Motion Planning of Wheeled Mobile Robots Subject to Slipping

Katarzyna Zadarnowska, Adam Oleksy

Abstract:

Most dynamic models of wheeled mobile robots assume that the wheels undergo rolling without slipping, thus in general wheeled mobile robots are considered as nonholonomic systems. This paper deals with the problem of motion planning of mobile robots, whose nonholonomic constraints have been violated during the motion. The slipping phase is studied in details. A static model of interaction forces between wheels and ground is adopted by means of the singular perturbation approach [2]. A novel control theoretic framework for mobile robots subject to slipping is defined: both kinematics and dynamics of a mobile robot are modeled as a control system with outputs, the performance of a locally controllable system is nontrivial, the Jacobian of the mobile robot is defined in terms of the linear approximation to the system [36]. A novelty of the methodology consists in respecting of the analogy between the stationary and mobile robots and deriving performance characteristics from local controllability. In this paper we address the problem of motion planning by means of the Jacobian pseudo inverse algorithm. The effectiveness of the Jacobian pseudo inverse motion planning algorithm is demonstrated with reference to differential drive type robot (Pioneer 2DX) subject to slipping.

Keywords: *dynamics, kinematics, motion planning, slipping effect, wheeled mobile robot.*

1. Introduction

Wheeled mobile robots are most often assumed to be capable of rolling without slipping, and modeled as nonholonomic systems. Numerous literature sources deal with mobile robots subject to nonholonomic constraints (namely pure rolling and nonslipping condition) [1, 5, 14, 36, 40]. Such an assumption is, however, far from realistic in practice. In fact, due to various effects such as slipping, sliding, compliance of the wheels, the ideal constraints are never strictly satisfied. Since friction is the major mechanism for generating forces acting on the vehicle's wheels, the problem of modeling and predicting tire friction has always been an area of intense research in the automotive industry. However, accurate tire–ground friction models are difficult to obtain symbolically, the most common tire friction models used in the literature are static [22, 30]. The dynamic friction models attempt to capture the transient behaviour of the tire–road contact forces under time–varying velocity conditions [7, 13]. The modeling of friction forces exerted at the wheels has been used for various studies [3, 29, 35, 39, 41]. A trajectory tracking problem for wheeled mobile robots subject to some drift and slipping effects, when the velocities become significant and therefore do not satisfy perfectly the ideal constraints, was undertaken by d'Andrea-Novel and coauthors [2] by means of the singular perturbation approach. A robust controller, that handles uncertain soil parameters, for trajectory tracking of four–wheel differentially driven, skid–steering vehicles has been presented in [8]. A traction control strategy minimizing slip in rough terrain has been presented in [6, 18, 24].

In this paper we are concerned with a problem of motion planning of wheeled mobile robots, for which nonholonomic constraints of pure rolling and nonslipping have been violated during the motion. The problem can be stated as follows: given a desirable location of the mobile robot in the taskspace, find a configuration in which the robot reaches the desirable position and orientation in a prescribed time horizon. The inverse kinematic algorithm based on the inverse of the Jacobian is the most popular method [4, 27]. Limitations of the methods presented in the abovementioned literature have been discussed in [42]; the inverse Jacobian method is not applicable to mobile robots.

As an alternative, the endogenous configuration space approach [36], which applies to mobile robots, has been proposed. This alternative has been inspired by the continuation method paradigm in motion planning of nonholonomic systems [34], and by the existing theory of stationary manipulators. As a starting point the methodology assumes a control system representation of the mobile robot and postulates that the performance of a locally controllable system should be non-trivial. The *end point map* of the system plays the role of the kinematics and dynamics of the mobile robot. Control actions exerted on the system are regarded as the *endogenous configurations* of the mobile robot. The *Jacobian* of the mobile robot is defined in terms of the linear approximation to the control system. As far as the Jacobian equation is concerned, a distinction between *singular* and *regular configuration* has been made. The motion planning problem for mobile robots is formulated as a control problem with prescribed control time horizon. The endogenous configuration space approach applies to any mobile robot. Inverse kinematics algorithms based on the pseudo inverse of the Jacobian for mobile robots have been examined in [20, 37]. The literature dealing with extension of the endogenous configuration space approach to the motion planning of nonholonomic mobile robots whose control system representation includes not only kinematics but also dynamics of the robotic system is rather modest [32]. The motion planning problem of mobile robots subject to slipping effects examined using traditional methods have been presented in [2, 9, 19, 31, 33].

Assuming the control theoretic point of view, our main objective will be to show that the motion planning problem for a mobile robot subject to slipping effects can be solved within the endogenous configuration space approach [36]. We shall begin with a description of the robot dynamics. To this aim we shall assume a linear dependence of the traction forces on the slip of the wheels, and then incorporate the obtained traction forces into the singular perturbation model introduced in [2], see also [12, 15]. This procedure will provide a control system representation of the robot motion, comprising a slow and a fast subsystem. The motion planning problem will be solved by means of a Jacobian inversion of the task map. To this aim we shall employ the Jacobian pseudo inverse. We show the functionality of the Jacobian pseudo inverse motion planning algorithm elaborated within the endogenous configuration space approach. To our best knowledge, an application of the endogenous configuration space methodology to the motion planning problem for mobile robots subject to slipping have not been tackled yet. We believe that the Jacobian motion planning algorithm designed by combining the singular perturbation modeling and the endogenous configuration space approach is a specific contribution of this paper. By design, the motion planning algorithm proposed in this paper applies to any control system representation having controllable linear approximation. The proposed motion planning method provides the open–loop control functions. Nevertheless, there exist some modifications of the method, which take into consideration the uncertainties of the model (e.g. Iterative Learning Control Strategy or Nonlinear Model Predictive Control Algorithm) [21, 26].

The paper is composed as follows. Section 2 presents an analysis of robotic systems subject to slipping effects. The analysis, patterned on [2], makes use of an explicit modeling of the dissipative nature of the interaction forces applied to the system by the external world. Section 2 also summarizes basic concepts of the endogenous configuration space approach; it introduces the Jacobian pseudo inverse motion planning algorithm. Section 3 presents the results related to the application of the Jacobian pseudo inverse motion planning algorithm to the Pioneer 2DX mobile robot moving with slipping. The paper is concluded with section 4.

