

APPLICATION OF DEPENDENCE GRAPHS AND GAME TREES FOR DECISION DECOMPOSITION FOR MACHINE SYSTEMS

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Abstract:

The paper concerns application of dependence graphs and parametric game trees for analysis and synthesis of dynamic properties of machine systems. Different graph solutions mean connections of input and output quantities as well as constructional parameters. The method of dependence graph distribution into the game tree structure was described. Multiple vertex numeration was introduced in order to determine subordination of elements in the system, and the additional time vertex allowed to obtain the structure with the closed loop of the feedback. The obtained graph solutions were evaluated taking into account decision decomposition into single constructional and/or service parameters.

Keywords: dependence graph, vertex complexity, system structure, game tree, decision decomposition, multiple vertex numeration.

1. Introduction

Engineering practice requires correct evaluation of the mathematical model describing a given system with some variables. A proper mathematical model contains a group of functions joining different variables and describing connections between quantities in the system. Decision tables [1] and logical functions [8], [18], [20], [22] can be applied in simulation of machine systems, for example described by ordinary or partial differential equations. It results from the fact that the occurring nonlinear elements can be divided into the finite number of linear elements (parts), and in a consequence we obtain some linear systems as for simulation from the primary single nonlinear system [13], [18]. The occurring constructional parameters influence courses of unknown functions dependent on time. Traditional analysis such as Input → Output and Output → Input of the given system with the dependence graph method allows to obtain the vertex groups of the following properties: [2], [3]:

- elements inside the given group have many information connections,
- particular groups have a little reciprocal information connections.

In this way we obtain constructional recommendations. Let us see that it is possible to obtain multiple solutions from the dependence graph, so selection of suitable subsolutions can be done with use of tree classifiers from the neuron networks [7], [14], [19].

Let us consider a different approach, being translation of the dependence digraph into the tree game structure

[9], [10]. Constructional and service parameters can be joined in different mathematical ways, it is necessary to determine precise decomposition of the dependence graph in order to define decision variables. Such procedure allows to determine the area of feasible solutions and to select a proper optimization procedure.

2. Application of the dependence graph for analysis of dynamic properties in the case of machine systems

The equations of dynamics can be used for determination of mutual connections of all the functions dependent on time. As a result of notation and decomposition of the dependence graph of those functions we obtain the groups of distribution which describe properties of successive subsystems of the considered machine system and a set of suitable constructional and service parameters.

Example

Fig. 1 [12], [18] presents a simplified hydraulic system including a gear pump, an overflow valve, a divider and a motor loaded by a big mass moment of inertia. The paper does not include numerical values of constructional and service parameters because diagrams will not determined while simulation.

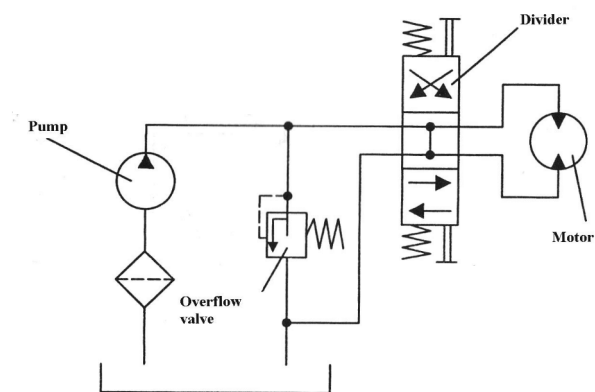


Fig. 1. Hydraulic scheme of the system.

The mathematical model of the above system has the following form:

1. equation of intensity of the flow from the pump

$$Q_S = Q_p 1(t) - \frac{1}{R_{p0}} P_p - Q_{zp}, \text{ where}$$

$$\frac{1}{R_{p0}} = \frac{1}{R_{pp}} + \frac{1}{R_{pz}} + \frac{1}{R_r} + \frac{1}{R_S} \quad (1)$$

and:

- P_p - pressure in the forcing line of the pump,
- Q_p - theoretical capacity of the pump,
- Q_{zp} - flow intensity through the overflow valve,
- Q_s - flow intensity given to the receiving part of the system;

2. equation of the overflow valve

$$Q_{zp} = 0 \text{ for } P_p \leq P_0; \quad \frac{dQ_{zp}}{dt} = \frac{K}{T} P_p - \frac{1}{T} Q_{zp} \text{ for } P_p > P_0 \quad (2)$$

3. equation of losses of pressure

$$P_p = R_l Q_s + P_s \quad (3)$$

where P_s - pressure drop between working chambers of the motor;

4. flow equation of the motor

$$\frac{dP_s}{dt} = \frac{1}{C} Q_s - \frac{D}{C} \omega \quad (4)$$

where ω - angular velocity of the motor shaft;

5. equation of the motor moments

$$\frac{d\omega}{dt} = \frac{D}{J} P_s - \frac{R}{J} \omega, \text{ i.e. } J \frac{d\omega}{dt} = DP_s - R\omega. \quad (5)$$

The unknown functions $P_p, P_s, Q_s, Q_{zp}, \omega$ are calculated on the basis of the given input of the system Q_p , so there is a system structure (Fig. 2). Thus, the following notations of the dependence graph are resulting:

1. The signals from which the given signal formed:

$$Q_{zp}(P_p), P_p(Q_s, P_s), P_s(Q_s, \omega), \omega(P_s), Q_s(Q_p, P_p, Q_{zp});$$

2. The signals forming the given signals:

$$Q_s(P_p, P_s), P_p(Q_{zp}, Q_s), Q_{zp}(Q_s), P_s(P_p, \omega), Q_p(Q_s).$$

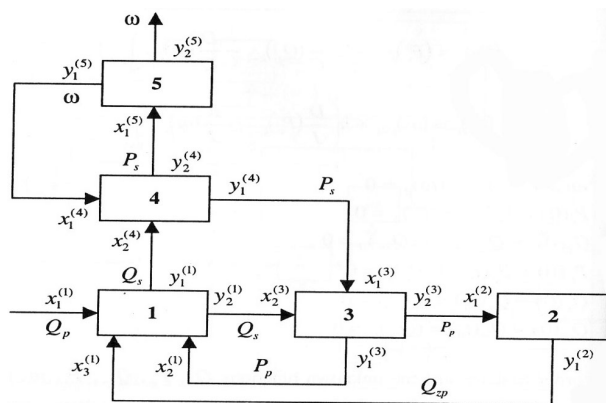


Fig. 2. System structure of the hydraulic system.

