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ORIGINAL PAPER

# Ponderomotive self-focusing of a short laser pulse under a plasma density ramp

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**Abstract.** The ponderomotive self-focusing of a short laser pulse in an underdense plasma under a plasma density ramp is analyzed. The pulse may acquire a minimum spot size due to the ponderomotive self-focusing. Beyond the focus, the nonlinear refraction starts weakening, and the spot size of the laser pulse increases, resulting in an oscillatory self-focusing and defocusing behavior of the beam with the propagation distance. In order to minimize the defocusing, we introduce a localized upward plasma density ramp. Due to the upward plasma density ramp, the laser beam retains a minimum spot size. Self-focusing becomes stronger with a mild ripple as the propagation distance increases. The conditions for the ponderomotive self-focusing for suitable parameters of the laser beam and the plasma are determined. The plasma density ramp of the considered type may be observed in gas jet plasma experiments.

**Key words:** self-focusing • underdense plasma • plasma density ramp • ponderomotive force • short laser pulse • Gaussian beam

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## Introduction

Recent advances in the short pulse laser technology opened the way to experiments involving laser pulses focused to extremely high intensites, in the range of $I \geq 10^{19}$ W/cm$^2$. This made possible the exploration of a variety of phenomena in both the atomic and plasma physics. Propagation of intense laser beams in plasma is a phenomenon relevant for many applications, such as X-ray lasers, laser-driven particle accelerators, soft X-ray generation etc. [1–3]. The laser-plasma interactions are also a subject of worldwide research due to their importance for the laser-induced nuclear fusion. In these applications there is a need for the laser pulse to propagate over several Rayleigh lengths, while preserving an efficient interaction with the plasma. At high intensities the relativistic effects in the laser pulse propagation in the plasma lead to the self--focusing because the dielectric constant of the plasma is an increasing function of the intensity. Additionally, the ponderomotive force of the focused laser beam pushes the electrons out of the region of high intensity, reducing the local electron density, which leads to the further increase of the plasma dielectric function and consequently an even stronger self-focusing of the laser pulse [4, 6, 9].

The ponderomotive self-focusing and relativistic self-focusing had been observed in many experiments and were proven to be efficient mechanisms to guide a laser pulse over long distances. Sarkisov *et al.* [14] observed relativistic self-focusing and channel formation by using an intense laser pulse of an axial intensity of

$6 \times 10^{18}$ W/cm$^{-2}$. Osman *et al.* [11] presented numerous theoretical discussions on the concept of the relativistic self-focusing of a high-power laser and its applications in the fast ignition. Esarey *et al.* [4] presented an extensive review of the self-focusing and self-guiding of short laser pulses in ionized gases and plasmas. Hafizi *et al.* [8] studied the propagation of an intense laser beam in plasma, including the relativistic and ponderomotive effects. They described the oscillations of the laser spot size in terms of an effective potential and observed that in the case of very intense laser beams the ponderomotive force displaces electrons in the radial direction, which may lead to the cavitation in the plasma. They found that a significant reduction in the spot size and a corresponding increase in the intensity become possible for laser powers exceeding the critical power for the relativistic self-focusing. In the previous studies it was observed that the laser can acquire a minimum spot size due to the relativistic self-focusing in the plasma. Beyond the focus, the nonlinear refraction starts weakening and the spot size of the laser increases, showing oscillatory behavior as a function of the propagation distance. To overcome the diffraction and the high-amplitude oscillation of the spot size, Gupta *et al.* [7] proposed to introduce a slowly increasing plasma density. They studied the effect of a plasma density ramp on the relativistic self-focusing of a laser beam and found that the self-focusing is increased in such a case. Later on they studied the propagation of a high power Gaussian laser beam through plasma with a density ramp where a magnetic field is present. They showed that the combined effect of the plasma density ramp and the magnetic field is to further enhance the self-focusing of the laser beam. Not only the spot size of the laser beam is reduced, but also it is maintained with only a mild ripple over several Rayleigh lengths [5]. The self-focusing of cylindrical beams due to the ponderomotive force is quite different because it depends on the full exponential nonlinearity rather than the cubic nonlinearity in the weak-coupling limit. A steady self-trapped propagation becomes then possible for an arbitrary incident power. The self-focusing becomes a periodic oscillatory phenomenon [10].