2. Basic concepts

Let us consider a class of wheeled mobile robots, whose nonholonomic constraints are not satisfied. Let $q \in \mathbb{R}^n$ denote generalized coordinates of the mobile robot. We shall assume that l $(l < n)$ velocity constraints $\mathsf{A}(q)\dot{q} = 0$, imposed on the robot motion can be violated. Intuitively, we are expecting that the violation of constraints measured by the norm $||\mathsf{A}(q)\dot{q}||$ is small. Using the canonical decomposition of $\mathbb{R}^n = Ker\mathsf{A}(q) \oplus Im\mathsf{A}^T(q)$, we introduce quasi-velocities $\eta \in \mathbb{R}^m$, $\mu \in \mathbb{R}^l$, $m + l = n$, such that the mobile robot kinematics are represented as

$$\dot{q} = G(q)\eta + \mathsf{A}^T(q)\varepsilon\mu, \tag{1}$$

where columns $g_1(q), \ldots, g_m(q)$ of the matrix $G(q)$ span the null space $Ker\mathsf{A}(q)$, so $\mathsf{A}(q)G(q) = 0$. $\eta \in \mathbb{R}^m$ - vector of auxiliary velocities and $\mu \in \mathbb{R}^l$ - vector of slips velocities. The quasi-velocity μ has been scaled by a small parameter ε representing the violation of constraints. Observe, that $\mathsf{A}(q)\dot{q} = \mathsf{A}(q)\mathsf{A}^T(q)\varepsilon\mu$, so the ideal case corresponds to $\varepsilon = 0$. The introduction of quasi-velocities and the scaling parameter has been originally proposed in [2] as a key element of the singular perturbation approach. The Lagrange equations of the mobile robot dynamics assume the following form [2]

$$\mathsf{Q}(q)\ddot{q} + \mathsf{C}(q,\dot{q}) = F(q) + \mathsf{B}(q)\mathsf{u}. \tag{2}$$

with $\mathsf{Q}(q)$ and $\mathsf{C}(q,\dot{q})$, denoting, respectively the inertia matrix, and the vector of centrifugal, Coriolis, frictional and gravity forces. $\mathsf{B}(q)$ stands for the control matrix. The vector $F(q)$ denotes the interaction forces (exerted on the system by the external world). u represents the control functions. Having rewritten the equations of motion (2) and with $\ddot{q}$ expressed from (1) as

$$\ddot{q} = \begin{bmatrix} G(q) & \mathsf{A}^T(q) \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \varepsilon\dot{\mu} \end{bmatrix} + \dot{G}(q)\eta + \dot{\mathsf{A}}^T(q)\varepsilon\mu,$$

where $\dot{G}(q) = \frac{\partial G(q)}{\partial q}(G(q)\eta + \mathsf{A}^T(q)\varepsilon\mu)$ $\dot{\mathsf{A}}^T(q) = \frac{\partial \mathsf{A}^T(q)}{\partial q}(G(q)\eta + \mathsf{A}^T(q)\varepsilon\mu)$, and added of an output function characterizing the task of the mobile robot, we obtain an affine control system representation of the kinematics and the dynamics of the mobile robot

$$\begin{cases} \dot{q} = G(q)\eta + \mathsf{A}^T(q)\varepsilon\mu \\ \begin{bmatrix} \dot{\eta} \\ \varepsilon\dot{\mu} \end{bmatrix} = P(q,\eta,\varepsilon\mu) + R(q)\mathsf{u} \\ y = k(q,\dot{q}). \end{cases} \tag{3}$$

The terms appearing in eq. (3) are defined in the following way

$$P(q,\eta,\varepsilon\mu) = \begin{bmatrix} P_1(q,\eta,\varepsilon\mu) \\ P_2(q,\eta,\varepsilon\mu) \end{bmatrix} =$$
$$\begin{bmatrix} G(q) & \mathsf{A}^T(q) \end{bmatrix}^{-1} \Big(-(\dot{G}(q)\eta + \dot{\mathsf{A}}^T(q)\varepsilon\mu) -$$
$$Q^{-1}(q)\mathsf{C}(q, G(q)\eta + \mathsf{A}^T(q)\varepsilon\mu) \Big) + H^{-1}(q) \begin{bmatrix} 0 \\ \mathsf{A}(q)F \end{bmatrix},$$

$$R(q) = \begin{bmatrix} R_1(q) \\ R_2(q) \end{bmatrix} = \begin{bmatrix} G(q) & \mathsf{A}^T(q) \end{bmatrix}^{-1} Q^{-1}(q)\mathsf{B}(q).$$

The matrix

$$H(q) = \begin{bmatrix} G^T(q)\mathsf{Q}(q)G(q) & G^T(q)\mathsf{Q}(q)\mathsf{A}^T(q) \\ \mathsf{A}(q)\mathsf{Q}(q)G(q) & \mathsf{A}(q)\mathsf{Q}(q)\mathsf{A}^T(q) \end{bmatrix}$$

is symmetric and positive definite, while the output function

$$y = k(q,\dot{q})$$

may describe the position coordinates or velocities of the mobile robot in its motion plane.

If ϵ is sufficiently small, the state variables of the control system (3) can be divided into slow (q,η) and fast μ, corresponding to the slow and the fast dynamics. Furthermore, it is easy to show, that after taking $\varepsilon = 0$, the control system (3) reduces to the kinematics and dynamics model of the mobile robot subject to the nonholonomic constraints. In this case, the third line represents the traction forces ($\in ImA^T(q)$) that enforce satisfaction of the nonholonomic constraints, see [2].

Since the admissible control functions of the mobile robot (3) belong to the Hilbert space, they will be assumed Lebesgue square integrable over the time interval $[0,T]$,

$$\mathsf{u}(\cdot)\in L^2_m[0,T].$$

They have sense of forces or torques, and constitute *the dynamic endogenous configuration space* $\mathscr{X}\cong L^2_m[0,T]$, [42]. $\mathscr{X}$ is a Hilbert space.

Let $z=(q,\eta,\mu)\in\mathbb{R}^{n+m+l}$ denotes the state vector of eq.(3), and suppose that for a given initial state z_0 and every control $\mathsf{u}(\cdot)$ there exists a state trajectory $z(t)=\varphi_{z_0,t}(\mathsf{u}(\cdot))=(q(t),\eta(t),\mu(t))$ and an output trajectory $y(t)=k(q(t))$ of system (3). In accordance with the endogenous configuration space approach the task map of the mobile robot will be identified with the end point map of the control system (3)

$$\mathsf{T}_{z_0,T}(\mathsf{u}(\cdot))=y(T)=k(\varphi_{z_0,T}(\mathsf{u}(\cdot))) \tag{4}$$

and computes the system output at T, when driven by $\mathsf{u}(\cdot)$.

