STABILITY OF MOTION OF THE CREWED AUTONOMOUS UNDERWATER VEHICLE

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Abstract

In this article I investigate the issues related to the motion stability of a crewed autonomous underwater vehicle of the 'wet' type. In vehicles of this kind, the necessity to perform decompression imposes certain requirements relating to the parameters of the motion. In particular it requires maintaining a constant depth regardless of the operating conditions. For this reason, the investigation is limited to an analysis of a two-dimensional motion within the vertical plane. The analysis of motion stability is based on the classic motion equations. In formulating the relations of forces, along with the hydrostatic influences, viscosity and inertia of the water environment are considered. Forces relating to the viscosity are examined through the resistance coefficients. The specific calculations draw from the results of model tests. Forces related to the inertia of the environment are accounted for considered through the water added-mass coefficients. In order to assess the quality of motion, a coefficient of damping and a degree of stability are used as values indicating the character of the results of the dynamic motion equations. The results of the example calculations are presented in the form of diagrams, complete with the lines representing the constant values of the above indicators of the motion stability for different structural solutions under various conditions in which the vehicle is operated.

Keywords: *Fluid Mechanics, ship hydrodynamics, stability of motion, underwater vehicle*

1. Introduction

We are going to look at the manoeuvrability of the crewed autonomous underwater vehicle of the 'wet' type. By the term manoeuvrability we understand the ability of the vehicle to move along the trajectory set by the helmsman. It is therefore a general term referring to the vehicle's ability to perform rectilinear motion as well as a full range of manoeuvres and turns. Depending on the required trajectory the characteristics of motion stability, and of agility may by isolated. The motion stability indicates the underwater vehicle's ability to maintain a rectilinear motion without the helmsman's intervention. The agility of an underwater vehicle refers to the ability of the vehicle to change direction by any angle at the shortest possible distance and with the slightest possible rudder movement.

From the above definitions it may seem that the characteristics of the motion stability and the vehicle agility are partially contradictory. Both of these characteristics are crucial in the 'wet' type underwater vehicle, which is not equipped with a watertight hull and where the crew is exposed to the direct impact of the underwater environment. The necessity to retain a constant cruising depth when a frogman is undergoing decompression, while maintaining the ability to perform rapid turns due to the limited visibility under water, makes manoeuvrability of an underwater vehicle one of its most important operating attributes.

The examination of the manoeuvrability of any object moving through fluid in fact comes down to dealing with the following issues: determining forces acting on the vehicle moving in fluid, forming equations for the motion, and testing results of these equations in order to identify the dynamic characteristics of the vehicle moving in fluid. The methodology generated for determining the forces acting on surface ships, however, cannot be directly applied to underwater vehicles. This is due to the characteristics of the environment in which they are operated, for example, the impact of the free water surface on the added-mass coefficient. In the light of the above contentions this paper presents an analysis of the forces acting on the underwater vehicle, formulates motion equations, and assesses the impact of selected structural parameters on the motion stability.

2. Coordinate systems and motion equations of an underwater vehicle

The position of an underwater vehicle in the water space is examined in reference to the inertial coordinate system X_g Y_g Z_g with its origin at the centre of gravity of the vehicle at the moment t=0. Additionally, two coordinate systems are introduced: the coordinate system X_1 Y_1 Z_1 bounded to the velocity vector of the vehicle's centre of gravity, and the mobile coordinate system rigidly bounded to the vehicle X Y Z. We are going to analyse the two-dimensional motion within the vertical plane overlapping with the diametral plane of the vehicle, assuming that the deflection of a rudder is not causing any change in the angle of roll or a change in the angle of yaw. Under such conditions the positioning of the vehicle in motion is characterised by the following parameters: the coordinates of the centre of gravity G, the angle of pitch φ, the angle of attack α, the trajectory angle γ, and the radius of rotation in the vertical plane R. The positioning of the depth rudders is described by the deflection angle δ.

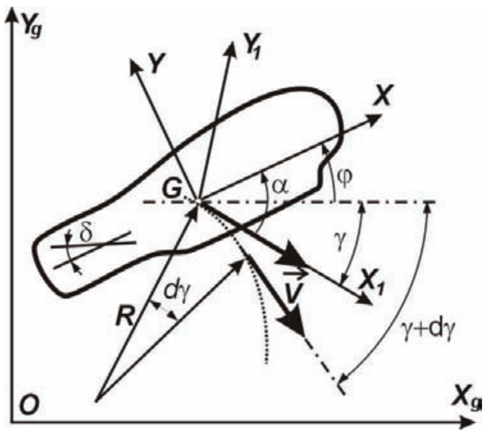


Fig. 1. Coordinate systems

Among these six parameters only three are independent. Assumed as independent are V, α, and φ interchangable with ω, the angular velocity understood as $\omega = \frac{da}{dt} + \frac{V}{R}$.