The plasma density transition plays an important role in laser-plasma interactions. Suk *et al.* [15] proposed a scheme for plasma electron trapping involving the density transition. Such a density ramp may be relevant for the self-focusing of a high-power laser pulse for particular laser and plasma parameters. This kind of plasma density ramp may be observed in a gas jet plasma experiments and resembles a plasma lens [12]. Sadighi-Bonabi *et al.* [13] showed that with proper plasma density ramp the spot size oscillations of the laser beam intensify and their amplitude decreases. This causes the laser beam to become more focused, which facilitates the penetration deeper into the plasma due to the reduced diffraction.

Our motivation in the present work is to study the nonlinear propagation of a high power laser in a slowly varying upward plasma density ramp, when the ponderomotive effects are operative. Generally, the self-focused laser diffracts and focuses periodically because of the mismatch between the channel and the spot size. For a given laser spot size, the oscillation amplitude becomes larger for a higher plasma density due to the enhanced relativistic effect. However, by slowly increasing the density, the oscillation amplitude of the laser spot size can be significantly reduced. As the laser propagates through the density ramp region, it sees a slowly narrowing channel. In such an environment, the oscillation amplitude of the spot size shrinks, while its frequency increases, which is a result of the adiabatic invariance theorem. The laser tends to become therefore more focused during the propagation in a plasma density ramp. Since the equilibrium electron density is an increasing function of the distance of propagation of the laser, additionally the diffraction length decreases rapidly as the beam penetrates deeper and deeper into the plasma. Consequently, the diffraction is reduced and the laser beam becomes more focused. This work has direct application to plasma-based accelerators, where a plasma density transition is very important for plasma electron injection into the acceleration stage.

In section *Nonlinearity induced by the ponderomotive force* we calculate the nonlinearity due to the ponderomotive effect. In section *Self-focusing equation in an inhomogeneous plasma* we discuss the evolution of the beam-width parameter and we find the condition for laser self-focusing. The numerical results are discussed in section *Numerical analysis* and the conclusions are presented in *Conclusion*.

## Nonlinearity induced by the ponderomotive force

Let us consider the propagation of a Gaussian laser beam through an unmagnetized cold plasma with the electron density $n$ and a density gradient (ramp) along the $z$ direction. The electric field of the laser beam is given by

$$\vec{E} = \vec{A}(z,r)\exp[i(\omega t - kz)] \quad (1)$$

where $\omega$ is the frequency of laser beam, $c$ is the speed of light, and $k(z) = (\omega/c)\varepsilon_0^{1/2}$.

The group velocity of the laser increases with laser intensity; hence, the rear portion of the pulse overlaps with the front portion, causing stronger focusing and sharpening of the pulse as investigated by Upadhyay *et al.* [16]. At $z = 0$, the intensity distribution of the beam in this situation is expressed as $EE^* = E_0^2\exp(-r^2/r_0^2f^2)$ where $r$ is the radial component of the cylindrical coordinate system and $r_0$ is the initial beam width or spot size of laser, and $f(z)$ is the so-called beam-width parameter.

We assume that the pulse propagates without changing shape. We also limit our attention to the case of a linearly polarized laser pulse. A linearly polarized pulse is more complex to study because the analytic simplifications that are possible in the case of circularly polarized laser pulses, which lack higher harmonic content, do not apply. In addition, for a transverse circularly polarized electromagnetic wave, the electrons in the laser field move along a circular trajectory. Its longitudinal momentum is equal to zero, and the transverse momentum is nonzero. For a linearly polarized electromagnetic wave propagating in the plasma the transverse and longitudinal motions of the electrons are always coupled and have a nonzero value. Also, an intense circularly polarized laser light is unstable in the Kerr media, as investigated in an experiment by Close *et al*. [2].