Given the system (3), we shall study the following motion planning problem for the mobile robot: find a control function $\mathsf{u}(\cdot)$ such that, at a given time instant T, the system output reaches a desirable point y_d of the task space, so $y(T)=y_d$. The motion planning problem becomes equivalent to the inversion of the task map: find a control function $\mathsf{u}(\cdot)$ such that $\mathsf{T}_{z_0,T}(\mathsf{u}(\cdot))=y_d$. This inversion can be achieved using a Jacobian algorithm [36]. To this aim, we define the Jacobian of (4) as the derivative

$$\mathsf{J}_{z_0,T}(\mathsf{u}(\cdot))\mathsf{v}(\cdot)=D\,\mathsf{T}_{z_0,T}(\mathsf{u}(\cdot))\mathsf{v}(\cdot),$$

$\mathsf{v}(\cdot)\in\mathscr{X}$. The Jacobian map transforms tangent vectors to the dynamic endogenous configuration space into $\mathbb{R}^3$, and describes how an infinitesimal change in the input force is transmitted into a change of the position and orientation of the mobile robot at T.

In order to compute the Jacobian map at a given configuration $\mathsf{u}(\cdot)\in\mathscr{X}$ we introduce the variational system associated with (3)

$$\dot{\xi}=A(t)\xi+B(t)\mathsf{v},\ \zeta=C(t)\xi \tag{5}$$

as the linear approximation to (3) along $z(t)$, initialized at $\xi_0=0$, where

$$A(t)=$$
$$\begin{bmatrix}\frac{\partial(G(q(t))\eta(t)+\mathsf{A}(q(t))\varepsilon\mu(t))}{\partial q} & G(q(t)) & \mathsf{A}^T(q(t))\varepsilon\\ \frac{\partial(P(q(t),\eta(t),\varepsilon\mu(t))+R(q(t))\mathsf{u}(t))}{\partial q} & \frac{\partial P(q(t),\eta(t),\varepsilon\mu(t))}{\partial\eta} & \frac{\partial P(q(t),\eta(t),\varepsilon\mu(t))}{\partial\mu}\end{bmatrix},$$

$$B(t)=\begin{bmatrix}0\\R(q(t))\end{bmatrix},\quad C(t)=\frac{\partial k(z(t))}{\partial z}.$$

$\xi\in\mathbb{R}^n$ denotes the state of the variational system; its evolution is described by the eq. (5).

Taking into account (3) and (5), we can compute the Jacobian map as the output trajectory $\zeta(T)$ of the variational system (5) [36]

$$\mathsf{J}_{z_0,T}(\mathsf{u}(\cdot))\mathsf{v}(\cdot)=C(T)\int_0^T\Phi(T,t)B(t)\mathsf{v}(t)dt, \tag{6}$$

where $\Phi(t,\mathsf{s})$ denotes the fundamental matrix of system (5), that satisfies the evolution equation

$$\frac{\partial\Phi(t,\mathsf{s})}{\partial t}=A(t)\Phi(t,\mathsf{s}),\quad \Phi(\mathsf{s},\mathsf{s})=I_{n+m+l}\ .$$

Observe that the Jacobian (6) corresponds to the compliance map introduced in [42].

Given the end point map (4) of the mobile robot, and a desirable point y_d in the taskspace, the motion planning problem consists in defining an endogenous configuration $\mathsf{u_d}(\cdot)\in\mathscr{X}$ such that $\mathsf{T}_{z_0,T}(\mathsf{u_d}(\cdot))=y_d$. Usually, this problem may be solved numerically, by means of a Jacobian pseudo inverse motion planning algorithm. Let $\mathsf{u}_\vartheta(\cdot)\in\mathscr{X},\vartheta\in\mathbb{R}$ denote a smooth curve in the dynamic endogenous configuration space, passing through an initial configuration $\mathsf{u}_0(\cdot)$. The taskspace error $e(\vartheta)=\mathsf{T}_{z_0,T}(\mathsf{u}_\vartheta(\cdot))-y_d$ along this curve, describing the difference between actual and desirable mobile robot locations at T should decrease along with ϑ, in a prescribed way, e.g. exponentially. By requiring that the error should decrease exponentially, $\frac{d}{d\vartheta}e(\vartheta)=-\gamma e(\vartheta)$, with decay rate $\gamma>0$, we derive the Ważewski-Davidenko equation

$$\mathsf{J}_{z_0,T}(\mathsf{u}_\vartheta(\cdot))\frac{d\mathsf{u}_\vartheta(\cdot)}{d\vartheta}=-\gamma e(\vartheta). \tag{7}$$

Finally, using the Jacobian pseudo inverse operator

$$\left(\mathsf{J}^{\#}_{z_0,T}(\mathsf{u}(\cdot))\zeta\right)(t)=B(t)\Phi^T(T,t)\mathsf{M}^{-1}_{z_0,T}(\mathsf{u}(\cdot))\zeta, \tag{8}$$

where

$$\mathsf{M}_{z_0,T}(\mathsf{u}(\cdot))=\\ C(T)\int_0^T B(t)\Phi(T,t)\Phi^T(T,t)B^T(t)dtC^T(T)$$

is the mobility matrix of the endogenous configuration $\mathsf{u}(\cdot)$, we arrive at the dynamic system defining the Jacobian pseudo inverse algorithm

$$\frac{d\mathsf{u}_\vartheta(t)}{d\vartheta}=-\gamma\left(\mathsf{J}^{\#}_{z_0,T}(\mathsf{u}_\vartheta(\cdot))e(\vartheta)\right)(t). \tag{9}$$

Solution of the motion planning problem is obtained as the limit at $+\infty$ of the trajectory of (9), $\mathsf{u}(t)=\lim_{\vartheta\to+\infty}\mathsf{u}_\vartheta(t)$. Once the inverse operator in (8) is chosen, the solution of eq. (9) is unique.

