Adaptive Control of Frictional Contact Models for Nonholonomic Wheeled Mobile Robot

Submitted 14th March 2011; accepted 25th August 2011

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Abstract:

Mobility of the robot depends on the vehicle dimensions, locomotion principles and wheel characteristics. The function of the wheel is to carry the load and to produce the traction force. The main factors of wheel terrain interaction are bearing capacity of ground, traction performance of the wheel and geometry of terrain profile. In this paper the system and control concepts of the wheeled robot is discussed in more detail, within the framework provided by the wheel terrain contact model. The dynamic model of the wheeled robot is presented by considering contact forces of the wheel due to their relative motion of the wheel and contact plane. Finally, a dynamic relation is introduced and results are presented in terms of forces, torques and displacements related to wheel terrain interaction. To estimate the forces in the system arising from the interaction between a deformable wheel and rigid terrain using the software package Ansys 10.0. Simulations were performed using Matlab-Simulink program and the results are shown that the proposed controller can overcome the influences the effect of contact forces in order to achieve the desired trajectory.

Keywords: *wheeled robot, dynamic model, wheel terrain interaction, Ansys analysis, Matlab-Simulink.*

1. Introduction

The required condition of wheeled mobile robot in an environment is stable and fast navigation to reach the target. But the change of characteristics of the robot movement will cause unstable drive according to the relationship between the driving wheel and ground. Most of the control algorithms do not consider the physical dimensions and capabilities of the mobile robot within its environment. When the wheel torque generates a turning momentum along the wheel rim, it develops resistive forces on the motion. The integration of longitudinal shear stresses over the entire contact path represents the tractive force. The tractive force can be used to overcome the rolling resistance and to generate pulling force. The actual wheel ground interaction needs to be considered in order to improve the robot motion control. Wheeled robots are almost always designed so that all wheels are in ground contact at all times. Thus, three wheels are sufficient to guarantee stable balance. Instead of worrying about balance, wheeled robot research tends to focus on the problems of traction and stability, maneuverability, and control which can provide sufficient traction and stability for the robot to cover all of the desired.

Mobile robots have actuated wheels whose slip rate, rolling, inertia moments, and mass distribution contribute to the forces exerted on the structure of the vehicle thus affecting the accuracy and maneuverability of the robot [1]-[3]. In the model [4], the steady state wheel forces and torques are generated as functions of the longitudinal and lateral slip. Depending on the slip definition the dynamics of a wheel depends on the vehicle velocity or the angular velocity of the wheel. The Hertzian pressure distribution was assumed for the normal surface contact load over a contact area. The tangential forces in both the rolling and lateral directions were considered and were assumed to be proportional to the Hertzian pressure. Theory of vehicle dynamics [3], a well established discipline in automobiles dealing with dynamic properties of rolling motion, has revealed that different nonlinear dynamic effects and disturbances will be generated in different wheel ground interaction conditions.

Motion planner minimizes the distance between the present robot location and the desired end location. A local level motion planner attempted to attain the goals set by the higher level [5]. This was done by computing wheel accelerations, contact forces, equations of motion and the new state of the deformable regions in the terrain [6]-[8]. This algorithm also incorporated the wheel ground interaction and a bounded control torque constraint.

Nilanjan Chakraborty and Ashitava Ghosal [9] developed a hybrid parallel mechanism with the wheel ground contact described by differential equations which take into account the geometry of the wheel, the ground and the nonholonomic constraints of no slip. The workspace for the WMR is not always ideal and usually packed with various forms of disturbances including frictions, irregular terrains, obstacles in robot's path, parametric changes and uncertainties within and outside the system, making it almost impossible to model all these disturbances and incorporate them into the dynamics of the WMR [10], [11]. Recently adaptive methods are used to compensate the effect of uncertainties in dynamic model and to configure the vehicle to adapt to terrain variations and allow rolling of wheels. Thus, in order to ensure a more robust and accurate operation of the mobile robot, a disturbance compensation scheme should be incorporated into the operation of the WMR.

2. Wheeled Mobile Robot Model

This paper analyzes the vehicle dynamics of wheeled mobile robots with contact forces. Condition that describes the limits of contact stability in terms of contact forces, it is derived from the interaction between a deformable wheel and rigid terrain. The best model for the continuous nature of the deformation and contact area is non linear finite element method. Implicit finite element methods (FEM) have traditionally been used to determine contact parameters during static and quasi-static loading conditions. The first one is based on a formulation using displacements and the second one is based on a mixed formulation using displacements and contact forces. The normal force is generated by the supporting normal load. The tractive force is generated by the forward friction force and the lateral force, which is existed.