The expression for the dielectric constant has the following form

(2) $$\varepsilon = \varepsilon_0 + \Phi(EE^*)$$

where $\varepsilon_0$ is the linear part of the dielectric constant and $\Phi(EE^*)$ is the term that arises due to the ponderomotive nonlinearity.

The linear part of the dielectric constant has the form

(3) $$\varepsilon_0 = 1 - \frac{\omega_p^2(z)}{\omega^2}$$

where $\omega_p(z) = (4\pi n(z)e^2/m)^{1/2}$ is the electron plasma frequency. Here $e$ is the charge of the electron, $m$ is its rest mass and $n(z)$ is the electron density. The upward plasma density ramp profile can be modeled by a simple expression $n(\xi) = n_0 \tan(\xi/d)$, where $\xi = z/R_{d0}$, $R_{d0} = kr_0^2$ is the characteristic length of the diffraction divergence, $n_0$ is the initial electron density and $d$ is an adjustable constant. This kind of a density profile can be achieved in an experiment by using transient laser boring on the gas jet that is relevant to laser wake field acceleration (LWFA) [1].

In the perturbative approximation

(4) $$\Phi(EE^*) \approx \tfrac{1}{2}\varepsilon_2 EE^*$$

where: $\varepsilon_2 = 3\omega_p^2(z)m\alpha/2\omega^2 M$, $\alpha = e^2M/6m^2\omega^2KT_0$, and $T_0$ is the electron temperature at the equilibrium. Therefore the dielectric constant of the plasma is given by the formula $\varepsilon = \varepsilon_0 + \Phi(EE^*)$, where $\varepsilon_0 = 1 - \omega_p^2(z)/\omega^2$ and $\Phi(EE^*) \approx \tfrac{1}{2}\varepsilon_2 EE^*$.

The density inhomogeneity enters into these formulae via the $z$ dependence of $\omega_p$.

## Self-focusing equation in an inhomogeneous plasma

The wave equation governing the electric vector of a laser beam in a plasma with the effective dielectric constant can be written as

(5) $$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}\varepsilon \vec{E} = 0$$

We are considering the propagation of a Gaussian laser beam along the $z$ direction, starting at $z = 0$. The electric field of the laser has the form

(6) $$\vec{E} = \vec{A}(z,r)\exp[i(\omega t - kz)]$$

where $k(z) = (\omega/c)\varepsilon_0^{1/2}$.

In the cylindrical coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

Applying that to Eq. (5), we obtain

(7) $$-2ik\frac{\partial \vec{A}}{\partial z} + \frac{\partial^2 \vec{A}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{A}}{\partial r} + \frac{\omega_0^2}{c^2}\Phi(EE^*)\vec{A} = 0$$

We now introduce an eikonal, $\vec{A} = A_0(z,r)\exp[ikS(z,r)]$, where $A_0(z,r)$ and $S(z,r)$ are real functions of space variables. We substitute the expression for $A$ into Eq. (7) and separate the real and imaginary parts of the resulting equation. From the real part we obtain the equation

(8) $$\frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) - \left(\frac{\partial S}{\partial r}\right)^2 - 2\frac{\partial S}{\partial z} + \frac{\omega^2}{k^2 c^2}\Phi(EE^*) = 0$$

From the maginary part we obtain

(9) $$\frac{\partial A_0^2}{\partial z} + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0$$

To find a solution of these equations describing a Gaussian beam and satisfying the appropriate initial conditions we expand $S$ as follows:

(10a) $$S = \frac{r^2}{2}\beta(z) + \varphi(z)$$

For an initially Gaussian beam, we may write

(10b) $$A_0^2 = \frac{E_0^2}{f}\exp\left[-\frac{r^2}{r_0^2 f^2}\right]$$

In Eq. (10) $\beta$ represents the inverse of the radius of curvature of the wave front and $r_0 f$ represents the width of the beam. In the geometrical optics approximation $r = r_0 f(z)$ represents the ray trajectory in the $(r, z)$ plane.