Since the dynamic endogenous configuration space $\mathscr{X}$ is infinite-dimensional, in order to carry out effective computations we shall use a finite parameterization of the control functions $\mathsf{u}(\cdot)$ in (3) by truncated orthogonal expansions (Fourier series)

$$\mathsf{u}_{ci}(t)=\sum_{j=0}^{k}c_{i2j-1}\sin j\omega t+c_{i2j}\cos j\omega t,\\ i=1,2,\dots,m\quad \omega=\frac{2\pi}{T},\quad c_{i-1}=0. \tag{10}$$

Subscript c in (10) means that the control functions are parameterized. After the parameterization, the dynamic endogenous configuration $\mathsf{u}\in\mathscr{X}$ is represented by a vector $c\in\mathbb{R}^s$, $s=m(2k+1)$ and the Jacobian (6) becomes a Jacobian matrix $J_{z_0,T}(c)$. A discretization of the motion planning algorithm (9) results in changing the dynamic endogenous configuration $c\in\mathbb{R}^s$ iteratively, with iterations indexed by an integer ϑ, i.e.

$$c_{\vartheta+1}=c_\vartheta-\gamma J^{\#}_{z_0,T}(c_\vartheta)e_\vartheta,\quad \vartheta=1,2,\dots, \tag{11}$$

where $J^{\#}_{z_0,T}(c) = J^T_{z_0,T}(c)\left(J_{z_0,T}(c)J^T_{z_0,T}(c)\right)^{-1}$, and $e_\vartheta = \mathsf{T}_{z_0,T}(u_{c_\vartheta}(\cdot)) - y_d$.
The discrete Jacobian pseudo inverse algorithm (11), which relies on the Euler approximation of the differential equations (9), is sensitive to its parameters. In particular, by increasing γ we may lose convergence, on the other hand, by decreasing it excessively we may slow down the convergence, i.e. computational complexity of the algorithm increases. Additionally, the final solution depends strongly on the initial controls. The proposed motion planning algorithm is local with respect to the initial controls. This is a common feature of the Newton-like algorithms.

3. Case study

As an example, let us consider Pioneer 2DX – the differential drive type mobile robot (a mobile robot of this class has been studied in [2]; fig. 1). The robot is

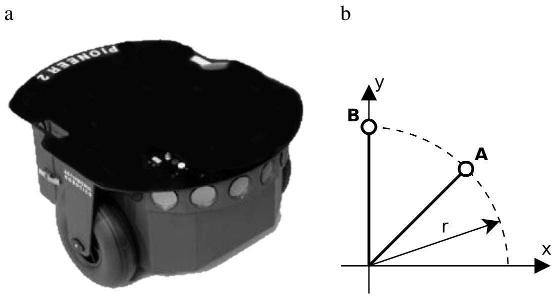

Fig. 1. a – Mobile robot Pioneer 2DX. b – Desirable positions of the robot.

equipped with two independently actuated, identical wheels (of radius r) mounted on a common axle (of length $2l$) (fig. 2). Let us assume that, as the result of a deformation at the contact point, the wheels may slip, both longitudinally as well as laterally. The generalized platform coordinates are chosen as $q = (x, y, l\theta, r\varphi_1, r\varphi_2)^T$, where x, y are position coordinates of the center of wheel axle, θ denotes the orientation of the platform, and φ_1, φ_2 are revolution angles of the wheels. The meaning of geometric parameters of the robot is explained in fig. 2. The mathematical model

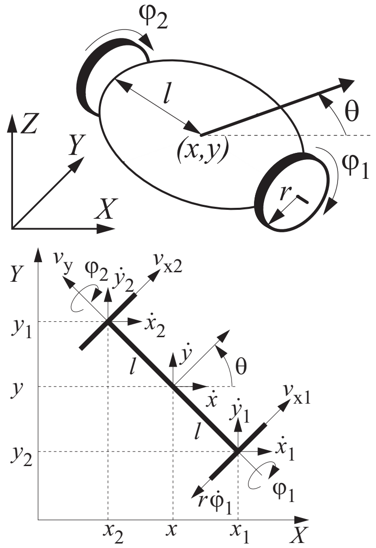

Fig. 2. Mobile robot.

of the kinematics and dynamics of the mobile robot is represented by the control system (3),

$$\mathsf{A}(q) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 & 0 \\ \cos\theta & \sin\theta & 1 & -1 & 0 \\ \cos\theta & \sin\theta & -1 & 0 & -1 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ -1 & 1 \\ 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathsf{B} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{r} \end{bmatrix}^T, \tag{12}$$

thus

$$\begin{bmatrix} \dot{\eta} \\ \varepsilon\dot{\mu} \end{bmatrix} = \begin{bmatrix} G(q) & \mathsf{A}^T(q) \end{bmatrix}^{-1} \Big(-\dot{G}(q)\eta - \dot{\mathsf{A}}^T(q)\varepsilon\mu + Q^{-1}(q)B(q)\mathsf{u} \Big) + H^{-1}(q) \begin{bmatrix} 0 \\ \mathsf{A}(q)F \end{bmatrix}.$$

The actuation of the wheels is achieved by two motors that generate two torques defining the inputs $u = (u_1, u_2)$ to the system. The output function describes the position coordinates and the orientation angle of the mobile robot in its motion plane $(y(q) = (x, y, \theta) \in \mathbb{SE}(2) \cong \mathbb{R}^2 \times \mathbb{S}^1)$.
Assuming that the center of mass of the robot is located at the middle point of the axle of wheels, we get the following form of the inertia matrix $\mathsf{Q} = diag\left\{m, m, \frac{I_\theta}{l^2}, \frac{I_\varphi}{r^2}, \frac{I_\varphi}{r^2}\right\}$, m – the mass of the robot, I_θ – the moment of inertia of the robot around the vertical axis, and I_φ – the moment of inertia of each wheel.

Now let us derive a model for the generalized interaction force F. As in [2], we shall adopt a "pseudo–slipping" model [29], according to which the lateral and longitudinal forces applied by the ground to the wheels are proportional, respectively, to the slip angle and the slip coefficient.
Let v denotes the velocity of the center of the wheel, that rotates with the angular velocity $\dot{\varphi}$. Let v_x and v_y be, respectively, the longitudinal and lateral components of v. The longitudinal slipping velocity of the wheel is equal to the difference $v_x - r\dot{\varphi}$, and the longitudinal slip s is given by $s = \frac{v_x - r\dot{\varphi}}{||v||}$. Eventually, the longitudinal traction force applied by the ground is characterized as

$$f_x = -Cs = -\frac{C}{||v||}(v_x - r\dot{\varphi}),$$

where C is a slip stiffness coefficient, depending on the nature of the wheel and the ground. The force f_x opposes the longitudinal slip.
The slip angle δ, defined as the angle between the plane of the wheel and the velocity of its center is given by $\delta \simeq \sin\delta = \frac{v_y}{||v||}$. The lateral traction force applied by the ground, opposed to the lateral velocity component v_y, takes the form

$$f_y = -D\delta = -\frac{D}{||v||}v_y$$

with D denoting a cornering stiffness coefficient depending on the nature of the wheel and the ground.
Finally, we get the following form of the interaction force acting on the wheel

$$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = -\frac{1}{||v||} \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} v_x - r\dot{\varphi} \\ v_y \end{bmatrix}. \tag{13}$$