2.1. Geometric model of the robot

The actual wheel-ground interaction needs to be considered in order to improve the robot motion control. Here the terrain assumed to be rigid and the wheel deformable. Consider a wheel that rolls on a plane while keeping its body vertical as shown in Fig.1. Configuration of the robot can be described by a vector $q = (x, y, \theta, \varphi)$ of generalized coordinates, where x, y are the Cartesian coordinates of center of the rear axle, θ measures the orientation of the robot body with respect to the X axis, and φ is the rolling angle of the wheel. At the wheel ground contact point, the holonomic constraint is $\upsilon_{zc} = 0$, which ensures wheel ground contact is always maintained. Moreover, at each instant, nonholonomic constraints which prevents instantaneous sliding and these are $\upsilon_{xc} = 0$ and $\upsilon_{yc} = 0$.

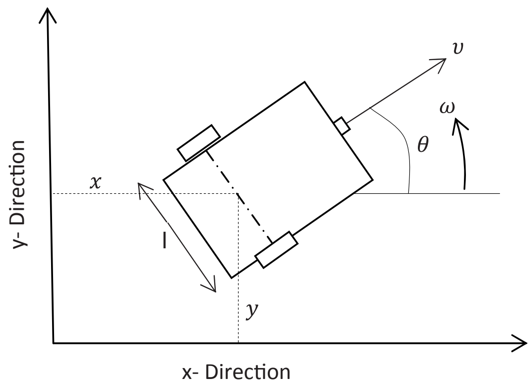

Fig.1. Three wheeled differentially driven mobile robot.

For simple dynamic model of the wheel is a thick cylinder that represents the middle cross section of the wheel and the linear velocity of the wheel center lies in the body plane of the wheel. The general dynamic equation of the wheel robot is given below,

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) + \tau_d = B(q)\tau + A^T(q)\lambda \quad (1)$$

where $M(q)$ is the inertia matrix, $C(q,\dot{q})$ is a matrix containing the centrifugal and coriolis terms, $G(q)$ is the gravity force matrix matrix, $B(q)$ is the input transformation matrix, τ is the input torque, $A^T(q)$ is the Jacobian matrix associated with the constraints, λ is the constraint force vector and q is the state vector representing the generalized coordinates. τ_d denotes the bounded unknown external disturbance. For the continuous nature of the deformation and contact, the non-linear finite element method is selected for the best model and the contact force is measured from the built in geometric model of a wheel and a terrain. When considering the motion resistances, the dynamics of a single wheel as shown in Fig. 2 is written as,

$$I_r\dot{\varphi}_w = \tau_r - M_y - F_x r_e \quad (2)$$

$$I_s\dot{\delta} = \tau_s - M_z \quad (3)$$

where I_r moment of inertia of the wheel about rolling, I_s moment of inertia of the wheel about turning, φ_w rolling velocity of the wheel, $\dot{\delta}$ turning velocity of the wheel, τ_r rolling torque, τ_s steering torque, M_y moment of rolling resistance, M_z moment of turning resistance and r_e effective radius of the wheel.

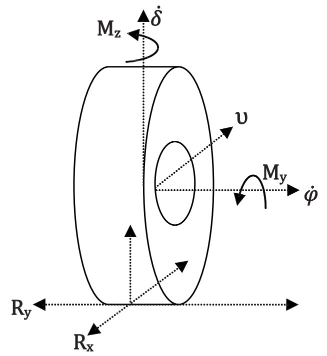

Fig. 2. Wheel ground interactions.

