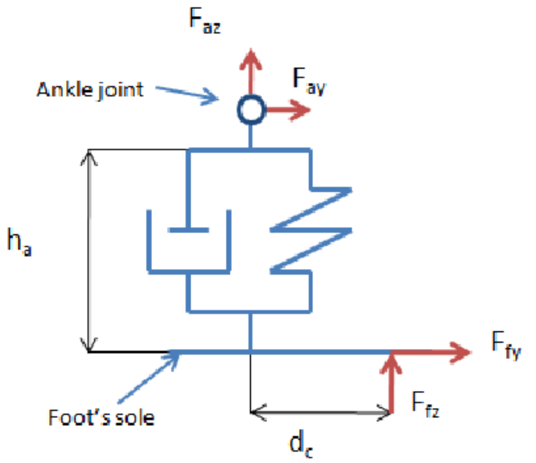


Fig. 6. Compliant foot model with spring-damper system

In an analogous manner, we propose to find point 'C' where the sum of moments caused by the forces acting on the foot is equal to zero (see Fig. 6). When assuming a foot with no mass, the location of point 'C', measured from the ankle (d_c) is given by (6).

$$d_c = -\frac{F_{ay}}{F_{az}} h_a \quad (6)$$

2.3. Compliant foot model

We propose to model the foot's compliance by means of a spring-damper system located between the ankle and the sole (see Fig. 6). We assume that deformation will occur only along the *z-axis* and that this segment is always perpendicular to the ground.

The walker's motion may then be modeled as shown in Fig. 7. In order to solve the ordinary differential equa-

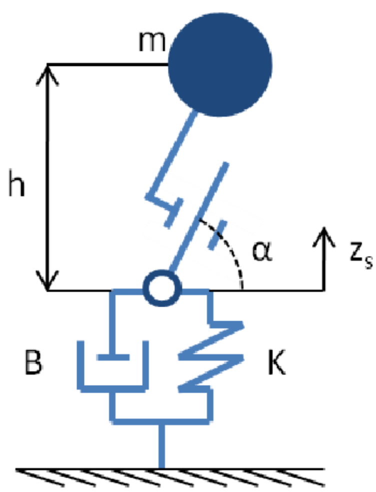


Fig. 7. Spring-damper simplified model

tion (7) and find the deformation of the spring element, we take the vertical acceleration of the walker's center of mass as input to the system. With the vertical position of the center of mass being denoted as h; we find its acceleration to be $\ddot{h}$.

Relation (7) is the classic definition of the spring-damper system, where $z_s = 0$ corresponds to the equilibrium position.

According to Geyer *et al.* a stable walking gait may be obtained with a spring constant ('K') close to 18 kN·m^{-1}. This value nears the highest stiffness registered for running [5] and was used here for calculations.

The value of the damper constant ('B') is taken to be 100 N·s·m^{-1}. This value was chosen in order to see the effect of the spring damper system while offering a natural-looking gait.

$$\begin{bmatrix} \dot{z}_s \\ \ddot{z}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/m & -B/m \end{bmatrix} \begin{bmatrix} z_s \\ \dot{z}_s \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \ddot{h} \tag{7}$$

In order to find the deformation in each foot (z_r and z_l) during double support phase, we propose to make use of the *shift function* introduced in Section 2.2 and the resulting deformation (z_s), as obtained by solving (7). The largest deformation should be measured on the spring bearing the highest load. For this we can suggest relations (8).

$$\begin{aligned} z_r &= f_r \cdot z_s \\ z_l &= f_l \cdot z_s \end{aligned} \tag{8}$$

3. Results

After reconstructing the gait in simulation, we compare the position of point 'C' (application point of ground reaction forces) in the reference frame of the supporting foot (with its origin under the ankle, as given in Fig. 6).