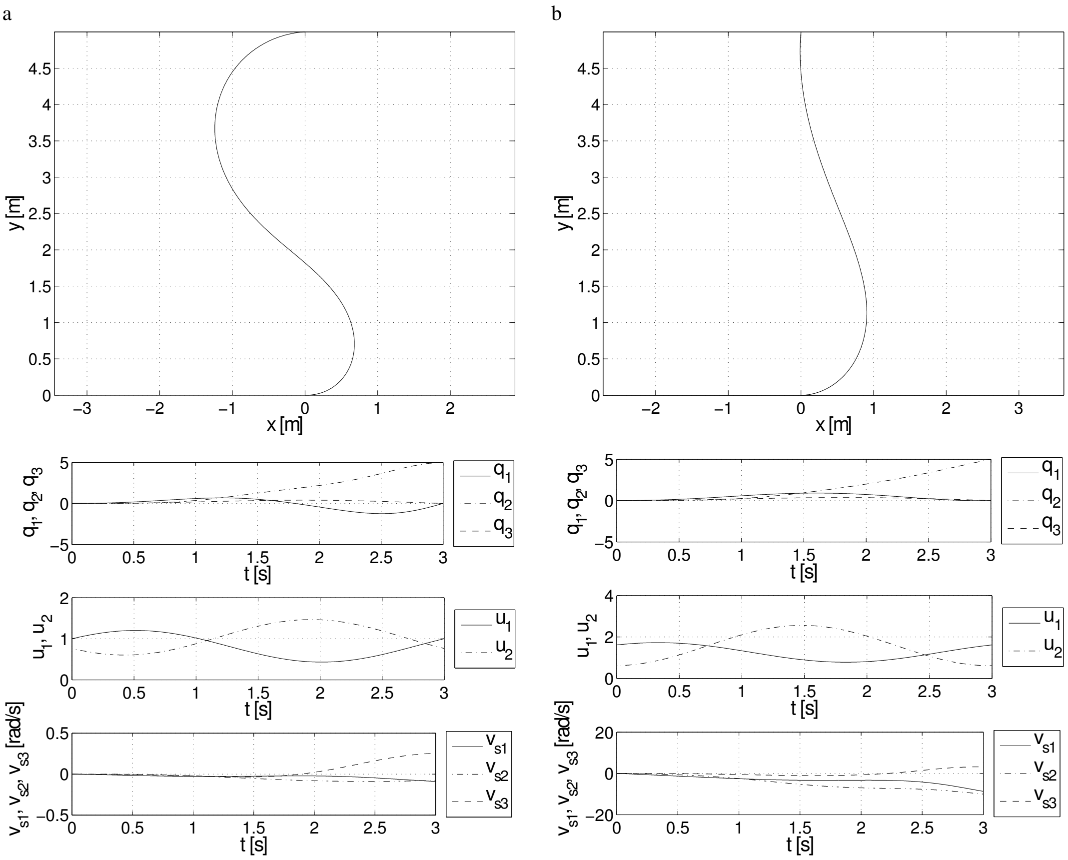

Fig. 3. Long distance, point B: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$*, and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$*, a –* $\varepsilon = 10^{-3}$*, b –* $\varepsilon = 10^{-1}$*.*

From the definition (12) of $\mathsf{A}(q)$ we deduce that for the wheel i

$$\begin{bmatrix} v_{ix} - r\dot{\varphi}_i \\ v_{iy} \end{bmatrix} = L_i(q)\mathsf{A}(q)\dot{q}, \tag{14}$$

where $L_i(q)$ is a 2×3 matrix, $i = 1, 2$. Specifically, the matrix $L_1(q) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, while $L_2 = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$.

The generalized interaction force of the wheel i is computed using the principle of virtual work

$$F_i^T \dot{q} = f_i^T v_i,$$

implying that

$$F_i = \mathsf{A}^T(q) L_i^T(q) f_i.$$

Using (13) and (14), we get

$$F_i = -\mathsf{A}^T(q) L_i^T(q) \frac{1}{||v_i||} \begin{bmatrix} C_i & 0 \\ 0 & D_i \end{bmatrix} L_i(q)\mathsf{A}(q)\dot{q}. \tag{15}$$

Finally

$$F = -\mathsf{A}^T(q) \sum_{i=1}^{k} \left(\frac{1}{||v_i||} L_i^T \begin{bmatrix} C_i & 0 \\ 0 & D_i \end{bmatrix} L_i \right) \mathsf{A}(q)\dot{q}. \tag{16}$$

In computations we shall additionally assume that D_i and C_i have the same value for both wheels. In order to avoid numerical problems that may appear for small values of $||v_i||$ we slightly modify the model introducing a saturation. In particular, if $||v_i|| < \delta$ then $||v_i||$ is replaced by δ, where δ is a small positive constant. To the needs of computer simulations we assume the following real geometric and dynamic parameters of the mobile platform Pioneer 2DX: $l = 0.163$m, $r = 0.0825$m, $m = 8.67$kg, $I_\theta = 0.256\text{kgm}^2$, $I_\varphi = 0.02kgm^2$ [17]. Additionally, we assume the saturation coefficient $\delta = 10^{-6}$, the cornering stiffness coefficient $D_i = 0.4$ and the slip stiffness coefficient $C_i = 1$ for $i = 1, 2$ [2].