The rolling motion generates a horizontal reactive force R_x and a lateral reactive force R_y while the twisting motion generates a pure reactive turning moment M_z in the vertical direction. Assuming that the ground is flat and does not deform, the above three quantities are defined in vehicle dynamics as,

$$R_x = \mu\frac{mg}{2} sgn(\dot{x}_l) \quad (4)$$

$$R_y = \mu_t\frac{mg}{2} sgn(\dot{y}_l) \quad (5)$$

$$M_y = \mu\frac{mg}{2} (sgn(\dot{x}_l))r_e \quad (6)$$

$$M_z = \mu\frac{mg}{2} (sgn(\dot{z}_l))\frac{b}{2} \quad (7)$$

Where b – width of the wheel, μ – longitudinal friction coefficient, μ_t – lateral friction coefficient. Based on the force and moment analysis for wheel in Fig. 2, the total resistive force acting on each wheel R_{eq} can be derived as:

$$R_{eq}(\dot{q}) = \begin{bmatrix} R_x \\ R_y \\ M_{eq} \end{bmatrix}$$

The dynamic model is obtained from dynamic properties of mass, inertia moments, friction force, gravitation and wheel ground interaction. The orthogonal force components are vertical, longitudinal and lateral. The lateral frictional forces also prevent the vehicle from sliding to unwanted directions. Several parameters of the terrain are used to estimate normal, lateral and longitudinal forces at the wheel contact patch. If the frictional force is less than the maximum value, the wheel position is not changed, if it is greater than or equal to maximum value, wheel is pulled in direction opposite to the friction force from the wheel position. The total resistive quantities are defined in vehicle dynamics as:

$$R_X = \mu \frac{mg}{2} (sgn\ (\dot{x}_1) + sgn\ (\dot{x}_2)) \tag{8}$$

$$R_Y = \mu \frac{mg}{2} (sgn\ (\dot{y}_1) + sgn\ (\dot{y}_2)) \tag{9}$$

$$M_T = \left[\mu \frac{mg}{2} (sgn(\dot{x}_1) - sgn(\dot{x}_2))\right] l/2 + \left[\mu_l \frac{mg}{2} (sgn(\dot{y}_1) - sgn(\dot{y}_2))\right] d \tag{10}$$

2.2. Contact model of the robot

The resulting frictional forces can be defined by integration of all forces acting on the contact surface. The pressure distribution resulting from the normal contact can be calculated in the local reference. As a consequence, the tangential and the normal forces in the global reference can be calculated by integrating the contact pressures on contact of the *X* and *Y* axes for the tangential forces and for the normal force on the Z axis.

$$F_x = \iint p_x dxdy \tag{11}$$

$$F_y = \iint p_y dxdy \tag{12}$$

$$M_z = \iint (xp_y - yp_x) dxdy \tag{13}$$

At a point of the contact surface the projected force F_y on the *Y* axis is zero due to the symmetry of the vehicle structure. As a result, contact friction leads not only to a resultant force applied to the center of the area but also to a non-vanishing moment about the normal axis through the center of that area. This moment, M_z is a function of the size of the contact area *A*, wheel material, type of wheel ground contact, weight of the vehicle, etc. Since M_z opposes the steering motion, it should be added to Eq. (18) using a sign function. At the contact point, the contact force can be decomposed into normal and tangential components. Let F_x be the horizontal component of contact force and F_z be the normal component of contact force. Assume that the coordinate frame and centre of gravity are lying in symmetry axis of the wheels. So that the contact force $F_y = 0$ and F_z is expressed as the function of contact pressure.

This resultant frictional force is still acting, but the new distribution of the normal forces creates a net torque opposing the rotational contribution of the friction and causing an overall deceleration of the wheel's forward velocity. The lateral wheel friction is a coloum friction, whose force takes two sign opposite values depending on the direction of turning of the vehicle. Therefore M_z can be rewritten as,

$$M_z = 2(-\int p_x dxdy) \frac{l}{2} sgn(\dot{z}) = -F_C \frac{l}{2} sgn(\dot{\omega})$$

$$m\dot{v} = (F_{xl} + F_{xr}) - R_x\, sgn(\dot{x}) \tag{14}$$

$$I\dot{\omega} = (F_{xl} - F_{xr}) l/2 - M_z \tag{15}$$

$$\tau_{li} = m\dot{v}_d + k_l(v_d - v) \tag{16}$$

$$\tau_{an} = I\dot{\omega}_d + k_a(\omega_d - \omega)$$

$$m\dot{v} = \frac{1}{r} * (\tau_l + \tau_r) = m\dot{v}_d + k_l(v_d - v)$$

$$I\dot{\omega} = \frac{l}{r} * (\tau_l - \tau_r) = I\dot{\omega}_d + k_a(\omega_d - \omega) \tag{17}$$

Considering the motion resistances, the dynamic model of the robot Eq.1 is rewritten as:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = B(q)\tau + \tau_c \tag{18}$$

where is the torques generated by the contact forces.