Inserting the expressions (10a) and (10b) into Eqs. (8) and (9), we get the self-focusing equation in the form

(11) $$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \frac{R_d^2}{2}\frac{\varepsilon_2}{\varepsilon_0}\frac{E_0^2}{r_0^2 f^3}$$

Equation (11) describes the variation of the beam width parameter $f$ with the distance of propagation $\xi$. The first term on the right-hand side corresponds to the diffraction divergence of the beam and the second term corresponds to the convergence resulting from the nonlinearity. Equation (14) is a second-order differential equation, which we solve assuming the suitable laser and plasma parameters. In this way one can find a suitable condition for the self-focusing of the laser during the propagation through the density ramp. We use the initial conditions $f = 1$ and $\partial f/\partial z = 0$ at $z = 0$.

## Numerical analysis

To obtain concrete results, we solve the beam-width parameter quation for the Ti:sapphire-Nd:glass laser beam of intensity $I \sim 5.7 \times 10^{17}$ W/cm$^{-2}$, laser wavelength of 1 μm, the beam spot size of 40 μm, and the initial electron density $n_0 = 1.1 \times 10^{19}$ cm$^{-3}$. Figure 1 shows that the normalized plasma frequency (plasma density distribution function) increases linearly with the normalized propagation distance.

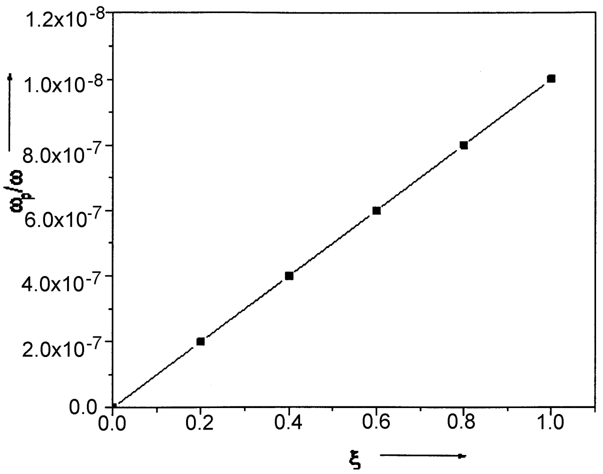


**Fig. 1.** Variation of the plasma density distribution function ($\omega_{p0}/\omega$) with the normalized distance ($\xi$). The normalized parameters are $d$ =10 and ($\omega_{p0}/\omega$) = 0.2.

Figures 2, 3, and 4 show the variation of the beam-width parameter $f$ as a function of the normalized propagation distance $\xi = z/R_{d0}$ in an underdense plasma with an upward density ramp and without the plasma density ramp. Now as the short laser pulse interacts with the plasma, the ponderomotive force comes into play. So in the region of a low plasma density the electrons are expelled from the high-intensity region by a ponderomotive force. If there is no density ramp, the beam-width parameter decreases monotonically up to a Rayleigh length because of the nonlinear effects. As the diffraction effects become predominant, the beam-width parameter increases after attaining a minimum value, and the laser beam starts diverging as a result of the saturation of the nonlinearity. Hence, the laser pulse goes through the stages of focusing and defocusing, and shows an oscillatory behavior. To avoid the laser pulse defocusing, we introduce an upward plasma density ramp. If there is a density ramp, the beam-width parameter decreases up to the Rayleigh length and then does not increase much, as in the case with no ramp, for the suitable parameters given above. After a couple of Rayleigh lengths the beam-width parameter attains a minimum value and maintains it for a longer distance. The saturation behavior of the beam-width parameter shows the strong self-focusing of the laser in a plasma with a density ramp. The physics behind the laser self-focusing during the propagation in an upward plasma density ramp may be understood as follows. After an initial focusing of the laser pulse, the relativistic mass effect will be much more pronounced in the region of increasing plasma density. Therefore, the laser focuses more during the propa-