The control functions will take the form (10) with $k = 1$, so $c \in \mathbb{R}^6$. The state space of the system (3) is 10-dimensional, $z = (q, \eta, \mu)^T \in \mathbb{R}^{10}$. The following motion planning problem will be examined: for given initial state $z_0 = 0$ of the robot, and three desirable positions (A, B) in the plane (see figure 1b)), find an endogenous configuration c guaranteeing that the positions are reached in time $T = 5s$ at the final robot orientation $\theta_d = 0$. A collection of final positions of the platform consists of 3 points distributed around a circle of radius r = 2m (short distance) or r = 5m (long distance). The initial controls of the mobile platform are fixed as $(c_{10}, c_{20}) = (1, 1)$, the remaining initial values of coefficients being zero. The error decay rate $\gamma = 0.75$. The motion planning problem is regarded as solved when the taskspace error norm $||e|| = ||y_d - k(q(T))||$ drops below 10^{-6} within ≤ 500 iterations. Every solution of the motion planning problem is accompanied with a computation of the platform trajectory length d, the

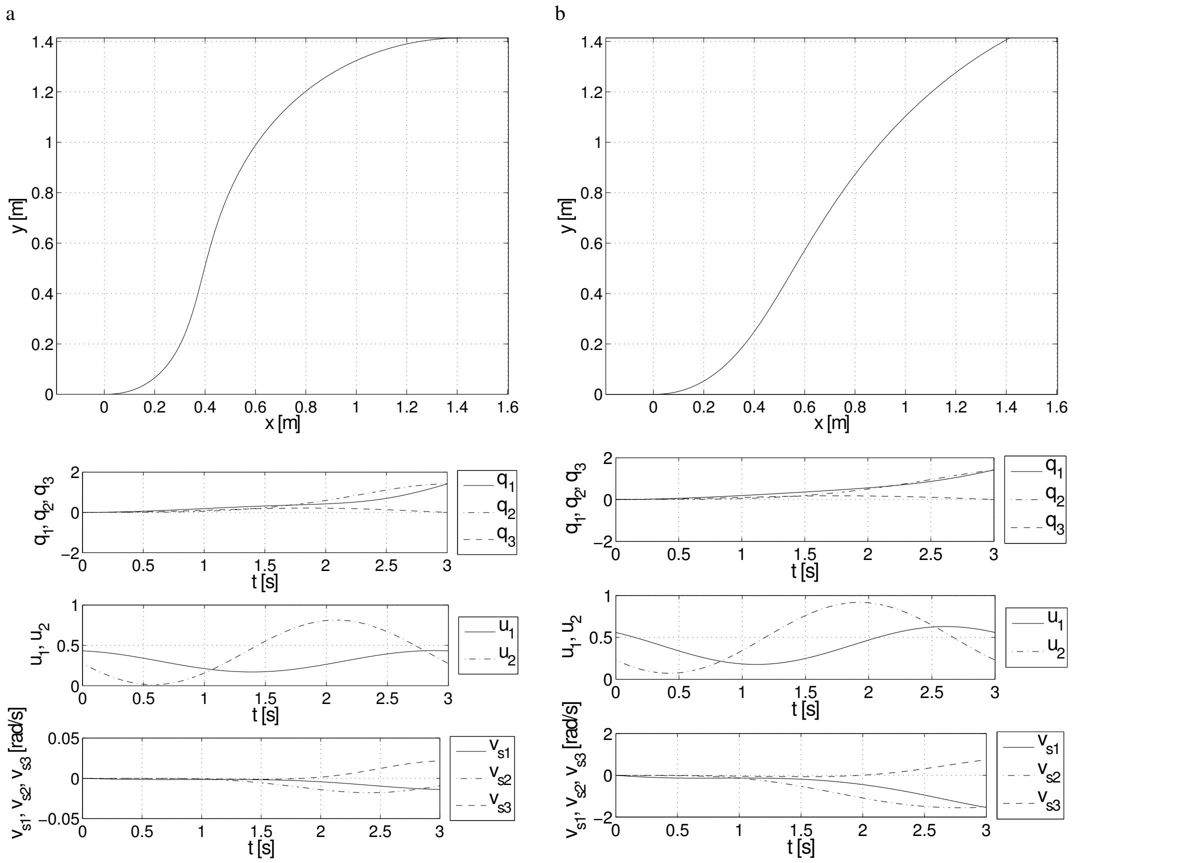

Fig. 4. Short distance, point A: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$*, and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$*, a –* $\varepsilon = 10^{-3}$*, b –* $\varepsilon = 10^{-1}$

control energy $\mathsf{e} = \int_0^T u(t)^T u(t)dt$, and the number of iterations necessary to find the solution i.

The results are summarized in tab. (1) ($\varepsilon_1 = 10^{-1}$ and $\varepsilon_2 = 10^{-3}$) and in fig. (3–4).

Tab. 1. Motion planning of the mobile robot Pioneer 2DX.

		position A			position B		
		i	$d[m]$	e	i	$d[m]$	e
r_1	ε_1	24	2,06	1,57	25	2,22	2,43
	ε_2	24	2,19	1,05	25	2,80	1,78
r_2	ε_1	24	5,15	7,51	26	5,62	14,01
	ε_2	24	5,41	3,58	25	6,76	5,71

As the next step ε have been assumed to be equal 1. Now the traction properties in the tire – road contact zone are illustrated by the changes of the parameters C_i, D_i. Naturally, the bigger C_i and D_i are, the better traction properties we get. Let us assume additionally $D_i = 2C_i$ (the lateral slip is twice larger than the longitudinal slip) and C_i equal, respectively, $C_{i_1} = 10^0$, $C_{i_2} = 10^2$, $C_{i_3} = 10^4$ and $C_{i_4} = 10^6$. The results are summarized in tab. 2 and in figs. 3, 8.

It is worth observing, that in most cases the energy lost due to slipping and skidding effects increases along with increasing ε. Point B is the most difficult final point for the algorithm: the acceptable trajectories are obtained with large energy expenditure. Finally, the closer to the ideal case we are, the less number of iterations i of the band-limited Jacobian pseudoinverse algorithm is needed and the longer

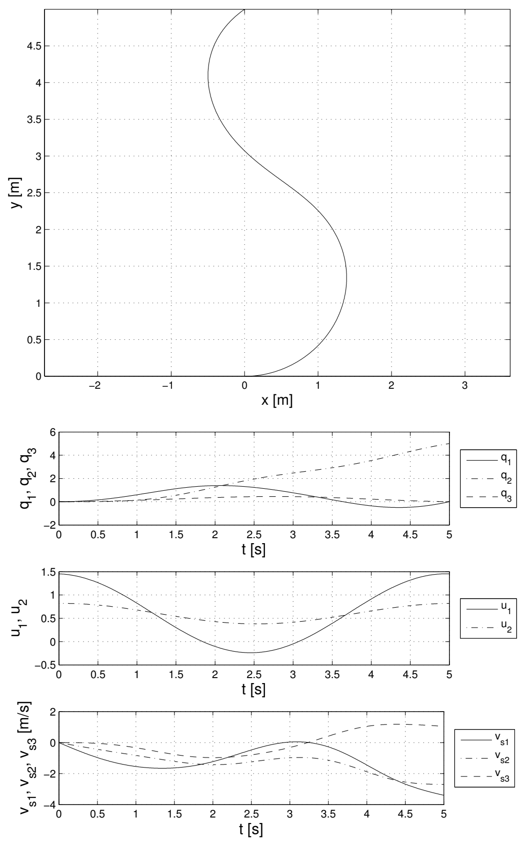

Fig. 5. Long distance, point B: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$*, and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$*,* $C_i = 10^0$*.*