The contact torques can be written under the following equation:

$$\tau_c = J(q)^T R_{eq}(q,\dot{q})$$

Where *J*(*q*) is the Jacobian matrix of the constraint on the position of the points on which these contact forces are applied. In real situations, motion resistance generated by the wheel ground interaction always exists, so the actual governing dynamic equations of motion of the robot are given by equation (19) rather than equation (1).

$$M(q)\ddot{q} + R(v,\omega) = B(q)\tau \tag{19}$$

where

$$M(q) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \quad R(v,\omega) = \begin{bmatrix} R_X & 0 \\ 0 & M_z \end{bmatrix} \begin{bmatrix} sgn\ (v) \\ sgn\ (\omega) \end{bmatrix}$$

$$B(q) = \frac{1}{r} \begin{bmatrix} 1 & 1 \\ l/2 & -l/2 \end{bmatrix}$$

In the case of trajectory tracking, control algorithms that consider wheel ground interaction are expected to demonstrate better tracking performance than those that do not consider the wheel ground interaction.

3. Simulation Results and Discussion

In this section, motion control scheme is considered with wheel ground interaction. Based on the dynamics of the mobile robot represented in equation (19), a control algorithm is proposed and modeled using Matlab-Simulink. Simulink model of the robot motion control is shown in Fig. 4. The model parameters taken for this simulation are $M = 100$ kg, $r = 0.1$ m, $b = 0.05$ m, $l = 0.5$ m, $I = 20$ kg/m^2, $v = 3$ m/s, $\omega = 0.5$ rad/s.

In this simulation the controller provides the desired trajectory, desired velocity and desired accelerations to the robot body. The 3D model of mobile robot wheel and terrain is created and analyzed using ANSYS10.0. All the external loads are applied at the wheel center. The contact was created by using Ansys software; here, wheel is contact element and terrain is target element. For contact CONTA174 and TARGET170 elements are used for 3D model. Friction effect is included into the material properties of the contact element. The material properties are listed in the Table 1. Then the contact region is finely meshed using a sub model approach. Next, quasi-static analysis is performed for the full model and the contact pressure results is plotted as shown in Fig. 3, and the value of contact force of wheel is calculated from the simulated results.

The developed fuzzy controller for this simulation is: angle and distance errors as inputs. During the robot movement, it moves whether in a straight line or circular arc, and creates the position and orientation errors which depend on the path. Designed FLC has three inputs and two outputs. Inputs are: linear distance errors and orientation angle error. Outputs are: the linear and angular

Table 1. Materials properties of the wheel.

Material	Young's modulus, N/mm^2	Poisson ratio	Density, kg/mm^3
Concrete	48x10^3	0.2	2.5x10^{-6}
Polyurethane	0.025x10^3	0.499	1.2x10^{-6}

velocities v_c and ω_c. The adaptive control of WMR with the dynamic model is to implement an adaptive control with the set of frictional contact force parameters in order to achieve the desired trajectory. To set the desired accelerations $\dot{v}_d$ and $\dot{\omega}_d$ by specifying required forces and torques by the equations 16 and 17.

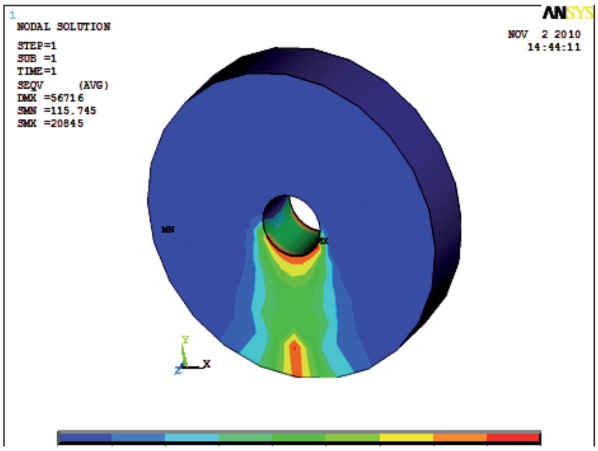

Fig. 3. Contact stress distribution on the robot wheel.