Generally, we can obtain a graph solution in the tree approach for the hydraulic system shown in Fig.1 taking into account constructional parameters and the following notations:

1. The signals from which the given signal formed:

$$Q_{zp}(P_p | K, T); P_p(P_s; Q_s | R_l); P_s(Q_s | C; \omega | C, D); \omega(P_s | D, J; R); Q_s(Q_p; P_p | R_{p0}; Q_{zp});$$

2. The signals forming the given signals:

$$Q_s(P_p, R_l; P_s, C); P_p(Q_{zp}, K, T; Q_s, R_{p0}); Q_{zp}(Q_s); P_s(P_p; \omega, D, J); Q_p(Q_s); \omega(P_s, D, C).$$

In this case the obtained graph solution (for the initial vertex Q_p) is polysemantic from the point of view of successive obtaining the subgroups:

$$\begin{aligned} & -(\{Q_p, Q_s, P_p, P_s, R_l, C\}), \quad --(\{T, Q_{zp}\}, \{K\}), \\ & --(\{Q_{zp}\}, \{K\}, \{T\}), \quad --(\{K, Q_{zp}\}, \{T\}), \quad ---\{R_{p0}\}, \\ & ----(\{\omega, J\}, \{D\}), \quad ----(\{\omega, D\}, \{J\}) \quad [2]. \end{aligned}$$

3. The dependence graph for tree game structures

The considered hydraulic system containing the elements joined in a way allowing for a flow of signals can be written with the dependence graph for tree game structures (Fig. 3). The graph distribution from any vertex in the first stage leads to a tree structure with cycles, and next to a general tree game structure. Each structure has a proper analytic notation (G_i^+ and G_i^{++} , where i is a vertex, from which the graph decomposition started) determining a way of transition from the dependence graph to the tree structure.

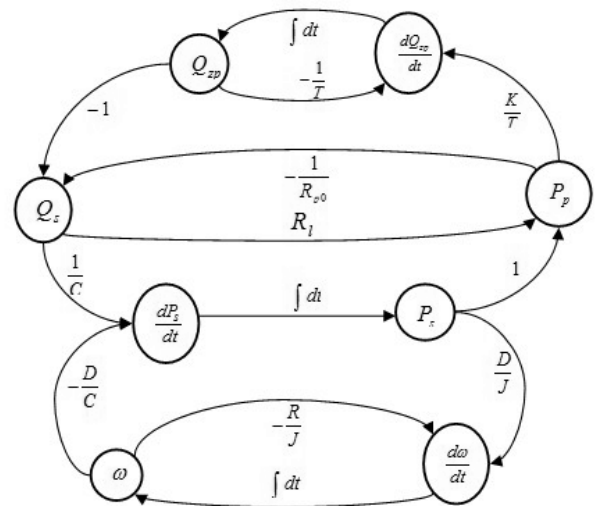


Fig. 3. Dependence digraph of the signal flow.

Let us present decomposition of the dependence graph of the signal flow from the defined initial vertex P_p [16]. First of all, we obtain a tree structure with cycles (Fig. 4), described by Eq. (6). Since it is possible to come back from a given vertex to the former vertex or even the initial vertex, we obtain the analytic notation (7) determining the obtained tree structures with cycles. At the next stage we obtain a tree game structure shown in Fig. 5.

$$G_{Pp}^+ = ({}^0 Pp (1 - \frac{1}{Rpo} Qs ({}^2 Rl Pp, \frac{1}{C} \frac{dPs}{dt} ({}^3 \int dt Ps ({}^4 1 Pp, \frac{D}{J} \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{D}{C} \frac{dPs}{dt}, -\frac{R}{J} \frac{d\omega}{dt} ({}^6) {}^5) {}^4) {}^3) {}^2), \frac{K}{T} \frac{dQzp}{dt} ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp}{dt}, -1 Qs) {}^3) {}^2) {}^1) {}^0 \tag{6}$$