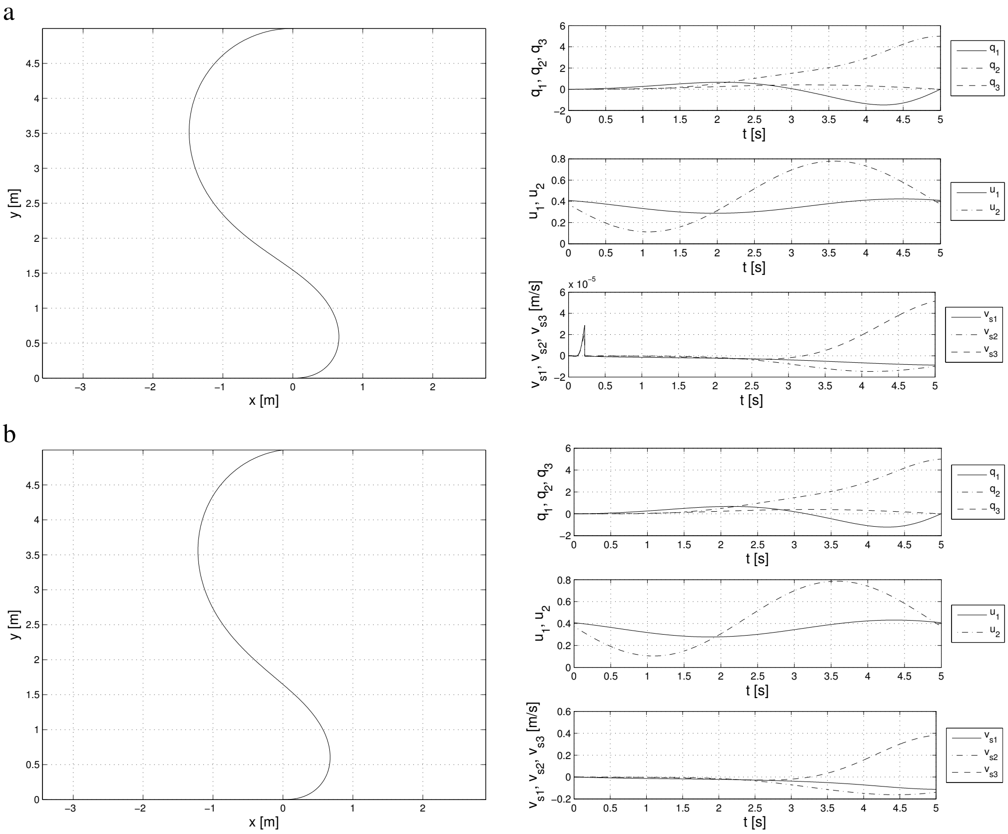

Fig. 6. Long distance, point B: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$, *and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$, *a –* $C_i = 10^6$, *b –* $C_i = 10^2$.

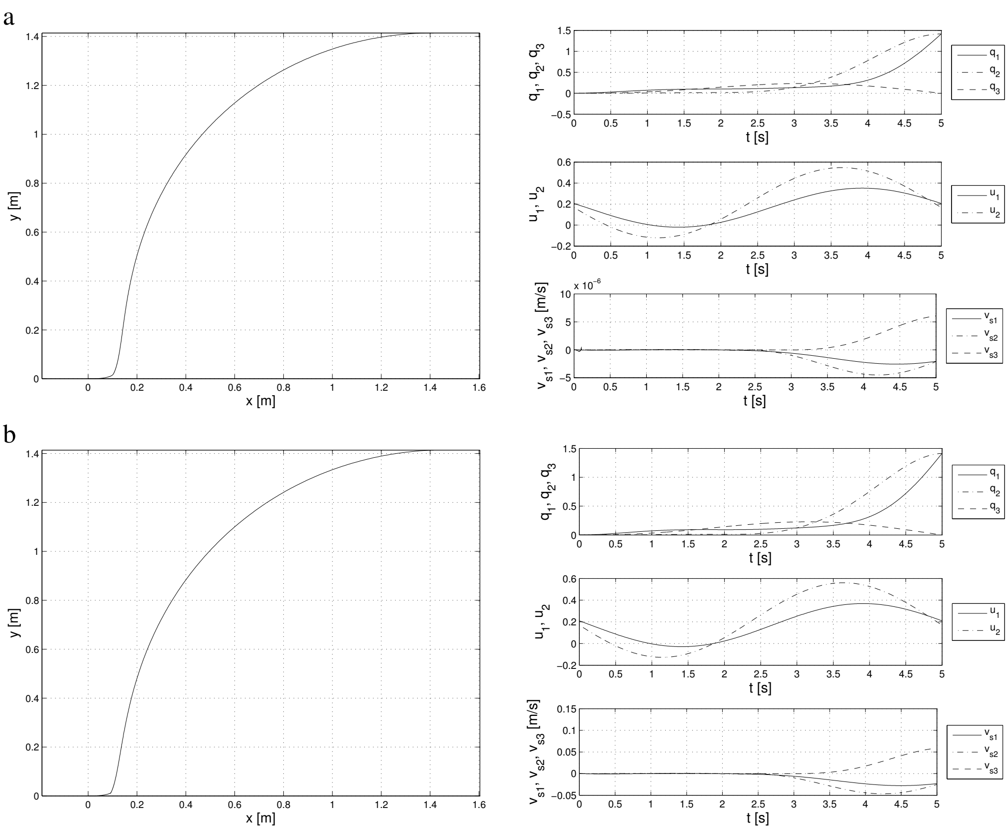

Fig. 7. Short distance, point A: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$, *and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$, *a –* $C_i = 10^6$, *b –* $C_i = 10^2$.

