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Effects of warmness and spatial nonuniformity of plasma waveguide on periodic absolute parametric instability

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Abstract. The periodic absolute parametric instability (API) of the low-frequency oscillations excited by a monochromatic pumping field of an arbitrary amplitude in a warm 1-D (one-dimensional) nonuniform magnetoactive plasma is investigated. The separation method can be used for solving the two-fluid plasma equations describing the system. By applying this method we were able to determine the frequencies and growth rates of unstable modes and the self-consistent electric field. Plasma electrons are considered to have a thermal velocity. Different solutions for the spatial equation can be obtained the following cases: A) API in a uniform plasma, B) API in a nonuniform plasma. The latter has been studied here for two cases: B.1) the exact harmonic oscillator and B.2) the bounded harmonic oscillator (a bounded plasma). An increment has been found in the build-up of the oscillations, and it has been shown that the spatial nonuniformity of the plasma exerts the stabilizing effect on the parametric instability. A reduced growth rate of API in the warm plasma, in comparison to the cold plasma, is reported. It has also been found that the warmness of the plasma has no effect on the solution of the space part of the problem (only through the separation constant).

Key words: periodic absolute parametric instability (API) • low frequency waves • warm plasma • growth rate • dispersion equation

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Introduction

The separation method was modified [9] to investigate the API in a magnetoactive nonuniform plasma by a monochromatic high frequency (HF) electric field of an arbitrary amplitude. The method was applied to solve different problems [13, 15, 16]: (1) the stabilization effect of a strong HF electric field on a two-stream (Buneman) instability in plane and cylindrical plasma waveguides, as discussed, (2) an analysis of the effect of spatial plasma nonuniformity on the parametric instability of electrostatic waves in a plasma waveguides subjected to an intense HF electric field, as performed, and (3) an analytical expression of the reflection coefficient for the electrostatic wave propagating along a nonuniform plasma slab immersed into the high-amplitude HF electric field, as presented. It has been shown [2] that the dispersion equation describing a parametric excitation of surface waves at the isotropic plasma boundary (vacuum) within the eigen frequency renormalization coincides with the equations that determine the parametric excitation of volumetric waves in a uniform unbounded plasma. Following this conclusion, a method for investigating the parametric interaction of the external HF electrical field with electrostatic oscillations in an isotropic bounded nonuniform plasma was proposed [5, 8]. The method facilitates separation of the problem into two parts. The "dynamical" part describes the parametric build-up of oscillations and the corresponding equations within the renormalization of eigen frequencies coincides with equations for the parametrically unstable waves in an infinite uniform plasma. Natural frequencies of oscillations and spatial distribution of the amplitude of the self-consistent electrical field are determined from the solution of a boundary-value problem (the "space" part) by taking into account the specific spatial distribution of the plasma density. The proposed approach ("the separation method") [11] is significantly simpler than the standard method employed in the theory of the parametric resonance in a nonuniform plasma [12]. Therefore, it is of special interest to apply the separation method to solving different problems which involve a parametric excitation of electrostatic waves in a bounded nonuniform plasma. Demchenko *et al.* [4] reported the analysis of the effect of the spatial plasma nonuniformity on the parametric instability of electrostatic waves in a magnetized cylindrical waveguides subjected to an intense HF electric field. It has been already known that nonuniformity of plasma density leads to: (i) an increase of the threshold value of the pump wave amplitude above which a parametric amplification occurs, and (ii) to the localization of an unstable waves in a finite region of plasma. This suggests that an absolute character of the instability had been assumed. It should be emphasized that from the experimental point of view, it is vital to know whether a given parametric instability is absolute or convective. This is so essential because the nature of the parametric instability determines the mechanism of its saturation. The convective instability reaches saturation at a comparatively low level, due to the convection of energy of the decay product (secondary waves) away from the three wave resonance region. The absolute instability saturates at a higher level under the action of various nonlinear effects. From this point of view the absolute parametric instability [7] plays a crucial role in the energy transfer process from the electromagnetic radiation to the plasma and it may have important consequences for experiments on radio frequency (RF) plasma heating in tokamaks and for the laser fusion.