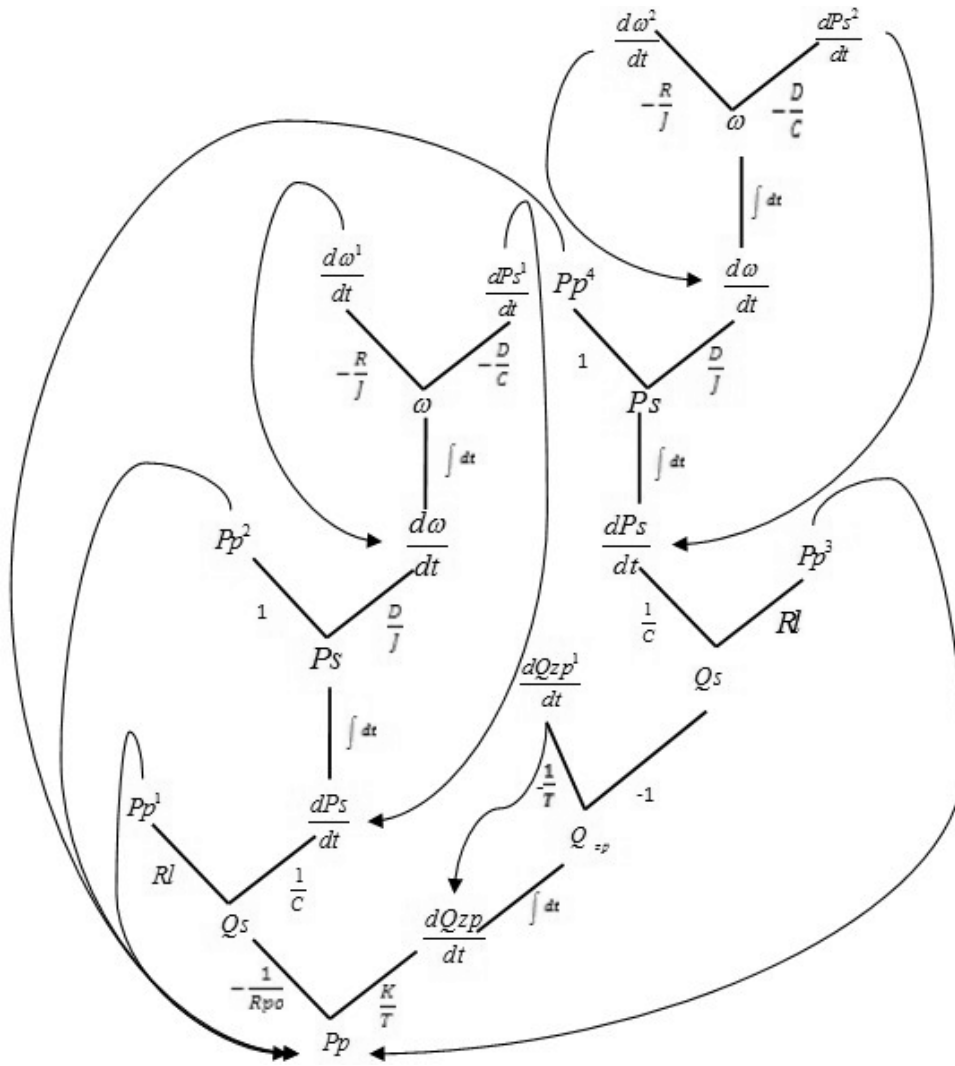


Fig. 4. Tree structure with cycles and the initial vertex P_p

$$G_{Pp}^{++} = ({}^0 Pp (1 - \frac{1}{Rpo} Qs ({}^2 Rl Pp^1, \frac{1}{C} \frac{dPs}{dt} ({}^3 \int dt Ps ({}^4 1 Pp^2, \frac{D}{J} \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{D}{C} \frac{dPs^1}{dt}, -\frac{R}{J} \frac{d\omega^1}{dt} ({}^6) {}^5) {}^4) {}^3) {}^2), \frac{K}{T} \frac{dQzp}{dt} ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp^1}{dt}, -1 Qs ({}^4 Rl Pp^3, \frac{1}{C} \frac{dPs}{dt} ({}^5 \int dt Ps ({}^6 1 Pp^4, \frac{D}{J} \frac{d\omega}{dt} ({}^7 \int dt \omega ({}^8 - \frac{D}{C} \frac{dPs^2}{dt}, -\frac{R}{J} \frac{d\omega^2}{dt} ({}^8) {}^7) {}^6) {}^5) {}^4) {}^3) {}^2) {}^1) {}^0 \tag{7}$$

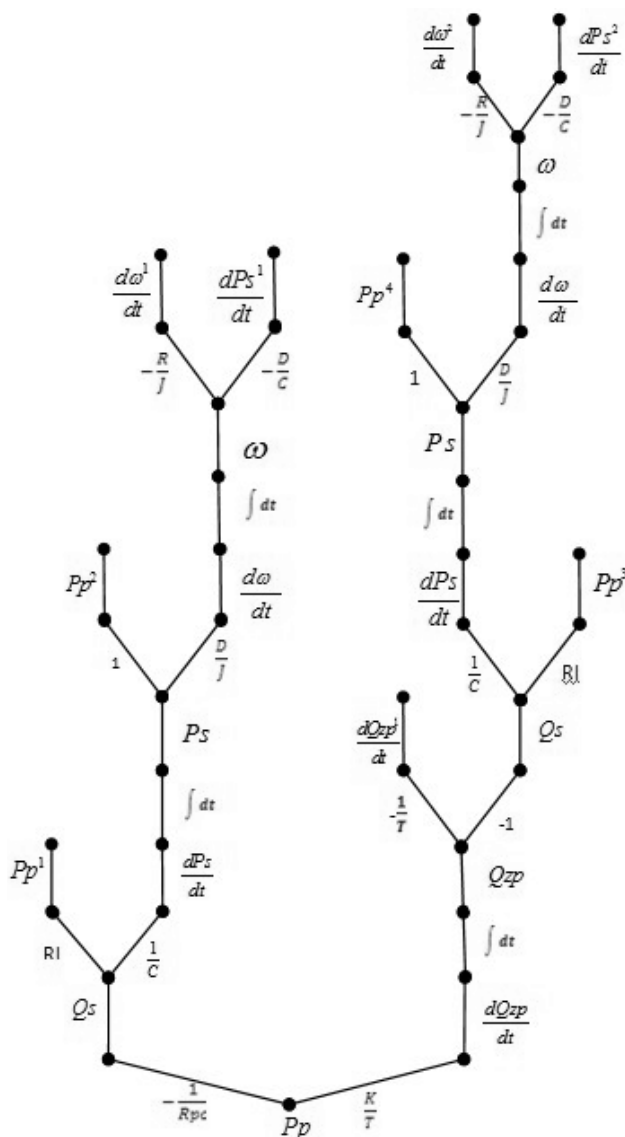


Fig. 5. Tree game structure from the initial vertex P_p .

According to the proper optimization procedure, the mathematical model describing the considered system should provide the maximum possible range of information. Thus, the dependence graph decomposition is performed from the given vertex, taking into account all

the constructional and/or service parameters separately written (not in the interaction method). Such decision decomposition leads to the tree structure with cycles and the tree game structure (Figs. 6 and 7) described by relationships (8) and (9).

$$\begin{aligned}
 G_{Pp}^+ = & ({}^0 Pp ({}^1 - \frac{1}{Rpo} Qs ({}^2 Rl Pp, \frac{1}{C} \frac{dPs}{dt} ({}^3 \int dt Ps ({}^4 1 Pp, D \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{1}{C} \frac{dPs}{dt}, \\
 & D \frac{dPs}{dt}, -\frac{1}{J} \frac{d\omega}{dt}, R \frac{d\omega}{dt})^6)^5), \frac{1}{J} \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{1}{C} \frac{dPs}{dt}, D \frac{dPs}{dt}, -\frac{1}{J} \frac{d\omega}{dt}, R \frac{d\omega}{dt})^6)^5)^4)^3)^2, \\
 & K \frac{dQzp}{dt} ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp}{dt}, -Qs)^3)^2, \frac{1}{T} \frac{dQzp}{dt} ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp}{dt}, -Qs)^3)^2)^1)^0
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 G_{Pp}^{++} = & ({}^0 Pp ({}^1 - \frac{1}{Rpo} Qs ({}^2 Rl Pp^1, \frac{1}{C} \frac{dPs}{dt} ({}^3 \int dt Ps ({}^4 1 Pp^2, D \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{1}{C} \frac{dPs^1}{dt}, D \frac{dPs^2}{dt}, \\
 & - \frac{1}{J} \frac{d\omega^1}{dt}, R \frac{d\omega^2}{dt})^6)^5), \frac{1}{J} \frac{d\omega}{dt} ({}^5 \int dt \omega ({}^6 - \frac{1}{C} \frac{dPs^3}{dt}, D \frac{dPs^4}{dt}, - \frac{1}{J} \frac{d\omega^3}{dt}, R \frac{d\omega^4}{dt})^6)^5)^4)^3)^2, K \frac{dQzp}{dt} \\
 & ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp^1}{dt}, -Qs ({}^4 Rl Pp^3, \frac{1}{C} \frac{dPs}{dt} ({}^5 \int dt Ps ({}^6 1 Pp^4, D \frac{d\omega}{dt} ({}^7 \int dt \omega ({}^8 - \frac{1}{C} \frac{dPs^5}{dt}, D \frac{dPs^6}{dt}, \\
 & - \frac{1}{J} \frac{d\omega^5}{dt}, R \frac{d\omega^6}{dt})^8)^7), \frac{1}{J} \frac{d\omega}{dt} ({}^7 \int dt \omega ({}^8 - \frac{1}{C} \frac{dPs^7}{dt}, D \frac{dPs^8}{dt}, - \frac{1}{J} \frac{d\omega^7}{dt}, R \frac{d\omega^8}{dt})^8)^7)^6)^5)^4)^3)^2, \frac{1}{T} \frac{dQzp}{dt} \\
 & ({}^2 \int dt Qzp ({}^3 - \frac{1}{T} \frac{dQzp^2}{dt}, -Qs ({}^4 Rl Pp^5, \frac{1}{C} \frac{dPs}{dt} ({}^5 \int dt Ps ({}^6 1 Pp^6, D \frac{d\omega}{dt} ({}^7 \int dt \omega ({}^8 - \frac{1}{C} \frac{dPs^9}{dt}, D \frac{dPs^{10}}{dt} \\
 & , - \frac{1}{J} \frac{d\omega^9}{dt}, R \frac{d\omega^{10}}{dt})^8)^7), \frac{1}{J} \frac{d\omega}{dt} ({}^7 \int dt \omega ({}^8 - \frac{1}{C} \frac{dPs^{11}}{dt}, D \frac{dPs^{12}}{dt}, - \frac{1}{J} \frac{d\omega^{11}}{dt}, R \frac{d\omega^{12}}{dt})^8)^7)^6)^5)^4)^3)^2)^1)^0 \quad (9)
 \end{aligned}$$