It is convenient to analyse the dynamics of a vehicle within the bounded coordinate system XYZ. Limiting the analysis to the two-dimensional motion we have:

$$m\frac{dV_x}{dt} - mV_y\omega_z = P_x,$$

$$m\frac{dV_y}{dt} - mV_x\omega_z = P_y. \tag{1}$$

$$I_z\frac{d\omega_z}{dt} - M_z,$$

where:

m - the mass of a vehicle combined with the mass of water filling the vehicle after submersion,

I_z - mass moment of inertia about the GZ-axis perpendicular to the motion plane.

Components of force P_x, P_y and the value of moment M_z are described by the following relations:

$$P_x = \tau_{1_1}\frac{dV_x}{dt} + \tau_{2_2}V_y\omega_z + \frac{1}{2}c_x\rho V^2 L_k B_k + T,$$
$$P_y = \tau_{2_2}\frac{dV_y}{dt} + \tau_{1_1}V_x\omega_z + \frac{1}{2}c_y\rho V^2 L_k B_k\,, \tag{2}$$
$$M_z = \tau_{6_6}\frac{d\omega_z}{dt} + \left(\tau_{1_1} - \tau_{2_2}\right)V_x V_y + \frac{1}{2}m_z\rho V^2 L_k^2 B_k + D\sin\phi h - Ta_y\,,$$

where:

τ_{ij} - water added-mass coefficients,

$c_x\, c_y\, m_z$ - resistance coefficients, lift (force), and the pitch moment,

D - hydrostatic force,

T - propeller thrust,

H - metacentric height,

a_y - distance from the vehicle's centre of gravity to the line of the propeller rod.

In general, the hydrodynamic coefficients are dependent upon the angel of attack, angular velocity of the vehicle, the deflection angle of the rudder, and the Froude number and the Reynolds number. However, the shape of a given vehicle changes, in the hydromechanics sense, with operation, only as a result of the deflection angle of the rudder. With the assumption that the free water surface has no impact, the characteristics of the fluid movement around the vehicle, is sufficiently described by the Reynolds number. If in addition we take into consideration the fact that during operation of the vehicle the value of its angular velocity is small, that the attack angles assume small values, and finally that the deflection of the rudder is structurally limited and rarely exceeds 0.25 rad, we can represent the functions describing the hydromechanics coefficients in the Taylor series leaving to further analysis only the terms of the first order. Therefore:

$$c_y = c_{yo} + c_y^a a + c_y^\delta \delta + c_y^\omega \omega\,, \tag{3}$$
$$m_z = m_{zo} + m_z^a a + m_z^\delta \delta + m_z^\omega \omega,$$

where:

$c_y^a, c_y^\delta, c_y^\omega, m_z^a, m_z^\delta, m_z^\omega$ - partial derivatives of the hydromechanics coefficients in relation to the attack angle, the deflection angle of the rudder, and the angular velocity of the vehicle.

In analysing the motion stability of an underwater vehicle we are going to measure the changes to the motion parameters in relation to their values in a time-independent motion. To this aim each of the motion parameters will be treated as a sum of their value in the time-independent motion, and a given change:

$$V = V_o + \Delta V\,,$$
$$\alpha = \alpha_o + \Delta\alpha\,, \tag{4}$$
$$\phi = \phi_o + \Delta\phi,$$
$$\omega = \omega_o + \Delta\omega.$$

Assuming that the changes of the motion parameters caused by chance occurrences have small values, it is possible to linearise the motion equations by omitting the non-linear terms as small values of the second and further orders. Simultaneously examining the motion of the underwater vessle with small angles of attack (less than 0.13 rad) we can assume $V_x = V$ and $V_y = -V\alpha$. Hence:

$$\frac{dV_x}{dt} = \frac{dV}{dt}\,, \tag{5}$$
$$\frac{dV_y}{dt} = \frac{dV}{dt}a - \frac{da}{dt}V.$$