Simulated calculations:

Contact force at node 1281 = 277.83 N
Contact force at node 2148 = 328.06 N
Contact force at node 1669 = 274.46 N
Total contact force = 277.83 +328.06 + 274.46
F_c = 880.35 N

It can be observed from Fig. 5 that the tracking errors of dynamic controller with wheel ground interaction are much less than that of simple dynamic controller. This result shows that both position and orientation tracking errors can be reduced substantially when the wheel

(a)

(b)

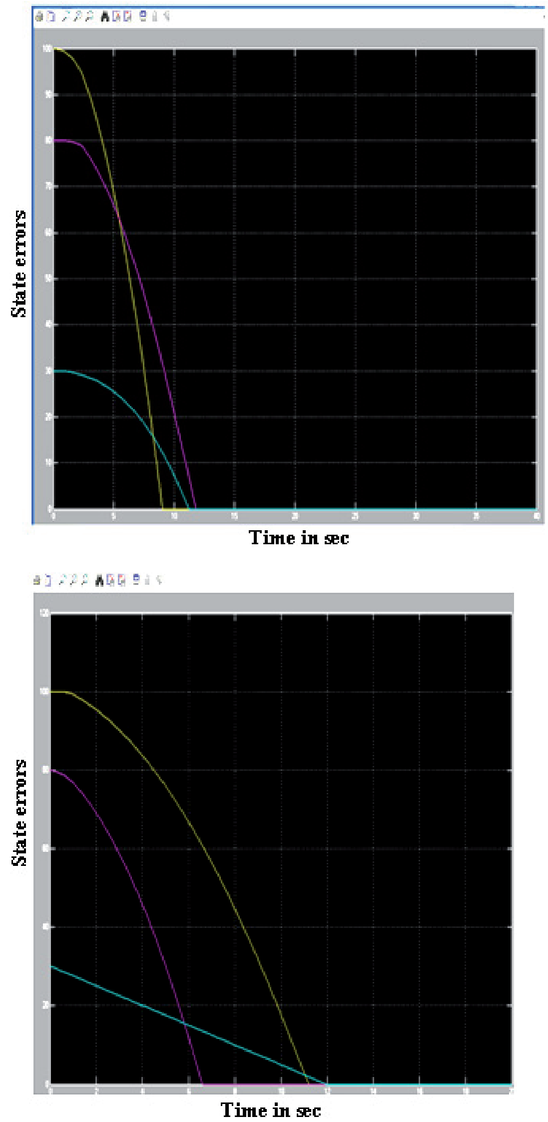

Fig. 5. The convergence of the state errors in trajectory tracking a) without contact force b) with contact force.