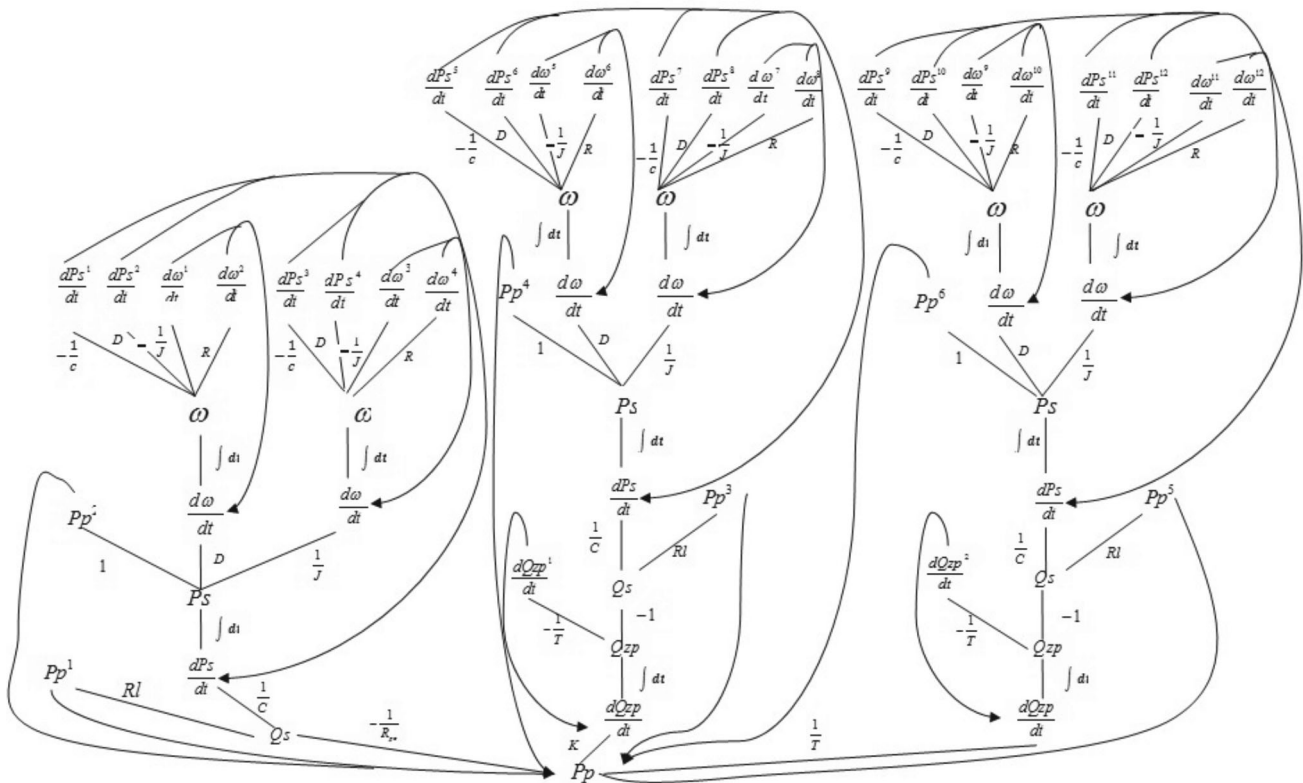


Fig. 6. Tree structure with cycles and the initial vertex P_p including decision decomposition.

Occurrence of numerous straight, oblique and complex feedbacks is one of dynamic properties of machine systems, resulting from their nature. The feedback provides influence of the results from the final block of the optimization process (in form of time characteristics of time-dependent quantities) on selection of and possibility of application of essential information flowing into the initial block. The given block can be considered as the closed system when there is a feedback causing that the input signal does not depend only on the internal state of the object, but on the present value of the output signal as well. Thus, it is necessary to introduce an additional initial vertex Q_p to the dependence graph notation. This addi-

onal vertex should be joined with the final vertex ω by the return decision of transition k . Finally, the mathematical model of the considered hydraulic system can be written with the dependence digraph of the signal flow shown in Fig. 8 [4], [5], [6].

Decomposition of the dependence graph from Fig. 8 from the initial vertex Q_s including decision decomposition leads to the tree structure with cycles (Fig. 9) and the game structure (Fig. 10), described by the relationship (10). The additional feedback on the tree structures is determined by suitable return arcs and decision allowing to change the input signal on the basis of the output signal [5], [6].