With such assumptions the motion equations will take the form of:

$$n_1 \frac{d\Delta V_y}{dt} + n_2 V_o a_o \Delta\omega = T_o^v \Delta V - c_{xo} \rho V_o L_k B_k \Delta V,$$

$$- n_2 a_o \frac{d\Delta V}{dt} - n_1 V_o \frac{d\Delta a}{dt} + n_1 V \Delta\omega =$$

$$0{,}5 c_y^a \rho V_o^2 L_k B_k \Delta a + c_{yo} \rho V_o L_k B_k \Delta V + 0{,}5 c_y^\omega \rho V_o^2 L_k B_k \Delta\omega + 0{,}5 c_y^\delta \rho V_o^2 L_k B_k \Delta\delta, \qquad (6)$$

$$n_3 \frac{d\Delta\omega}{dt} = 0{,}5 m_z^a \rho V_o^2 L_k^2 B_k \Delta a - T_o^v a_y \Delta V + m_{zo} \rho V_o L_k^2 B_k \Delta V - D_o h \cos\varphi \Delta\varphi +$$

$$+ 0{,}5 m_z^\omega \rho V_o^2 L_k^2 B_k \Delta\omega + 0{,}5 m_z^\delta \rho V_o^2 L_k^2 B_k \Delta\delta,$$

where:

$$n_1 = m + \tau_{1_1},$$

$$n_2 = m + \tau_{2_2},$$

$$n_3 = m + \tau_{3_3}.$$

In the above equations the elements referring to the time-independent motion were omitted. It means that they describe the motion which has been changed with respect to the time-independent motion. Safety considerations of the underwater vehicle operation and the need for a frogman to undergo decompression speak to the rationale of analysing the character of the changed motion in relation to the time-independent motion at a constant depth.

Under actual conditions, there are chance external forces caused by the change in the operating conditions acting upon the underwater vehicle. The experience suggests that the impact of the angular value changes on the translational velocity is negligible. The assumption that the velocity of the vehicle does not change as a result of the interfering forces, means that the thrust force should be treated as constant as well. The solution of the first equations with the above assumption gives:

$$\Delta V^\bullet = C \exp\left(\frac{2c_{xo}}{m_1^\bullet}\right)\tau, \qquad (7)$$

where:

$\Delta V^\bullet \frac{\Delta V}{\Delta_o}$ - relative increase of the linear velocity,

$m_1^\bullet = \frac{2(m + \tau_{1_1})}{\rho L_k^2 B_k}$ - non-dimensional mass of the vehicle together with the water added-mass coefficients,

$\tau = \frac{tV_o}{L_k}$ - non-dimensional time.

This means that the change of the velocity always diminishes according to the exponential function. If we insert this solution into the remaining motion equations we will therefore see that the formulas dependent on the relative increase of velocity work in relation to the changes of the Δα and Δφ parameters, just like certain external forces which diminish with time. We end up with:

$$\frac{d\Delta a}{d\tau} + B_{2_1} \Delta a + C_{2_1} \frac{d\Delta\varphi}{d\tau} = -D_{2_1} \Delta\delta,$$

$$\frac{d^2\Delta\varphi}{d\tau^2} + C_{3_1} \frac{d\Delta\varphi}{d\tau} + E_{3_1} \Delta\varphi + B_{3_1} \Delta a = -D_{3_1} \Delta\delta, \qquad (8)$$

where:

$$B_{2_1} = \frac{c_y^a \rho L_k^2 B_k}{2\left(m + \tau_{2_2}\right)},$$

$$C_{2_1} = \frac{c_y^\omega \rho L_k^2 B_k}{2\left(m + \tau_{2_2}\right)} - \frac{\left(m + \tau_{1_1}\right)}{\left(m + \tau_{2_2}\right)},$$

$$D_{2_1} = \frac{c_y^\delta \rho L_k^2 B_k}{2\left(m + \tau_{2_2}\right)},$$

$$B_{3_1} = \frac{m_z^a \rho L_k^4 B_k}{2\left(I + \tau_{6_6}\right)},$$

$$C_{3_1} = \frac{m_z^\omega \rho L_k^4 B_k}{2\left(I + \tau_{6_6}\right)},$$

$$D_{3_1} = \frac{m_z^\delta \rho L_k^4 B_k}{2\left(I + \tau_{6_6}\right)},$$

$$E_{3_1} = \frac{D_o h \cos\varphi_o L_k^2 B_k}{V_o^2\left(I + \tau_{6_6}\right)}.$$

3. Motion stability of an underwater vehicle

The change in the angular acceleration of the motion parameters of an underwater vehicle is described by the above equations of the altered motion. The character of such kinds of solutions is contingent upon the roots of a characteristic equation, which in this case is a polynomial of the third degree.

$$\begin{aligned} &r^3 + Ar^2 + Br + C = 0, \\ &A = B_{2_1} + C_{3_1}, \\ &B = C_{3_1} B_{2_1} - C_{2_1} B_{3_1} + E_{3_1}, \\ &C = E_{3_1} B_{2_1}, \end{aligned} \tag{9}$$

where:

r - complex number resulting from the Laplace transform.