In this paper we also developed a method which permits to reduce the problem of the absolute parametric instability [4, 15] excited by a monochromatic pumping field of an arbitrary amplitude in a nonuniform magnetoactive plasma to the problem of a parametric excitation of spatial oscillations in a uniform isotropic plasma. Below, the parametric excitation of low-frequency waves, with their dispersion completely determined by a high-frequency field, in a strong magnetic field is discussed for cases where the cyclotron frequency of ions significantly exceeds the frequency of the excited oscillations. The separation method [17] is used to investigate the API in a bounded nonuniform plasma under the effect of a pump field, static magnetic field and warmness of the plasma waveguide. Both the pump field $\vec{E}_p = \vec{E}_0 \sin(\omega_0 t)$ and the static magnetic field $\vec{B}_0$ are directed along the z-axis. Assuming the intensity of the magnetic field to be high enough ($\omega_{c_\alpha} >> \omega_{p_\alpha}$), the motion of plasma particles is considered to be confined along the z-axis only.

Mathematical model

Separation method in the problem of absolute parametric instability

The initial system of equations consists of the two-fluid equations combined with the Poisson's equation [15]:

$$(1)\qquad \frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \vec{\nabla})\vec{V}_\alpha = \frac{e_\alpha}{m_\alpha}(\vec{E}_P + \frac{1}{c}[\vec{V}_\alpha \vec{B}_0] - \vec{\nabla}\Phi) - \frac{1}{n_\alpha m_\alpha}\vec{\nabla}P$$

$$(2)\qquad \frac{\partial n_\alpha}{\partial t} + div(n_\alpha \vec{V}_\alpha) = 0$$

$$(3)\qquad \Delta\Phi = -4\pi\sum_\alpha e_\alpha n_\alpha$$

where n_α and $\vec{V}_\alpha$ are the density and velocity of particles of species; α, P is the scalar pressure and Φ is the potential of the self-consistent electric field, and $\alpha \equiv (e, i)$. We may relate the pressure and density through the equation of state (an alternative to truncating the moment equations) in the form: $p_\alpha = v_{s\alpha}^2 \rho_{1\alpha}$, where $v_{s\alpha}^2 = T_\alpha/m_\alpha$ is the square of the thermal velocity of species α and $\rho_{1_\alpha} = mn_{1_\alpha}$ is the mass density. In the equilibrium state, the particles velocity $\vec{u}_\alpha(0,0,u_\alpha)$ is determined by the following expression:

$$(4)\qquad \vec{u}_\alpha = -\frac{e_\alpha \vec{E}_0}{m_\alpha \omega_0}\cos(\omega_0 t)$$

The plasma is unperturbed at $t = 0$, so that for $t > 0$;

$$(5)\qquad n_\alpha = n_{\alpha_0}(x) + \delta n_\alpha;\ \vec{V}_\alpha = u_{\alpha_0} + \delta\vec{V}_\alpha$$

where the perturbations of velocity $\delta\vec{V}_\alpha(0,0,\delta V_\alpha)$, density δn_α and electrical potential φ can be represented in the form $(\delta\vec{V}_\alpha, \delta n_\alpha, \Phi) \sim \exp(i \cdot k_z \cdot z)$. Introducing a new variable:

$$(6)\qquad \nu_\alpha = e_\alpha\, \delta n_\alpha \cdot \exp(-iA_\alpha),$$

where: $A_\alpha = -a_\alpha \sin(\omega_0 t)$ and $a_\alpha = [(e_\alpha \cdot k \cdot E_0) / (m_\alpha \cdot \omega_0^2)] \approx a_e$ (as $[(a_i/a_e) \approx (m_e/m_i)] << 1$), then linearizing the system of equations of hydrodynamics (1) and (2) supplemented by Poission's Eq. (3) and using Eq. (6), we obtain:

$$(7)\qquad \nu_\alpha'' + v_{s\alpha}^2 k_z^2 \nu_\alpha = -\frac{e_\alpha^2}{m_\alpha} \cdot e^{-iA_\alpha(t)} \cdot \hat{L}_2\Phi$$

where: $\hat{L}_2 = -k_z^2 \cdot n_{0\alpha}$. Poission's Eq. (3) takes the operator form:

$$(8)\qquad \hat{L}_1\Phi = 4\pi\sum_\alpha \nu_\alpha \cdot e^{+iA_\alpha(t)},\ \hat{L}_1 \equiv -\frac{\partial^2}{\partial x^2} + k_z^2$$

Assuming $\nu_\alpha(x,t) = \nu_{\alpha 1}(t) \cdot \nu_{\alpha 2}(x)$, $\Phi(x,t) = \Phi_1(t) \cdot \Phi_2(x)$ and separating the variables in Eqs. (7) and (8), we obtain:

(9) $$\frac{d^2\Phi_2}{dx^2} - k_z^2 \varepsilon(x,p)\Phi_2 = 0$$

where: $\varepsilon(x,p) = 1 - [(\omega^2_{Pe}(x))/p^2]$ and p is a constant. It is found also that the final form of equations describing the "dynamical" (parametric) part of the problem:

(10) $$\frac{d^2\nu_{e_1}}{dt^2} + \eta^2\nu_{e_1} + p^2(\nu_{e_1} + \omega_{i_1} e^{-i(A_e - A_i)}) = 0$$

(11) $$\frac{d^2\omega_{i_1}}{dt^2} + \frac{m_e}{m_i} p^2(\omega_{i_1} + \nu_{e_1} e^{i(A_e - A_i)}) = 0$$

where: $\eta^2 = v_{se} \cdot k_2^2$, $\omega_i = [(m_e/m_i) \cdot (\alpha_i/\alpha_e)\, v_{i1}]$ and $\{\alpha_s\alpha_i = -[(m_i/m_e) \cdot (4\pi/p^2)]\}$. In this case, the system of Eqs. (10) and (11) coincides (within the redefinition of $\omega^2_{Pe} \to p^2$, $\omega^2_{Pi} \to (m_e/m_i) \cdot p^2$, $v_{se} = 0$ in a cold plasma) with the system describing HF suppression of Buneman instability in a uniform unbounded plasma [2] and also with the system of equations describing parametric excitation of spatial oscillations in a uniform isotropic plasma [9].

Solution of the "spatial Eq. (9)"

Now, let us examine Eq. (9) for different cases:

A) API in uniform plasma

A uniform plasma case corresponds to the propagation of volumetric waves ($\varepsilon_0 < 0$) with dispersion:

(12) $$\omega = \left(\frac{k_z}{k}\right) \cdot \omega_{p_0}, \quad k^2 = (k_x^2 + k_z^2)$$

B) API in nonuniform plasma

B.1) Exact harmonic oscillator

API in a nonuniform plasma is considered in which the density distribution is determined by the relation $n = n_0 [1 - (x^2/L^2)]$ [8]. In this case Eq. (9) takes the form:

(13) $$\frac{d^2\Phi_2}{dx^2} - k_z^2\Phi_2 + \frac{k_z^2}{p^2}\left(\frac{4\pi e^2 n_0}{m}\right) \cdot \left(1 - \frac{x^2}{L^2}\right)\Phi_2 = 0$$

Eq. (13) yields:

(14) $$\frac{d^2\Phi_2}{dx^2} + (A - Bx^2)\Phi_2 = 0$$

where: $A = -k_z^2\varepsilon_0$, $B = \dfrac{\omega_{p_0}^2 k_z^2}{p^2 L^2}$ and $\varepsilon_0 = 1 - \dfrac{\omega_{p_0}^2}{p^2}$.