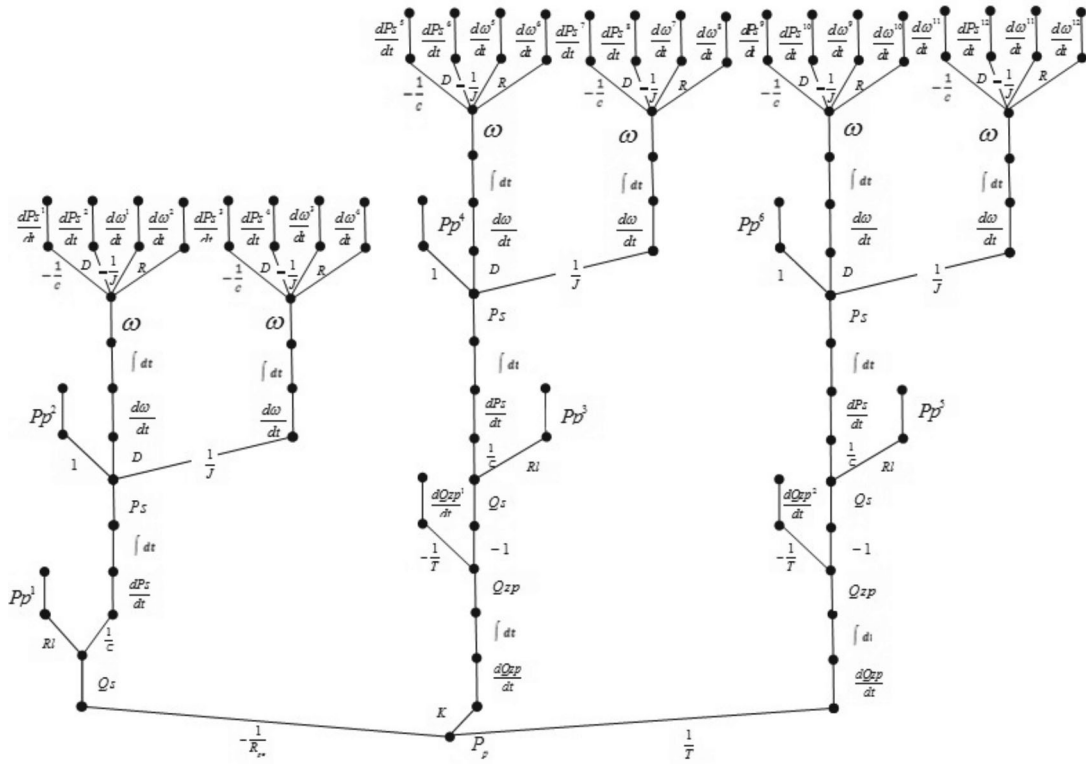


Fig. 7. Tree game structure from the initial vertex P_p including decision decomposition.

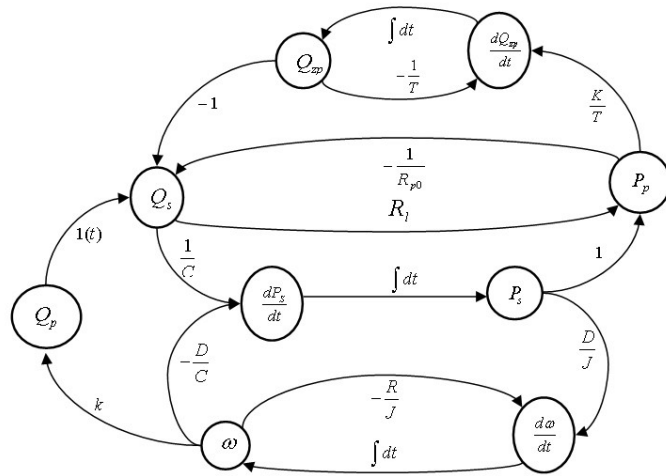


Fig. 8. Dependence digraph of the signal flow with the additional time vertex Q_p .

$$\begin{aligned}
 G_{Q_s}^{++} = & \left({}^0 Q_s \left({}^1 R P P \left({}^2 -\frac{1}{R_{p0}} Q_s^1, K \frac{dQ_{zp}}{dt} \left({}^3 \int dt Q_{zp} \left({}^4 -\frac{1}{T} \frac{dQ_{zp}}{dt}, -Q_s^2 \right)^4 \right)^3, \frac{1}{T} \frac{dQ_{zp}}{dt} \left({}^3 \int dt Q_{zp} \right. \right. \right. \\
 & \left. \left. \left({}^4 -\frac{1}{T} \frac{dQ_{zp}}{dt}, -Q_s^3 \right)^4 \right)^3 \right)^2, \frac{1}{C} \frac{dP_s}{dt} \left({}^2 \int dt P_s \left({}^3 1 P P \left({}^4 -\frac{1}{R_{p0}} Q_s^4, K \frac{dQ_{zp}}{dt} \left({}^5 \int dt Q_{zp} \left({}^6 -\frac{1}{T} \frac{dQ_{zp}}{dt}, \right. \right. \right. \right. \\
 & \left. \left. \left. -Q_s^5 \right)^6 \right)^5, \frac{1}{T} \frac{dQ_{zp}}{dt} \left({}^5 \int dt Q_{zp} \left({}^6 -\frac{1}{T} \frac{dQ_{zp}}{dt}, -Q_s^6 \right)^6 \right)^4, D \frac{d\omega}{dt} \left({}^4 \int dt \omega \left({}^5 k Q P \left({}^6 1 [t] Q_s^7 \right)^6, \right. \right. \\
 & \left. \left. -\frac{1}{C} \frac{dP_s^1}{dt}, D \frac{dP_s^2}{dt}, -\frac{1}{J} \frac{d\omega^1}{dt}, R \frac{d\omega^2}{dt} \right)^5 \right)^4, \frac{1}{J} \frac{d\omega}{dt} \left({}^4 \int dt \omega \left({}^5 k Q P \left({}^6 1 [t] Q_s^8 \right)^6, -\frac{1}{C} \frac{dP_s^3}{dt}, \right. \right. \\
 & \left. \left. D \frac{dP_s^4}{dt}, -\frac{1}{J} \frac{d\omega^3}{dt}, R \frac{d\omega^4}{dt} \right)^5 \right)^4 \right)^3 \right)^2 \right)^1 \right)^0
 \end{aligned}
 \tag{10}$$