The coefficients of the characteristic equation are functions of the underwater vehicle construction. The coefficients of the water added-mass coefficients as well as the derivatives of the hydrodynamic coefficients depend on the hull shape, and on the surface and the positioning of stabilisers. In order to measure the impact of these values on the character of the motion, appropriate function relations have been constructed. For the hydrodynamic coefficients the relations stemming from the theory of the small extension wings have been used:

$$c_y^a = \frac{\pi\varepsilon_k}{2} + \frac{\pi\varepsilon_s}{2}, \tag{10}$$

$$c_y^{\omega\bullet} = \frac{\pi\varepsilon_k}{4} + \frac{\pi\varepsilon_s}{4},$$

$$m_z^a = \frac{\pi\varepsilon_k}{4} + \frac{\pi\varepsilon_s}{2} x_s^\bullet,$$

$$m_z^{\omega\bullet} = \frac{\pi\varepsilon_k}{8} + \frac{\pi\varepsilon_s}{4} x_s^\bullet,$$

where:

ε_k, ε_s - extensions of the wings equivalent to the hull and the horizontal stabilisers,

$x_s^\bullet$ - relative distance between the stabiliser and the vehicle's centre of gravity in reference to the hull length.

The water added-mass coefficients were determined by treating the hull as an ellipsoid of revolution. In the case of the horizontal stabilisers the cross sections hypothesis was used. Assuming that the mass of the vehicle equals mass of water contained in the volume of the equivalent ellipsoid, which is justified under conditions of zero buoyancy, we have:

$$m_1^\bullet = \frac{\pi}{3}\left(1 + K_{1_1}\right)\varepsilon_k \ , \tag{11}$$

$$m_2^\bullet = \frac{\pi}{3}\left[1 + \left(K_{2_2}^k + K_{2_2}^s\right)\right]\varepsilon_k \, ,$$

$$m_3^\bullet = \frac{\pi}{3} i^{\bullet 2}\left[1 + \left(K_{6_6}^k + K_{2_2}^s\right)x_s^{\bullet 2}\right]\varepsilon_k \, ,$$

where:

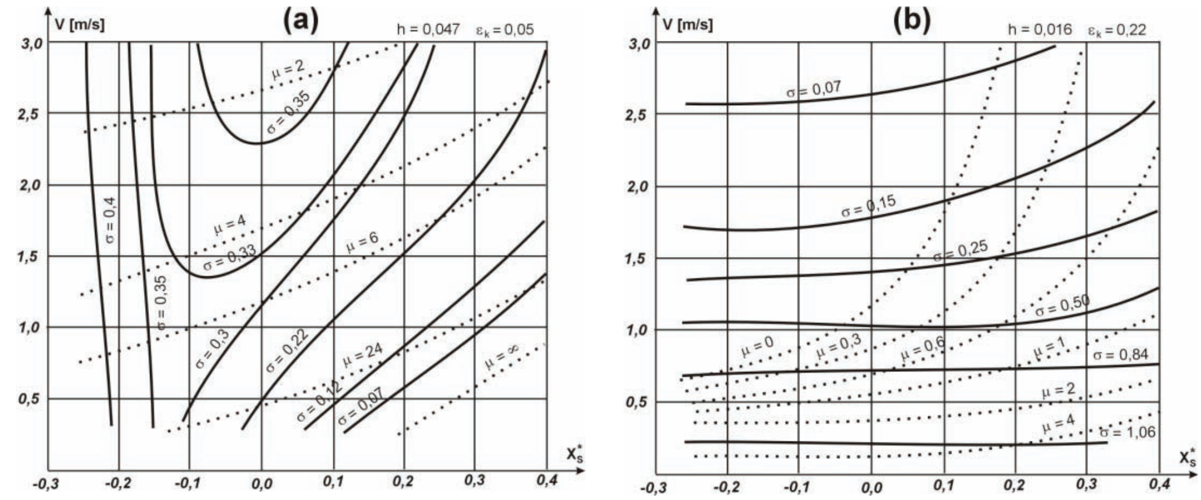
$K_{1_1}, K_{2_2}^k, K_{6_6}^k, K_{2_2}^s$ - non-dimensional coefficients of the water added-mas coefficients in relation to the mass (of moment of inertia) of the underwater vehicle,

$i^\bullet$ - non-dimensional radius of inertia in relation to the vehicle's hull length.