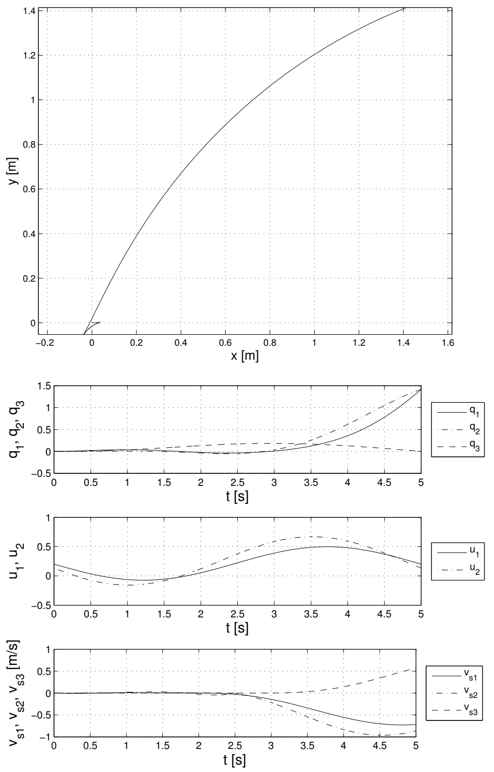

Fig. 8. Short distance, point A: Robot paths and posture trajectories, control functions $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$*, and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$*,* $C_i = 10^0$.

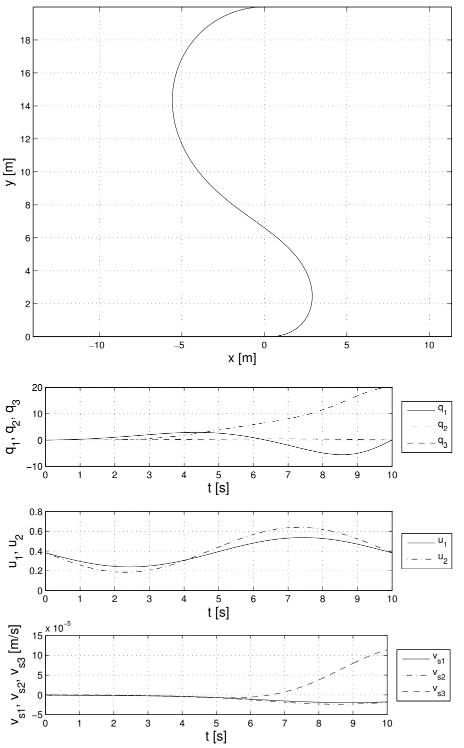

Fig. 9. Long distance $r = 20m$*,* $T = 10s$*, point B: Robot paths and posture trajectories, control functions* $\mathsf{u} = (\mathsf{u}_1, \mathsf{u}_2)$*, and slips* $(v_{s1}, v_{s2}, v_{s3}) = (v_x - r\dot{\varphi}_1, v_x - r\dot{\varphi}_2, v_y)$*,* $C_i = 10^6$*,* $\mathsf{i} = 18$*,* $d = 26,7597[m]$*,* $\mathsf{e} = 3,6421$.

distance the platform covers while executing the task of motion planning (fig. 3a and 4a). The band-limited Jacobian pseudoinverse algorithm copes well with long distances (see fig. 9). Preliminary results (see tabs. 1 and 2) confirm, that substituting the role of the perturbation parameter ε for the parameters C_i and D_i expressing the traction properties of the wheels (violation of the nonholonomic constraints) reduces considerably the computational complexity of the Jacobian pseudoinverse algorithm.

Tab. 2. Motion planning of the mobile robot Pioneer 2DX.

		position A			position B		
		i	$d[m]$	e	i	$d[m]$	e
r_1	C_{i_1}	17	2,1480	1,1809	15	2,3371	1,4171
	C_{i_2}	14	2,2156	0,7733	15	2,8208	0,9656
	C_{i_3}	14	2,2357	0,7293	15	2,9319	0,9206
	C_{i_4}	14	2,2359	0,7289	15	2,9332	0,9201
r_2	C_{i_1}	14	5,1471	2,0313	19	6,6976	5,5604
	C_{i_2}	14	5,4266	1,4108	17	6,7147	1,9244
	C_{i_3}	14	5,4926	1,3584	17	7,1136	1,9099
	C_{i_4}	14	5,4934	1,3579	17	7,1190	1,9099

4. Conclusions and future works

The main objective of this paper has been to propose a novel approach to motion planning for wheeled mobile robots subject to slipping effects. We have focused on mobile robots, whose ideal pure rolling and nonslipping constraints have been violated during the motion. Following [2], this transgression has been modeled as a small perturbation of the ideal constraints. To describe the kinematics and the dynamics of the robot, the endogenous configuration space approach has been adopted. The basic concepts of mobile robots subject to slipping have been defined by correspondence to nonholonomic mobile robots. Thus, the proposed approach is a combination of the singular perturbation modeling and the endogenous configuration space approach. As the result, we have obtained a control system representation of the robot kinematics and dynamics, defined a task map as the end point map of this system, and reduced the motion planning problem to the inversion of the task map achieved by means of the Jacobian pseudo inverse operator. The motion planning algorithm devised in this paper relies on the linearization of the control system along a trajectory, and applies to any system whose linearization is controllable. The motion planning problem has been solved by means of the Jacobian pseudo inverse algorithm. As an illustration of the theory developed in the paper, the motion planning problem for the mobile robot Pioneer 2DX subject to slipping have been solved. Our computer experiments have confirmed that this algorithm is able to solve efficiently the motion planning problem of the mobile robot subject to slipping; it is computationally rather expensive, but has good convergence properties. The motion planning algorithm derived within the endogenous configuration space approach is computable and useful in application to mobile robots whose nonholonomic constraints are violated. Future research will be conducted towards experimental verification of this approach. Current works are conducted towards application more adequate models of interaction forces between wheels and the ground [7]. A construction of hybrid model of a mobile robot whose motion consists of slipping and nonslipping phases would be of interest.

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AUTHORS

Katarzyna Zadarnowska* – Institute of Computer Engineering, Control and Robotics; Wrocław University of Technology; ul. Janiszewskiego 11/17, 50-372 Wrocław, Poland, e-mail:
`Katarzyna.Zadarnowska@pwr.wroc.pl`

Adam Oleksy – Institute of Computer Engineering, Control and Robotics; Wrocław University of Technology; ul. Janiszewskiego 11/17, 50-372 Wrocław, Poland, e-mail: `Adam.Oleksy@pwr.wroc.pl`

* Corrresponding author

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