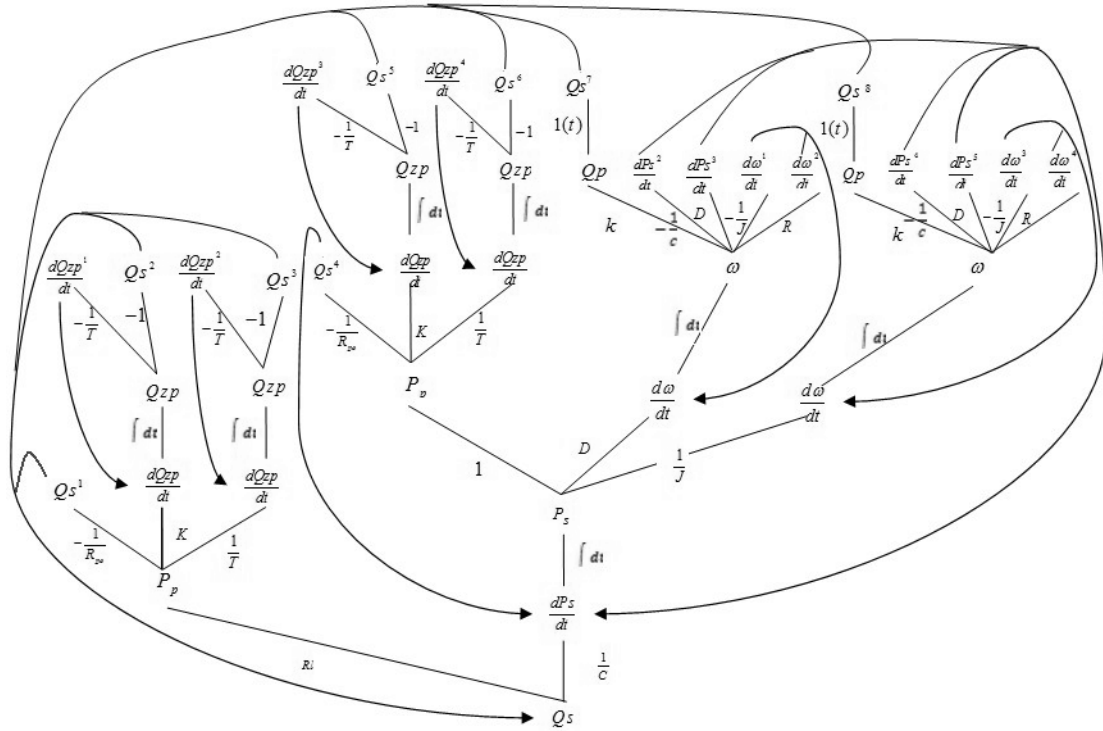


Fig. 9. Tree structure with cycles and the initial vertex Q_s with the additional time vertex Q_p .

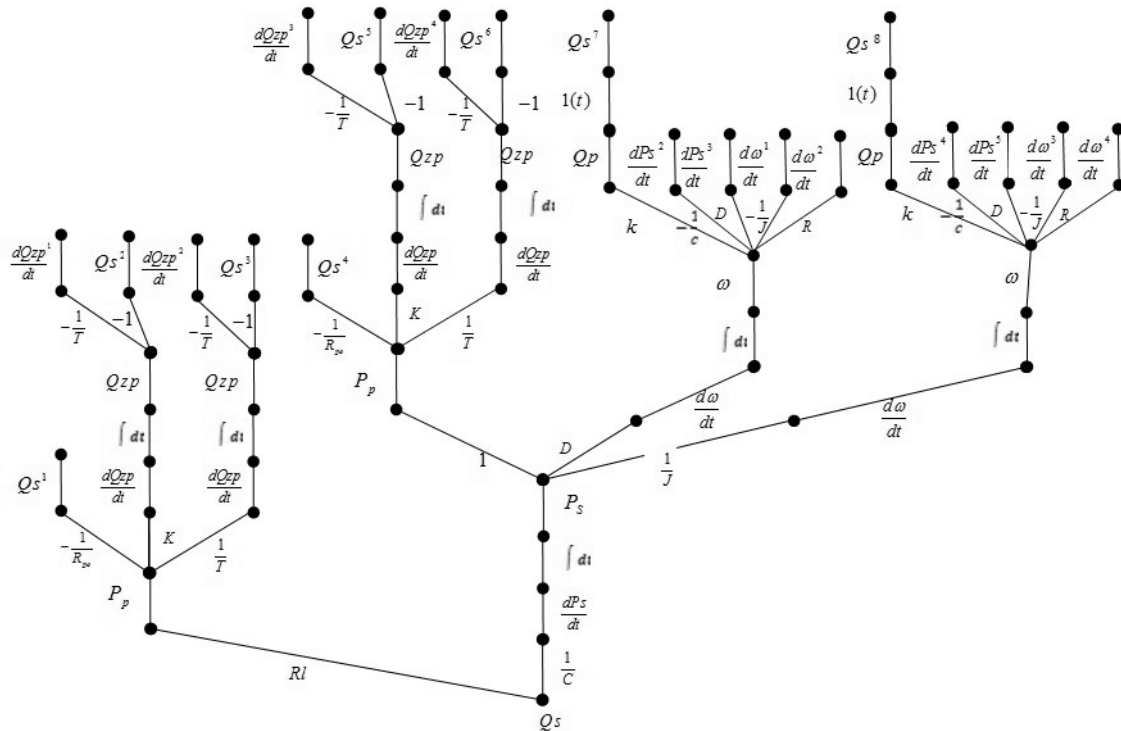


Fig. 10. Tree game structure from the initial vertex Q_s with the additional time vertex Q_p .

Considering tree structures we must determine element subordination in the system. Each structure has its proper analytic notation (G_i^+ and G_i^{++} , where i is the vertex from which the graph decomposition was started). Each element q_r has always subordinated elements q_i . Both q_r and q_i elements can occur many times in the expression G_i^{++} in

the brackets $(^k \dots)^k$ with different values of k , i.e. at various stages of the tree structure. Thus, it is necessary to introduce multiple numeration of vertices [4], [15]. Fig. 11 shows a tree structure from Fig. 10 including multiple vertex numeration described by relationship (11).