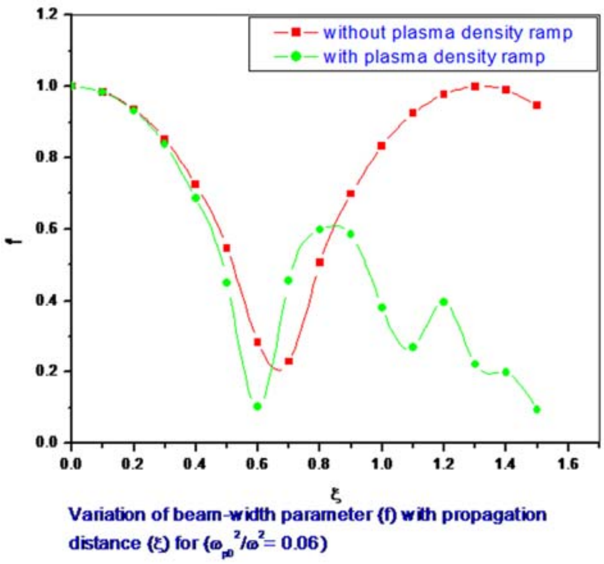


**Fig. 3.** Variation of the beam-width parameter ($f$) as a function of the normalized propagation distance ($\xi$) in the presence of the plasma density ramp (green curve) and without the plasma density ramp (red curve). The values of the parameters are ($\omega_{p0}^2/\omega^2$) = 0.06, $\omega r_0/c$ = 50, $v_{th} \sim 3 \times 10^{-2}\, c$ and $v_{osc} \sim 10^{-2}\, c$.

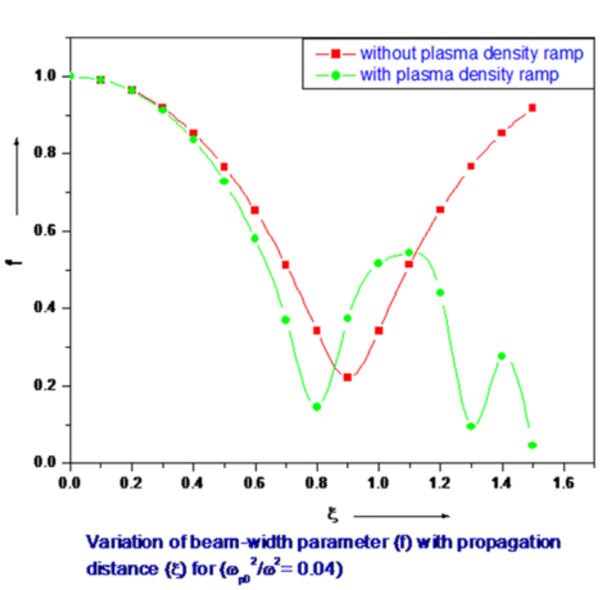


**Fig. 2.** Variation of the beam-width parameter ($f$) with the normalized propagation distance ($\xi$) in the presence of the plasma density ramp (green curve) and without the plasma density ramp (red curve). The values of the parameters are ($\omega_{p0}^2/\omega^2$) = 0,04, $\omega r_0/c$ = 50, $v_{th} \sim 3 \times 10^{-2}\, c$ and $v_{osc} \sim 10^{-2}\, c$.

