Possible accuracy of the Cotton-Mouton polarimetry in a sheared toroidal plasma

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Abstract. The Cotton-Mouton effect in the sheared plasma with helical magnetic lines is studied, using the equation for the complex amplitude ratio (CAR). A simple model for helical magnetic lines in plasma of toroidal configuration is suggested. Equation for CAR is solved perturbatively, treating the shear angle variations as a small perturbation, caused by the spiral form of the magnetic lines. It is shown that the uncertainty of the polarization measurements in the toroidal plasma with a spiral form of the magnetic lines does not exceed 1.0–2.0%, which determines the limiting accuracy of the Cotton-Mouton polarimetry. It is furthermore pointed out that the method of a priori subtraction of the "sheared" term may significantly improve the accuracy of the Cotton-Mouton polarimetry.

Key words: diagnostics plasma • Cotton-Mouton effect • complex polarization angle

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Introduction

In the Cotton-Mouton polarimetry one measures the phase difference

$$(1) \qquad \delta_{\mathrm{CM}}(L) = \int_0^L (k_1 - k_2)\,d\sigma$$

between two linearly polarized normal modes propagating through the magneto-active plasma [1–3, 5] of depth L. Here k_1 and k_2 are wave numbers, and σ is an arc length along the sounding ray. The phase difference (1) determines the shape of the polarization ellipse, which is subject to a direct measurement.

Equation (1) for the Cotton-Mouton phase difference is valid under the assumption that the orientation of the magnetic field B_0 does not change along the sounding ray. However, this is not the case in tokamak plasmas, where magnetic lines, formed by a superposition of the toroidal and the poloidal magnetic field, acquire a helical form [16].

Segre [14] has obtained an exact solution for the Stokes vector [6] in the homogeneous and uniformly sheared plasma. In distinction to [15], in this paper we describe the evolution of the electromagnetic wave in an inhomogeneous and non-uniformly sheared plasma. Instead of the Stokes vector formalism (SVF), which is widely used in the plasma polarimetry [14, 15], we describe the polarization state of the electromagnetic wave field using the technique of a CAR [6] which adequately characterizes various parameters of the polarization ellipse [2], including the commonly used Stokes vector and the complex polarization angle.

It is worth to emphasize that the polarimetry, along with the interferometry is the basic diagnostic method

in thermonuclear reactors, including the Joint European Torus (JET) and international thermonuclear experimental reactor (ITER). The plasma polarimetry provides essential information both on the plasma density and the magnetic field in thermonuclear reactors. Small changes in the polarization state, caused by the helical (spiral) form of magnetic field lines, are estimated in this paper using the perturbation theory. The perturbative method is applied to a first order ordinary differential equation for the polarization ratio $\xi = a_2/a_1$, which characterizes the shape of the polarization ellipse. Thereat advantage of the perturbative approach – as compared to the numerical solution of Eq. (4) – lies in the fact that the perturbation method immediately leads to an explicit expression for the deviation of the polarization ellipse from the idealized situation where the static magnetic field in plasma has only a toroidal component.

The quasi-isotropic approximation for electromagnetic waves in a weakly anisotropic plasma

The quasi-isotropic approximation (QIA) of geometrical optics approximation [4, 7, 8, 12, 13] provides an asymptotic solution of Maxwell equations for electromagnetic waves in a weakly anisotropic medium. In QIA, in the lowest (0-th) order the electromagnetic wave field is assumed to have the form:

$$(2) \qquad \mathbf{E} = \Gamma A(\mathbf{r})\exp[ik_0\Psi(\mathbf{r})]$$

where $\Psi(\mathbf{r})$ and $A(\mathbf{r})$ are respectively the eikonal and the amplitude of the scalar wave field in the isotropic medium with background permittivity $\varepsilon_0(\mathbf{r})$, and $\Gamma = \Gamma_1\mathbf{e}_1 + \Gamma_2\mathbf{e}_2$ is the polarization vector Γ orthogonal to the ray.