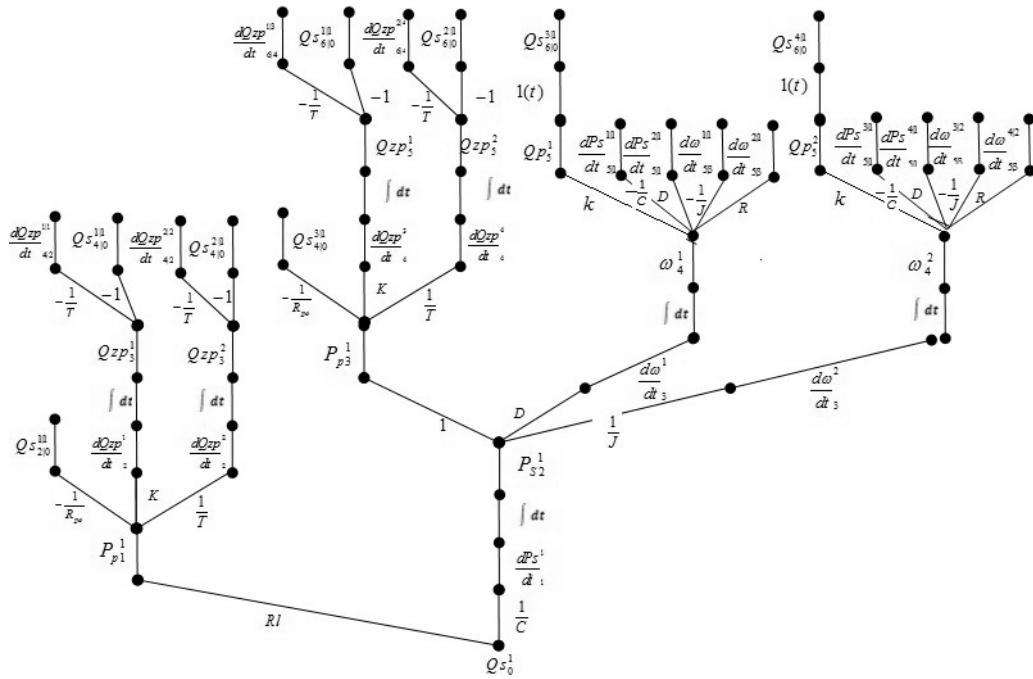


Fig. 11. Tree game structure from the initial vertex Q_s with the additional time vertex Q_p and multiple vertex numeration.

$$\begin{aligned}
 G_{Q_s}^{++} = & ({}^0Q_{s0}^1 ({}^1RIP_{p1}^1 ({}^2-\frac{1}{R_{po}} Q_{s20}^{11}, K \frac{dQ_{zp}^1}{dt} ({}^3 \int dt Q_{zp3}^1 ({}^4 -\frac{1}{T} \frac{dQ_{zp}^{11}}{dt}, -Q_{s40}^{11})^4)^3, \frac{1}{T} \frac{dQ_{zp}^2}{dt} ({}^3 \int dt Q_{zp3}^2 ({}^4 -\frac{1}{T} \frac{dQ_{zp}^{22}}{dt}, \\
 & -Q_{s40}^{21})^4)^3)^2, \frac{1}{C} \frac{dP_{s1}^1}{dt} ({}^2 \int dt P_{s2}^1 ({}^3 1P_{p3}^1 ({}^4 -\frac{1}{R_{po}} Q_{s40}^{31}, K \frac{dQ_{zp}^3}{dt} ({}^5 \int dt Q_{zp5}^1 ({}^6 -\frac{1}{T} \frac{dQ_{zp}^{33}}{dt}, -Q_{s60}^{31})^6)^5, \frac{1}{T} \frac{dQ_{zp}^4}{dt} ({}^5 \int dt Q_{zp5}^2 \\
 & ({}^6 -\frac{1}{T} \frac{dQ_{zp}^{24}}{dt}, -Q_{s60}^{21})^6)^5)^4, D \frac{d\omega^1}{dt} ({}^4 \int dt \omega_4^1 ({}^5 kQ_{p5}^1 ({}^6 [t]Q_{s60}^{31})^6, -\frac{1}{C} \frac{dP_{s1}^{11}}{dt}, D \frac{dP_{s1}^{21}}{dt}, -\frac{1}{J} \frac{d\omega^{11}}{dt}, R \frac{d\omega^{21}}{dt})^5)^4, \\
 & \frac{1}{Jdt} \frac{d\omega^2}{dt} ({}^4 \int dt \omega_4^2 ({}^6 [t]Q_{s60}^{41})^6, -\frac{1}{C} \frac{dP_{s1}^{31}}{dt}, D \frac{dP_{s1}^{41}}{dt}, -\frac{1}{J} \frac{d\omega^{31}}{dt}, R \frac{d\omega^{41}}{dt})^5)^4)^3)^2)^1) \quad (11)
 \end{aligned}$$

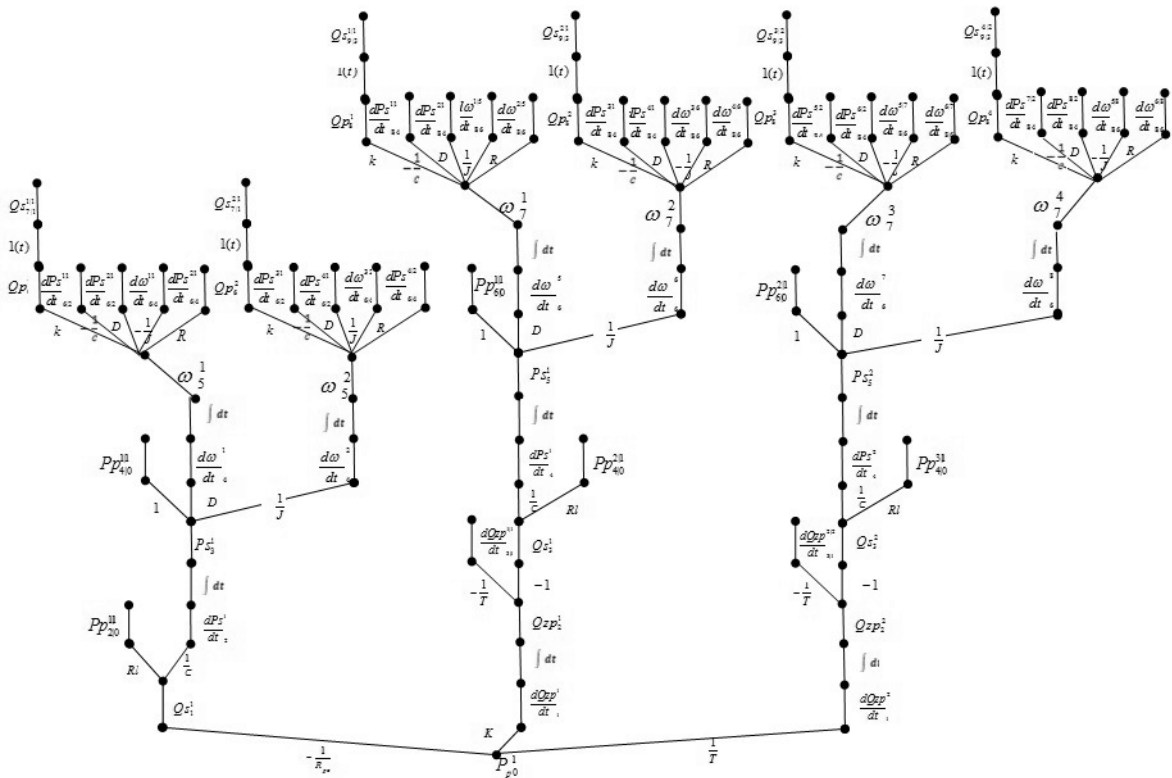


Fig. 12. Tree game structure from the initial vertex P_p including the time vertex Q_p and multiple vertex numeration.