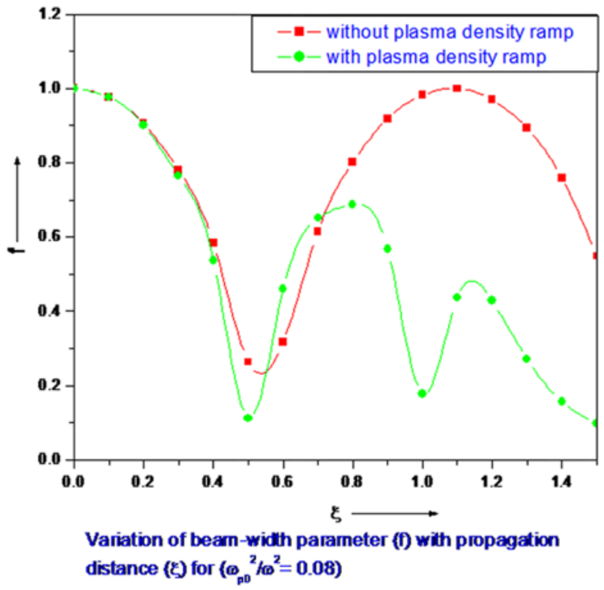


**Fig. 4.** Variation of the beam-width parameter ($f$) as a function of the normalized propagation distance ($\xi$) in the presence of the plasma density ramp (green curve) and without the plasma density ramp (red curve). The values of the parameters are ($\omega_{p0}^2/\omega^2$) = 0.08, $\omega r_0/c$ = 50, $v_{th} \sim 3 \times 10^{-2}\, c$ and $v_{osc} \sim 10^{-2}\, c$.

gation in a plasma density ramp. On the other hand, since the equilibrium electron density is an increasing function of the distance of propagation of the laser pulse, the plasma dielectric constant decreases rapidly as the beam penetrates deeper and deeper into the plasma. Consequently, the self-focusing effect is enhanced and the laser is more focused. Hence, the upward plasma density ramp plays an important role in enhancing the focusing of the laser beam.

The length of the plasma density ramp was chosen so as to avoid the maximum defocusing of the laser pulse. The laser pulse is being focused up to $0.2R_{d0}$ in both cases (with or without density ramp). Better focusing is observed by increasing the length of the density ramp. But the plasma density should not be too big, since otherwise the laser pulse would be reflected because of the overdense plasma effect. Here we consider the numerical parameters where the electron cavitation does not take place. If the electron cavitation would occur, the electron density will be zero inside the cavitation zone. In such a case the plasma effect would not be dominant. Hence the laser would be defocused during the propagation inside the channel. Therefore the parameters had been chosen in such a way that the plasma effect was operative and the electron cavitation was avoided. From the Figs. 2, 3, and 4 we see that in both cases (with and withouth the plasma density ramp), the laser spot size shows an oscillatory behavior as a function of the propagation distance. It means that the self-focusing increases in the presence of the plasma density ramp.

## Conclusion

The plasma density ramp plays an important role in the laser-plasma interaction. The density ramp may be imporatant for the self-focusing of short laser pulses if the laser and plasma parameters are chosen in an appropriate way. Here, we study the nonlinear propagation of a short laser pulse in a slowly varying upward plasma density ramp. The self-focused laser pulse diffracts and focuses periodically because of the mismatch between the channel size and the spot size. For a given laser spot size, the oscillation amplitude becomes larger for a higher plasma density due to the enhanced relativistic effect. By slowly increasing the density, the oscillation amplitude of the laser spot size can be significantly reduced. As the laser propagates through the density ramp region, it sees a slowly narrowing channel. In such an case the oscillation amplitude of the spot size shrinks, while its frequency increases. Therefore, the laser pulse propagating in a plasma density ramp tends to become more focused. If there is no density ramp, the laser pulse is defocused due to the dominance of the diffraction effect. As the plasma density increases, the self-focusing effect becomes stronger. Similarly as in the case of no density ramp, the beam-width parameter does not increase much. After several Rayleigh lengths, the beam-width parameter attains a minimum value and maintains it for a long distance. Consequently, the self-focusing effect is enhanced and the laser pulse is more focused.

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