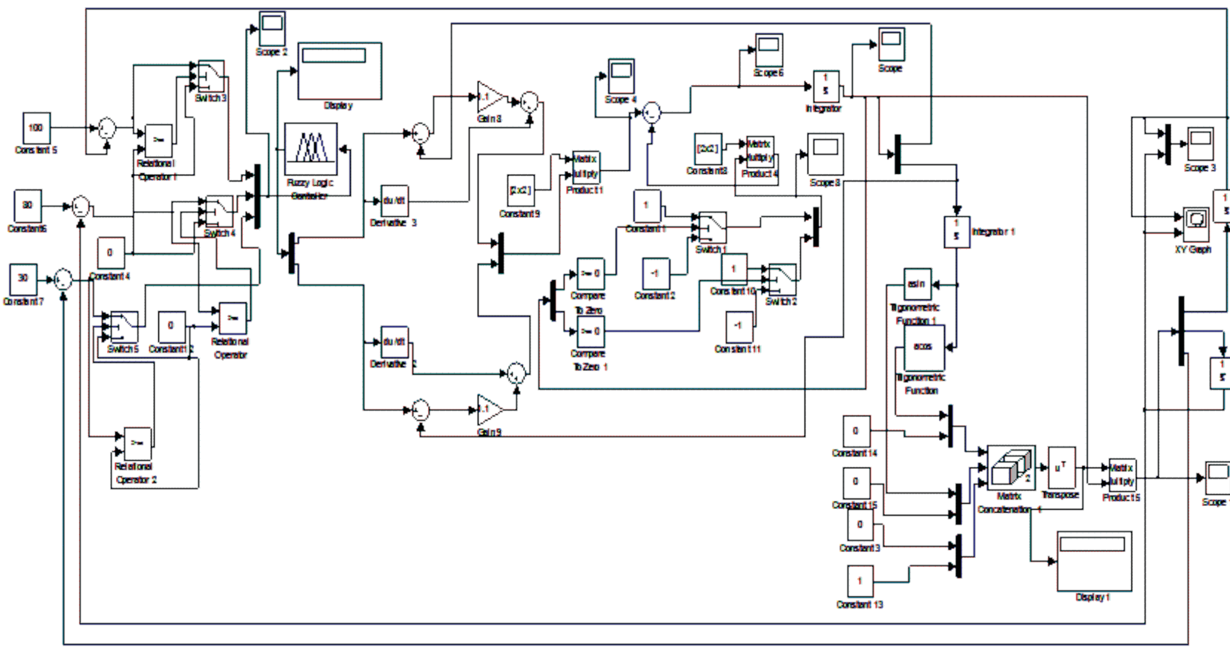

Fig.4. Simulink model of integrated kinematic and dynamic with Fuzzy controller.

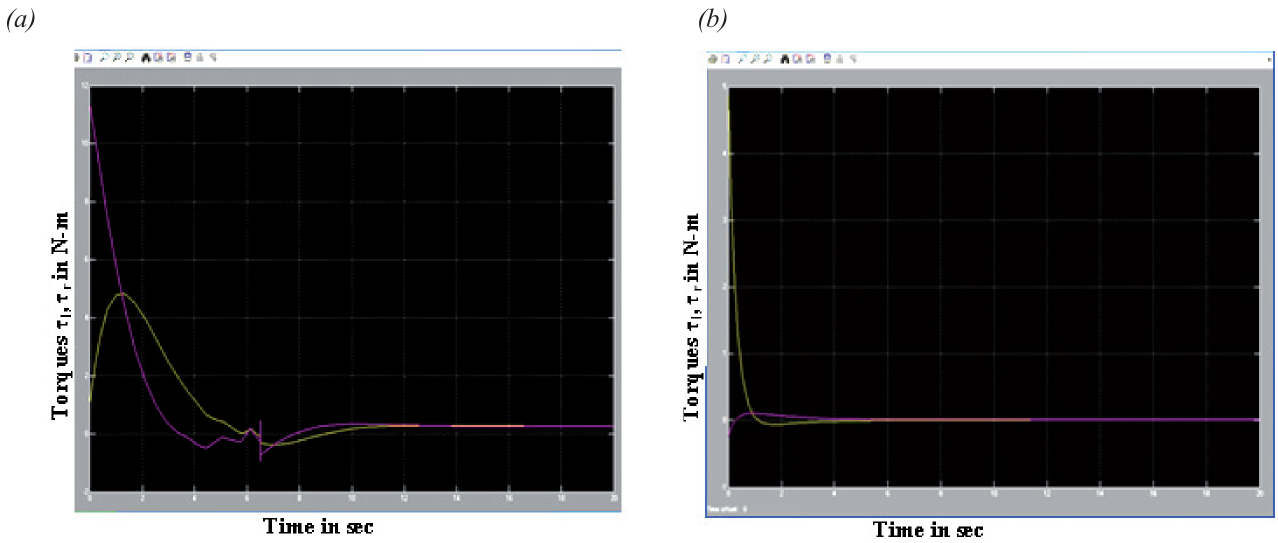

Fig. 6. The developed torques in left and right wheels a) without contact force b) with contact force.

ground interaction is considered in resistive forces. The controller is able to recover from the error and stabilizes the robot to the desired trajectory, even if wheel ground interaction parameters are variable during motion. When implementing the computation of F_c will add the contributions to the dynamic model. While position and velocity errors are rapidly compensated for very small changes in wheel ground interaction parameters and Fig. 6 shows the results of the controller that represents the torques and to the wheels of autonomous robot.

4. Conclusion

The dynamic model of the wheeled mobile robot was constructed with wheel ground interaction and the robot parameters were computed to provide robot motion. This paper analyzes the vehicle dynamics of wheeled mobile robots with resistive moment of the contact forces. Adaptive contact force distribution scheme is proposed to satisfy the stable contact condition. The genetic fuzzy controller is proposed is used to estimate the influences the effect of contact forces and its effectiveness is demonstrated by simulation. Future works may integrate the resistive contact forces into dynamic model to implement the proposed methods on the real robot.

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