In order to perform a quantitative assessment of the quality of the altered motion we are going to introduce the concepts of the degree of stability σ and of the coefficient of damping μ.
We consider the degree of stability to be the shortest distance of the real root or a pair of complex roots from the imaginary axis. The coefficient of damping is determined by selecting the highest quotient of the imaginary to the real part from among all quotients calculated for the complex roots.

The differential equations theory stipulates that the term described by the smallest root has the highest value and is damped slower than others. Hence, the dominant component in the results will be the one described by the degree of stability. Similarly, in a case where there are complex roots, the component described by the coefficient of damping will have the highest oscillation.

The results of the example calculations are shown in the diagrams, along with lines representing the constant values of the degree of stability and those of the coefficients of damping in the function of parameters characterising the vehicle structure and the conditions of its operation.



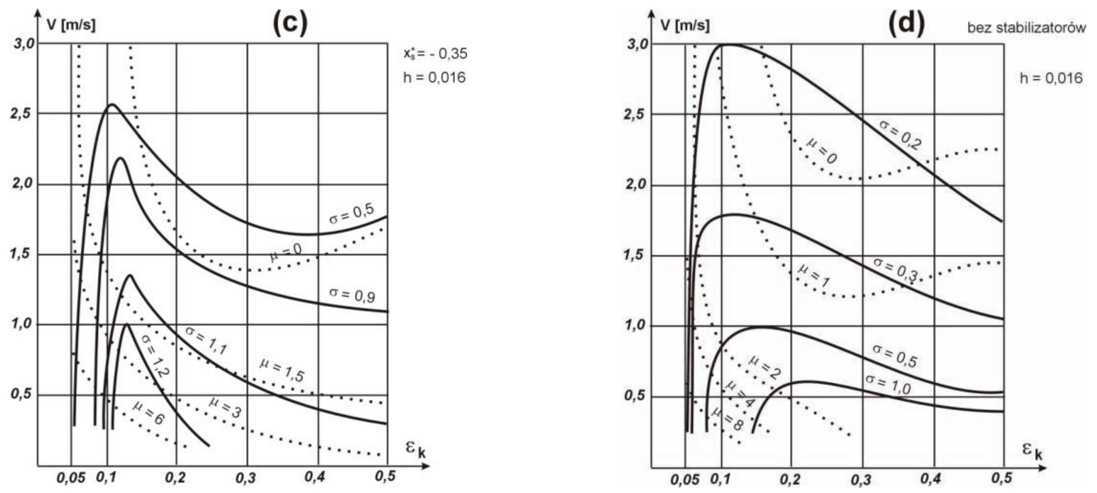


Fig. 2. Changes of the coefficient of damping and the degree of stability

Within the scope under investigation it is possible to specify four sub-fields of the arrangement of the characteristic equation roots. The first of the sub-fields is where one of the roots or the real part of the complex roots assumes a positive value. In this sub-field the motion of a vehicle is unstable. This is shown in the diagrams on the right of the line $\mu=\infty$ and $\sigma=0$, which corresponds to the range of low velocities and shifting of the stabilisers towards the bow.

In the sub-field where all roots are real and negative, the increases of the motion parameters change aperiodically. This sub-field of monotonous changes of the motion parameters corresponds to the range of high operation velocities with the stabilisers located on stern.

The remaining two sub-fields are characterised by the occurrence of pairs of complex roots in the results. It means, that the changes of motion parameters will be oscillating. Wherein, if from among all of the roots, the one nearest to the imaginary axis is a real root, then the dominant component of the results with be the aperiodic component. In the forth sub-field, the degree of stability results from the pair of complex roots. It is then the oscillating components that dominate, which may prove highly adverse for the frogman due to the rapid changes in the depth of submersion.

4. Conclusions

The analysis of changes of the coefficient of damping and of the degree of stability caused by a change of specified operational and structural values, allows for a straightforward assessment of the vehicle's dynamics as early as at the designing stage.

It is however obvious that the results obtained on the basis of examination of the roots of the characteristic equation will be correct only as long as the motion equations are expressing the real relations between the dynamic parameters. Because the roots of the characteristic equation do not overtly describe changes of the motion parameters, it is possible to assess the correctness of the assumed model through the classic method of solving the motion equations and a subsequent comparison of these results with the results of practical experiments. However, this is a lengthy and costly process. Therefore, it seems that one reasonable solution could be conducting comprehensive research only once (for a given type of underwater vehicle), while using the discussed indicators of the motion stability in design practice.

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