The complex amplitudes ratio ξ (CAR) is defined as [1, 11]

$$(3) \qquad \xi = \Gamma_2 / \Gamma_1$$

The evolution of CAR in a weakly magnetized plasma is described by an equation derived in [9] on the basis of QIA equations:

$$(4) \qquad \dot{\zeta} = \frac{1}{2}\Big[i\Omega_2(1-\zeta^2)-2i\Omega_1\zeta+\Omega_3(1+\zeta^2)\Big]$$

We are using here the plasma parameters Ω_1, Ω_2, Ω_3, $\Omega_\perp$, as suggested by Segre [14, 15].

A simple model of helical magnetic lines in the toroidal system

A convenient model of helical magnetic lines in the toroidal plasma was suggested in [10]. Let ϕ be a toroidal angle, measured relative to the z axis, let θ be the poloidal angle, measured relative to the vertical axis x, and let ρ be the distance from the circle $R = $ const., where R is the large radius of the toroidal surface (Fig. 1).

The Cartesian coordinates x, y and z, shown in Fig. 1, may be expressed via the toroidal variables θ, ρ
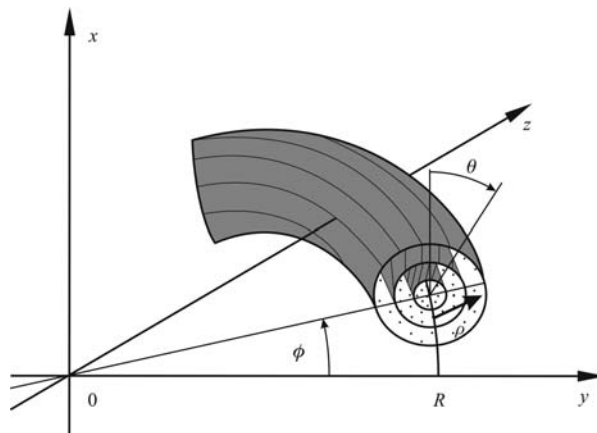
Fig. 1. Helical magnetic lines, formed by the superposition of the toroidal and poloidal magnetic fields in the toroidal plasma.

and ϕ as follows:

$$(5) \qquad x = \rho\cos\theta, \quad y = (R + \rho\sin\theta)\sin\phi,$$
$$z = (R + \rho\sin\theta)\cos\phi$$

Magnetic lines (5) are of a helical form, which may be modeled by assuming the following dependence of the poloidal angle θ on the azimuthal angle ϕ:

$$(6) \qquad \theta = \theta^\circ + \nu\varphi = \theta^\circ + \nu\frac{\mu}{R}$$

Here θ° is the initial poloidal angle. Instead of the toroidal angle ϕ we will use in the following the dimensionful parameter $\mu = \phi R$, which is an arc length along the circular axis $R = $ const. Finally, the parameter ν which describes the rate at which the poloidal angle θ does change along the circular axis, plays the role of a "helical" factor.

Let us assume that the sounding ray, crossing the plasma along the y axis, is represented by the equations

$$(7) \qquad z = -R + \sigma, \quad x = 0, \quad z = 0$$

Here arc length σ is measured from the point $z = -R$, where the ray crosses the circular axis. The point where the ray enters the plasma is given by $\sigma_0 = -a$, while $\sigma_{\text{exit}} = a$ is the point of exit. Correspondingly we have $z_0 = -(R + a)$ and $z_{\text{exit}} = -R + a$, as shown in Fig. 2.
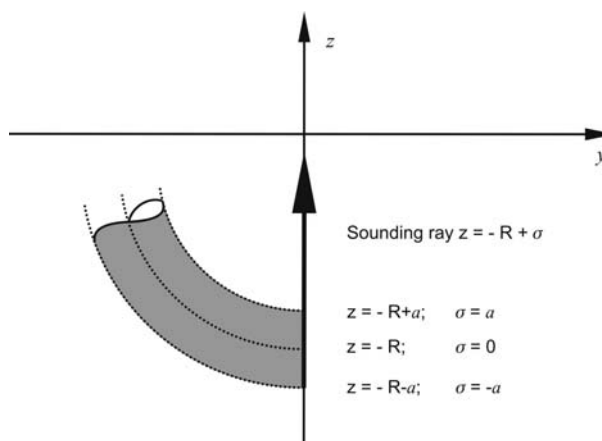
Fig. 2. The sounding ray, crossing the toroidal plasma along the z axis.