$$\begin{aligned}
G_{PP}^{++} = & ({}^0 Pp_0^1 ({}^1 - \frac{1}{Rpo} Qs_1^1 ({}^2 RlPp_{20}^{11}, \frac{1}{C} \frac{dPs^1}{dt} ({}^3 \int dtPs_3^1 ({}^4 1Pp_{40}^{11}, D \frac{d\omega^1}{dt} ({}^5 \int dt\omega_5^1 ({}^6 kQp_6^1 ({}^7 \int [t] Qs_{71}^{11})^7, \\
& - \frac{1}{C} \frac{dPs^{11}}{dt} ({}^6_2, D \frac{dPs^{21}}{dt} ({}^6_2, - \frac{1}{J} \frac{d\omega^{11}}{dt} ({}^6_4, R \frac{d\omega^{21}}{dt} ({}^6_4)^6)^5), \frac{1}{J} \frac{d\omega^2}{dt} ({}^5 \int dt\omega_5^2 ({}^6 kQp_6^2 ({}^7 \int [t] Qs_{71}^{21})^7, - \frac{1}{C} \frac{dPs^{31}}{dt} ({}^6_2, \\
& D \frac{dPs^{41}}{dt} ({}^6_2, - \frac{1}{J} \frac{d\omega^{32}}{dt} ({}^6_4, R \frac{d\omega^{42}}{dt} ({}^6_4)^6)^5)^4)^3)^2, K \frac{dQzP^1}{dt} ({}^2 \int dtQzP_2^1 ({}^3 - \frac{1}{T} \frac{dQzP^{11}}{dt} ({}^3, - Qs_3^1 ({}^4 RlPp_{40}^{21}, \frac{1}{C} \frac{dPs^1}{dt} ({}^5 \\
& \int dtPs_5^1 ({}^6 1Pp_{60}^{11}, D \frac{d\omega^5}{dt} ({}^7 \int dt\omega_7^1 ({}^8 kQp_8^1 ({}^9 \int [t] Qs_{93}^{11})^9, - \frac{1}{C} \frac{dPs^{11}}{dt} ({}^8_4, D \frac{dPs^{21}}{dt} ({}^8_4, - \frac{1}{J} \frac{d\omega^{15}}{dt} ({}^8_6, R \frac{d\omega^{25}}{dt} ({}^8_6)^8)^7, \\
& \frac{1}{J} \frac{d\omega^6}{dt} ({}^7 \int dt\omega_7^2 ({}^8 kQp_8^2 ({}^9 \int [t] Qs_{93}^{21})^9, - \frac{1}{C} \frac{dPs^{31}}{dt} ({}^8_4, D \frac{dPs^{41}}{dt} ({}^8_4, - \frac{1}{J} \frac{d\omega^{36}}{dt} ({}^8_6, R \frac{d\omega^{46}}{dt} ({}^8_6)^8)^7)^6)^5)^4)^3)^2, \frac{1}{T} \frac{dQzP^2}{dt} ({}^2 \\
& \int dtQzP_2^2 ({}^3 - \frac{1}{T} \frac{dQzP^{21}}{dt} ({}^3, - Qs_3^2 ({}^4 RlPp_{40}^{31}, \frac{1}{C} \frac{dPs^2}{dt} ({}^5 \int dtPs_5^2 ({}^6 1Pp_{60}^{21}, D \frac{d\omega^7}{dt} ({}^7 \int dt\omega_7^3 ({}^8 kQp_8^3 ({}^9 \int [t] Qs_{93}^{32})^9, \\
& - \frac{1}{C} \frac{dPs^{52}}{dt} ({}^8_4, D \frac{dPs^{62}}{dt} ({}^8_4, - \frac{1}{J} \frac{d\omega^{57}}{dt} ({}^8_6, R \frac{d\omega^{67}}{dt} ({}^8_6)^8)^7, \frac{1}{J} \frac{d\omega^8}{dt} ({}^7 \int dt\omega_7^4 ({}^8 kQp_8^4 ({}^9 \int [t] Qs_{93}^{42})^9, - \frac{1}{C} \frac{dPs^{72}}{dt} ({}^8_4, D \frac{dPs^{82}}{dt} ({}^8_4, \\
& - \frac{1}{J} \frac{d\omega^{58}}{dt} ({}^8_6, R \frac{d\omega^{68}}{dt} ({}^8_6)^8)^7)^6)^5)^4)^3)^2)^1)^0
\end{aligned} \tag{12}$$

Decomposition of the dependence digraph from Fig. 8 from the initial vertex including decision decomposition and multiple vertex numeration leads to the game structure (Fig. 12) expressed by relationship (12) [4].

4. Conclusions

Application of dependence graphs and game tree structures allows to present a sequence of changes of arithmetic values of constructional and service parameters in order to obtain the required behaviour of the system (for example machine system). Unlike traditional dependence graphs and tree classifiers, the dependence graph with game tree structures includes connection of importance rank of vertices (states) and height of the tree structure.

Application of decision decomposition for game graphs and trees does not change types and graphical shapes of such structures. They are more complex but they keep the given structural properties resulting from the initial dependence graph. Thus, a local role of decomposition can be distinguished. The introduced decision decomposition eliminates interaction of constructional and service parameters because a designer can make a decision about only single changes and observation at the successive stages.

The introduced additional time vertex resulting from the physical model well describes the feedback loop on the dependence graph. It appears from the following statements:

- an additional decision is a guideline for a designing engineer when he/she is going to change the output signal on the basis of the input signal,
- the tree structure with cycles and the tree game structure preserve identical shapes like before introduction of the additional time vertex, but they have complementary branches at suitable floors.

The introduced multiple vertex numeration allows to consider and distinguish the same elements occurring on different floors of the tree structure. That also allows to

preserve a general shape of the graphical structure existing even before decomposition.

The algorithmic method of formation of graphical structures from the mathematical model of the system describes the optimization method of systematic search. The game structure describes a space of possible solutions in order to find optimum objective functions. There is connection with other graphical structures which can be graphs in another sense, or even decision trees with node and/or branch coding. Such interpretation can lead to different types of logical trees [2], [11], [13], [17], [21], [23], [24].

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