Fig. 8 shows the location of point 'C' in the supporting foot's local frame; the solid line represents double support phase, while the dashed line indicates single support. When no deformation is allowed, we see large

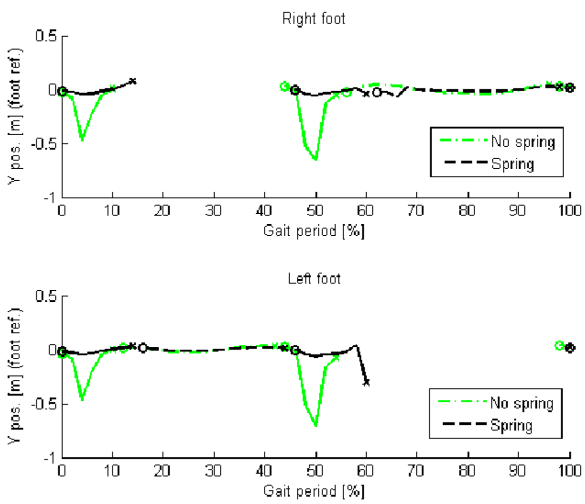


Fig. 8. Location of point 'C' expressed in the supporting foot's frame

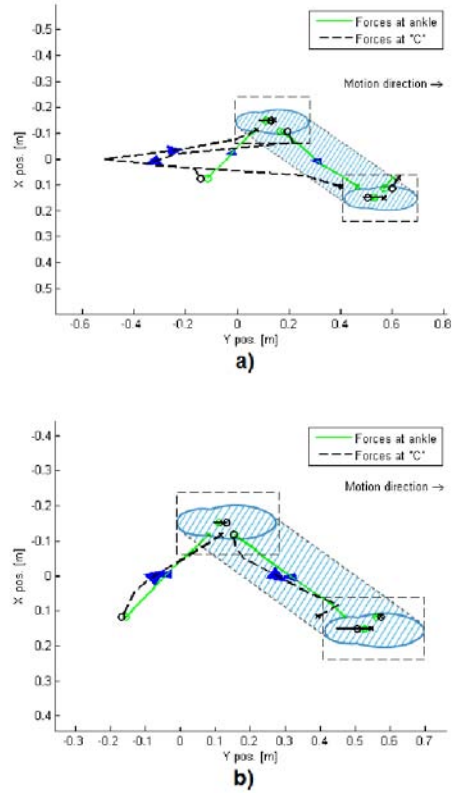


Fig. 9. Spatial motion of the CoP. a) shows a stiff foot, while b) shows the effect of the spring damper-system

peaks for the location of 'C'. This is due to the changes in forward velocity of the walker's center of mass when entering double support phase. In order to respect the equilibrium condition set by (6); 'C' must sometimes be located outside of the sole of the foot. This is of course, not desirable.

After allowing for deformation of the foot structure the moments caused by the forces at the ankle are diminished and 'C' remains closer to the ankle (see Fig. 8). Although not shown here, it is interesting to note that large vertical accelerations of the biped's center of mass ($\ddot{P}_{CM}$) are registered at these time instants. This is brought about by the change of trajectory followed by the center of mass, as indicated in the traditional inverted pendulum model for biped walking gait.

Fig. 9 shows the trajectory of the CoP point, projected on the ground plane. As a reference for both the non compliant and compliant foot (Fig. 9a and Fig. 9b respectively) we use the CoP as calculated for a single point contact located under the ankle for the corresponding foot (shown with a solid line). CoP point moves abruptly during the double support phase while using an un-deformable foot structure. This motion corresponds to the peaks observed in the position of 'C'.

After modeling the foot as a compliant element, we observe a smother trajectory; closer to that of the straight line, which would be expected under the assumed conditions.

By looking at the behavior of 'C' and its effect on the CoP criteria, we note that CoP travels to the heel of the foot before changing the support leg (see Fig. 9); in the same way as 'C' does (see Fig. 8).

4. Future work

Future work should focus on: i) the analysis of the evolution of CoP for different motion situations; ii) testing the proposed method by comparing CoP as obtained using direct force measurement; iii) studying the effect of toe joints and different spring-damper parameters whose respective actuation and values are linked to the desired motion and environment.

These investigations may lead to a better estimation of ground reaction forces, such as those obtained by Geyer *et al*. [5] and Ruiz Gárate [16]. They may also redefine the duration of the single and double support phases for the proposed model.

5. Conclusions

Criteria for stability evaluation, as given by Vukobratović [17, 18], were presented and evaluated for the single and double support phases. It is seen that for the reconstructions presented here, a compliant foot structure is preferred; since it keeps the ZMP/CoP within the support polygon (see Fig. 3 and Fig. 9). Such a compliant structure is proposed to be modeled by a spring-damper system with viscous friction.

Due to the characteristics of the biped model used here, the compliancy experienced on the foot may also be thought of as belonging to the ground.

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