Variations of the polarization angle in the sheared plasma

Neglecting Faraday term $\Omega_3 (1 + \xi^2)$, one can express Eq. (4) in the form

$$(8) \qquad \xi = -i\Omega_1\xi + f(\xi).$$

where the Cotton-Mouton term

$$(9) \qquad -i\Omega_1\zeta = i\Omega_\perp\zeta\cos 2\tilde{\alpha}_\perp$$

dominates, whereas the term $f(\xi) = -(1/2)i\Omega_\perp(1-\xi^2)\sin 2\tilde{\alpha}_\perp$, diminishing in the limit of $\tilde{\alpha}_\perp \to 0$, serves as a small perturbation due to the sheared plasma.

Equation (4) with the dominant term (8) admits an exact solution

$$(10) \qquad \zeta_0(\sigma) = \zeta(\sigma_0)\exp[i\delta_S(\sigma)]$$

where $\xi(\sigma_0)$ is the initial value of the complex amplitude ratio $\xi(\sigma)$ at $\sigma = \sigma_0$ and

$$(11) \quad \delta_S(\sigma) = -\int_{\sigma_0}^{\sigma}\Omega_1(\sigma')d\sigma' = \int_{\sigma_0}^{\sigma}\Omega_\perp(\sigma')\cos 2\tilde{\alpha}_\perp(\sigma')d\sigma'$$

is the Cotton-Mouton phase difference in the sheared plasma. When $\tilde{\alpha}_\perp = 0$, the phase difference δ_S coincides with the shear-free Cotton-Mouton shift, described by Eq. (1).

The phase shift $\delta_S(\sigma)$ differs from the shear-free phase shift $\delta_0(\sigma)$ by the value

$$(12) \quad \delta_1(\sigma) = \delta_S(\sigma) - \delta_0(\sigma) = -2\int_{\sigma_0}^{\sigma}\Omega_\perp(\sigma')\sin^2\tilde{\alpha}_\perp(\sigma')d\sigma'$$

The numerical modeling performed in [11] had shown that for the parameters similar to those of the ITER reactor the value $\delta_1(a)$ is small enough: $\delta_1(a) \approx 0.02$. This phase difference is approximately 50 times smaller than the Cotton-Mouton phase shift $\delta_0 = 2\Omega_\perp a$, which we have accepted here to be $\pi/4$. Thus the perturbation caused by the helical magnetic lines is of the order of about 1–2% of the Cotton-Mouton phase difference (1).

A small value of $\delta_1(a)$ justifies the commonly accepted practice of ignoring the influence of a sheared plasma. Despite the smallness of the perturbation term $\delta_1(a)$, it plays, however, an important role in the polarimetric measurements, since it determines the potential accuracy of Cotton-Mouton polarimetry in for the plasma with helical magnetic lines.

Improving the accuracy of the Cotton-Mouton polarimetry in the toroidal plasma

The small shear term $\delta_1(a)$ may be further reduced, if we first estimate the shear angle $\tilde{\alpha}_\perp(\sigma)$ on the basis of a model of helical magnetic lines and then subtract the estimated perturbation $\delta_1(a)$ from the measured phase $\delta_{measured}(a) = \arg\xi(a)$. The value $\delta_{measured}(a) - \delta_1(a)$ may then be assumed to represent a shear-free phase difference. Such a simple method is expected to decrease the

uncertainty in the Cotton-Mouton phase difference at least a factor of 5–10. The same is true for the product $N_eB_\perp^2$, which is estimated on the basis of the Cotton-Mouton phase shift $\delta_{CM}(a)$.

Conclusions

Influence of the sheared plasma on the Cotton-Mouton effect is analyzed, using the Eq. (4) for the complex amplitude ratio in a weakly anisotropic plasma, together with the simplified model for helical-like magnetic lines in a toroidal plasma, as suggested in [10]. Equation for the complex polarization angle (CPA) is solved using the perturbative approach, based on the assumption that the variable part of the shear angle is sufficiently small. It is shown that uncertainty of polarimetric measurements in the conditions of the ITER plasma might be of the order of few percent of the unperturbed (shear-free) solution. A simple algorithm for the processing of experimental data is suggested, which allows for a noticeable decrease in the uncertainty of the polarimetric measurements, caused by the sheared plasma.

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