The solution of Eq. (14), which describes trapped oscillations, are possible for $A < 0$ ($\varepsilon_0 < 0$) in the region $-\sqrt{|A|/B} < x < \sqrt{|A|/B}$. By substituting: $\xi = (k_z\omega_{p0}/L_p)^{1/2} \cdot x$ in Eq. (14) we obtained the equation:

(15) $$\frac{d^2\Phi_2}{d\xi^2} + \left(\frac{A}{\sqrt{B}} - \xi^2\right)\Phi_2 = 0 \quad \text{and} \quad \frac{|A|}{\sqrt{B}} = \frac{k_z|\varepsilon_0|\,pL}{\omega_{p_0}}$$

By substituting: $\Phi_2 = \psi(\xi) \cdot \exp(-\xi^2/2)$, in Eq. (15) and introducing the notation:

(16) $$2n + 1 = \frac{|A|}{\sqrt{B}} = \frac{k_z|\varepsilon_0|\,pL}{\omega_{p_0}}$$

the following equation is obtained:

(17) $$\psi''_{\xi\xi} - 2\xi\psi' + 2n\psi = 0$$

for the function $\psi(\xi)$. The solutions of this equation are Hermite polynomials [1]:

(18) $$\psi \sim H_n(\xi) = (-1)^n e^{\xi^2} \cdot \frac{d^n e^{-\xi^2}}{d\xi^n}$$

which satisfy the localizability condition (the width of the region of localizability of the oscillations is assumed to be significantly less than the width of the plasma layer) only for integral positive values of the number n (including zero). This fact permits considering Eq. (16) to be an analog of the quantization rule, which serves to determine the possible values of the quantity p (as it is a standing wave). Thus, the solution of Eq. (15) takes the form:

(19) $$\Phi_2 = c_n \cdot e^{-\xi^2/2} \cdot H_n(\xi)$$

From Eq. (16), we get:

(20) $$p_n^2 + (Q_n\omega_{p_0})p - \omega_{p_0}^2 = 0$$

where $Q_n = [(2n + 1)/(k_z L)]$. Thus, from Eq. (20), we obtain:

(21) $$p_n = \left(\frac{\omega_{p_0}}{2}\right) \cdot \left\{(Q_n^2 + 4)^{1/2} - Q_n\right\}$$

For lower integer n, at $Q_n << 1$, Eq. (20) takes the form:

(21a) $$p_n \approx \omega_{p_0} \cdot \{1 - (Q_n/2)\}$$

Thus, the oscillations are described by Eq. (17). The other solution of Eq. (20) is negative. At $L \to \infty$, $Q_n = 0$, (i.e., $n = 0$), $p \Rightarrow \omega_{p_0}$ (plasma waves in a uniform plasma), Eq. (20), takes the form:

(22) $$p_n^2 = \omega_{p_0}^2 \{1 - \delta\}, \quad \delta = \frac{1}{2} \cdot Q_n \left\{[4 + Q_n^2]^{1/2} - Q_n\right\}$$

This equation is the same as in a cold plasma case [9], i.e. the warm plasma has no effect on the space part of the problem (only through p).

B.2) Bounded harmonic oscillator (bounded plasma)

While solving the problem (B.1), it was assumed that:

(23) $$\Phi_2 \to 0 \quad \text{at} \quad x \to \pm\infty$$

(an unbounded plasma). But when considering it as a bounded plasma (metallic walls at $x = \pm b$, instead of Eq. (23) we should use:

(24) $$\Phi_2 = 0 \quad \text{at} \quad x = \pm b$$

Thus, Eq. (14), takes the form:

$$\frac{d^2\Phi_2}{dz^2}+\frac{h^2\omega_{p_0}^2}{p^2}\cdot\left(\frac{p^2}{\omega_{p_0}^2}\cdot|\varepsilon_0|-z^2\right)\Phi_2=0 \tag{25}$$

where $h \equiv (kL) >> 1$. Putting: $\frac{\delta N}{N}=\left(\frac{x}{L}\right)^2\cdot\left(1-\left(\frac{x}{L}\right)^2\right)$, in Eq. (25), we get:

$$\frac{d^2\Phi_2}{dz^2}+k^2\left\{\frac{\omega_{p_0}^2}{p^2}-1+\frac{\omega_{p_0}^2}{p^2}\cdot\left(\frac{x}{L}\right)^2\cdot\left(1-\left(\frac{x}{L}\right)^2\right)\right\}\Phi_2=0 \tag{26}$$

where: $\omega_{p_0} \approx p_0$, $\varepsilon_0 \approx 0$, ω_{p0} is the electron plasma frequency at $x = 0$ and p_0 is the separation constant at interger $n = 0$. In this case the solution of the above Eq. (26) gives us the wave oscillation described by the inequality:

$$kL = 3/2\,(2\pi n + \pi \pm \pi/4) >> 1, \quad kL \approx 3\pi n,\ n >> 1 \tag{27}$$

Thus, at an integer $n >> 1$, the plasma inhomogeneity is completely bounded by a metallic wall.

It can be concluded then that for the inhomogeneity plasma, the frequency and the growth rate of the wave oscillation are bounded by a metallic wall.

Solution of the “temporal” (time-dependent) equations

Following the procedure developed in Ref. [3] the dispersion equation of low-frequency oscillations ($\omega \approx (m_e/m_i)_s$) can be derived from Eqs. (7). Under the parametric resonance condition ($n\omega_0 \approx s$, n-integer), we get:

$$\omega^4-\frac{\Delta_n^2 s^2}{4}\omega^2-\frac{m_e}{2m_i}p^4\Delta_n J_n^2(a)=0 \tag{28}$$

where $\Delta_n = (s/n\omega_0)^2 - 1$, $s^2 = p^2 + \eta^2$, $\eta^2 = v_{se}^2\cdot k_z^2$ and it can be supposed here that the resonance “mismatch” Δ_n satisfies the inequalities $(m_e/m_i)\cdot(p/s)^4 << |\Delta_n| << 1$. From Eq. (28) we find the frequencies of parametrically excited plasma oscillations:

$$\omega^2=\frac{\Delta_n^2 s^2}{8}\left(1\pm\left[1+\frac{32}{\Delta_n^3}J_n^2(a)\frac{m_e}{m_i}\cdot\frac{p^4}{s^4}\right]^{1/2}\right) \tag{29}$$

with the Bessel function, $J_n(a)$.

Equation (29) yields an unstable solution with periodic instability ($\Delta_n < 0$). In this case $\gamma_{per} = I{\cdot}m{\cdot}\omega > 0$, i.e. small perturbations in the plasma grow exponentially in time, if the following condition is satisfied:

$$0>\Delta_n>-2\left(4J_n^2(a)\frac{m_e}{m_i}\cdot\frac{p^4}{s^4}\right)^{1/3} \tag{30}$$

The growth rate of the instability is determined by the expression:

$$\gamma_{per,\omega}=s\frac{|\Delta_n|}{4}\left[-1+\left(\frac{32J_n^2(a)}{|\Delta_n|^3}\frac{m_e}{m_i}\cdot\frac{p^4}{s^4}\right)^{1/2}\right]^{1/2} \tag{31}$$

The maximum value of the growth rate γ_{per} is reached at:

$$(n\omega_0)_{\max,\omega}=s\left[1+\left(\frac{1}{4}J_n^2(a)\frac{m_e}{m_i}\left(\frac{p}{s}\right)^4\right)^{1/3}\right] \tag{32}$$

By substituting Eq. (32) into Eq. (31) it can be found:

$$\gamma_{per,\omega}^{\max}=s\left(\frac{\sqrt{27}}{32}J_n^2(a)\frac{m_e}{m_i}\cdot\frac{p^4}{s^4}\right)^{1/3} \tag{33}$$

The main feature of Eqs. (31)–(33) relies on the existence of a separation constant p which enables us to account for the plasma nonuniformity.

From Eq. (31), it follows that the threshold value of the HF field amplitude in case of periodic instability is determined by the relation:

$$32J_n^2(a_{thr})\left(\frac{m_e}{m_i}\right)\left(\frac{p}{s}\right)^4\geq|\Delta_n|^3,\quad |\Delta_n|=-\frac{s^2}{(n\omega_0)^2}+1 \tag{34}$$

At small amplitudes of the pumping wave, from Eq. (34) we have:

$$a_{thr}^2=\frac{m_i}{8m_e}\frac{s^4}{p^4}\left(|\Delta_0|+\delta\frac{\omega_{P_0}^2}{(n\omega_0)^2}\right)^3,\quad |\Delta_0|=\frac{\omega_{P_0}^2}{(n\omega_0)^2}-1 \tag{35}$$

At $\eta^2 = v_{th}^2{\cdot}k_z^2 = 0$, Eq. (33) takes the form:

$$\gamma_{per,c}^{\max}=p\left(\frac{\sqrt{27}}{32}J_n^2(a)\frac{m_e}{m_i}\right)^{1/3} \tag{36}$$

$$32J_n^2(a_{thr})\left(\frac{m_e}{m_i}\right)\geq|\Delta_n|^3,\quad |\Delta_n|=\left|\frac{p^2}{(n\omega_0)^2}-1\right| \tag{37}$$

At small amplitudes of the pumping wave, from expression (37) we have:

$$a_{thr}^2=\frac{m_i}{8m_e}\left(|\Delta_0|+\frac{\omega_{P_0}^2}{(n\omega_0)^2}\cdot\delta\right)^3,\quad |\Delta_0|=\frac{\omega_{P_0}^2}{(n\omega_0)^2}-1 \tag{38}$$

which is in agreement with the work of Demchenko and Omel’chenko [8] for the cold plasma case.

Equations (33) and (36) for the periodic API become:

$$\gamma_{per,\omega}^{\max}=\left(\frac{1}{\sqrt{1+\left(\frac{\eta}{p}\right)^2}}\right)\cdot\gamma_{per,c}^{\max} \tag{39}$$

Thus, from Eq. (39), it can be concluded that in the warm plasma the growth rate of a periodic API is smaller when compared to the cold plasma [8].

Conclusion

The effect of a 1-D plasma nonuniformity on the API of elecrostatic waves in a magnetized pumped warm plasma has been investigated with the separation method. Different solutions for the spatial equation were examined for the following cases: A) API in a uniform plasma (Eq. (12)), B) API in a nonuniform plasma, where two cases were studied: B.1) the exact harmonic oscillator (Eq. (21)), and B.2) the bounded harmonic oscillator (a bounded plasma, Eq. (27)).

It follows from Eqs. (22), (33), (34) and (35) that taking into account a nonuniformity of the warm plasma density results in a decrease in the maximum values of the oscillation build-up increments and an increase in the threshold value of the electric field amplitude of the pumping wave in comparison with the case of a uniform plasma. These results are consistent with the results of Refs. [4, 8].

Equation (22) is the same as the corresponding equation in the cold plasma case [8]; i.e. the warm plasma has no effect on the space part of the problem (only through the separation constant p). The main feature of expressions (31)–(33) consists in the existence of the separation constant p which enables us to account for the plasma nonuniformity. It can be concluded from Eq. (39) that the growth rate of periodic API is smaller in a warm plasma than in a cold plasma, which was considered by Demchenko *et al*. [8].

It should be noted that our approach is significantly simpler than the method ordinarily employed in the theory of a parametric excitation of waves in a nonuniform plasma [10]. Therefore, it is of practical interest to apply the method to solving different problems in a parametric resonance in a nonuniform plasma taking into account finite plasma temperature and nonuniformities of the HF electric and static magnetic fields. The method developed in Refs. [6, 14] is best suited for investigating the parametric effects under a high-amplitude pump wave, $W = n_0T_e$. This method was modified [18] for investigating the API in a magnetoactive nonuniform plasma by a monochromatic HF electric field of an arbitrary amplitude.

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