Different Kinematic Controllers Stabilizing Nonholonomic Mobile Manipulators of (nh, nh) Type About Desired Configuration

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Abstract:

In the paper, a problem of selection of kinematic control algorithm for nonholonomic manipulator has been considered. For a nonholonomic manipulating arm, two kinematic algorithms – Astolfi algorithm (working in closed-loop) and Nakamura, Chung and Sørdalen algorithm (working in open loop of control) have been compared. Simulation results have shown that influence of the dynamics on the behavior of mobile manipulator of (nh, nh) type is huge. It means that only kinematic control algorithms using feedback in the control loop are sufficiently robust to apply them in practical applications. Then, for the nonholonomic wheeled mobile platform, Astolfi algorithm, which belongs to discontinuous class of kinematic algorithms, has also been compared to a discontinuous algorithm proposed by Zhang & Hirschorn.

Keywords: *nonholonomic systems, point stabilization, discontinuous control.*

1. Introduction

In the paper, we will discuss a possible choice of control algorithms preserving point stabilization for mobile manipulators. A mobile manipulator, we will call a robotic system – rigid manipulator mounted on a mobile platform.

The stabilization problem of nonholonomic systems has attracted a lot of attention in the literature in recent years. It has been shown that it can't be solved using smooth and static feedback law, [2]. Thus, discontinuous or time-varying feedback law should be designed.

If we take into account the type of components mobility of mobile manipulators, we obtain 4 possible configurations: type (h, h) – both the platform and the manipulator are holonomic, type (h, nh) – a holonomic platform with a nonholonomic manipulator, type (nh, h) – a nonholonomic platform with a holonomic manipulator, and type (nh, nh) – both the platform and the manipulator are nonholonomic. The notion "doubly nonholonomic" manipulator was introduced in [7] for the type (nh, nh).

In the sequel, we will restrict our considerations to mobile manipulators of (nh, nh) type, and as an object of simulations we will take a vertical 3-pendulum mounted on the mobile platform of the class $(2, 0)$.

The problem of designing a control law for rigid robotic manipulators received much attention in late 80th and 90th years of the last century. There were many works on the control algorithms for the manipulators based on different level of knowledge about their dynamics, which assumed that each degree of freedom was controlled independently by actuator. Such system can be regarded as fully actuated mechanical system.

Recently, a new approach to the problem of robot drive has been proposed. In [5] Nakamura, Chung and Sordalen proposed a new nonholonomic mechanical gear, which is able to transmit velocities from the inputs to many passive joints, see Fig. 2. Usage of nonholonomic gears causes that nonholonomic manipulator is an example of underactuated system (at kinematic level). In [5] the prototype of the nonholonomic manipulator was introduced and discussed. The nonholonomic constraints of the gear were due to the rolling contact without slipping between balls of gear and special supporting wheels in the robot joints. The authors presented the prototype with 4 joints; a big advantage of such a construction is the possibility to drive many rotational joints (not necessary lying on a plane) with only two input engines.

2. Mathematical model of the doubly nonholonomic mobile manipulator

In the paper, we will consider a mobile manipulator that consists of two subsystems, namely the mobile platform (which is often called in the literature "a mobile robot"), playing a role of the transportation part, and the rigid manipulating arm equipped with specific nonholonomic drives. The schematic presentation of such an object is given in Fig. 1.

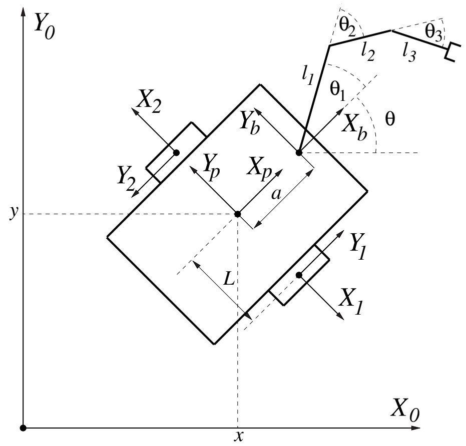

Fig. 1. Object of considerations: 3-pendulum mounted on the unicycle.

Each of the subsystems has specific nonholonomic constraints, which determine its motion. Such constraints have to be included in the description of the whole system, i.e. of the doubly nonholonomic mobile manipulator.

2.1. Nonholonomic constraints for the mobile platform

The behavior of the mobile platform can be described using generalized coordinates $q_m \in R^n$ and generalized velocities $\dot{q}_m \in R^n$. In our further considerations, we make an assumption that the slipping effect does not occur in the motion of the platform and its motion is pure rolling. It means that momentary velocity between each wheel and the surface equals to zero. The constraints introduced by this assumption are nonholonomic, and they can be expressed in the so-called Pfaffian form

$$A_1(q_m)\dot{q}_m = 0, \tag{1}$$

where $A_1(q_m)$ is a constraint matrix for the platform of $(l \times n)$ size. From equation (1) we can conclude that the generalized velocities of the platform $\dot{q}_m$ are always in the null space of A_1 matrix. It means that it is always possible to find a vector of auxiliary velocities $u \in R^m$, $m = n - l$ which define some expression (the kinematics)

$$\dot{q}_m = G_1(q_m)u. \tag{2}$$

Matrix $G_1(q_m)$ of $n \times m$ size is full rank matrix, which fulfills the following equality

$$A_1(q_m)G_1(q_m) = 0.$$

In the sequel, we restrict our consideration to the nonholonomic mobile platform of the (2,0) class, presented in Fig. 1. For such a platform, the nonholonomic constraints have the form

$$\begin{aligned} \dot{q}_m &= \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \\ w \end{pmatrix} = \\ &= \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} v \\ w \end{pmatrix} = G_1 u, \end{aligned} \tag{3}$$

where $q_m = (x, y, \theta)^T$ describe posture of the platform. (x, y) are cartesian coordinates of the platform's mass center relative to the inertial basic frame X_0Y_0, and angle θ describes the orientation of the platform (the angle between the local axis X_p and the global axis X_0), see Fig. 1. Symbols v and w denote linear and angular velocities of the platform, respectively.

2.2. Nonholonomic constraints for the manipulator

In [5], a new approach to the control problem for the manipulators has been presented. The authors have developed a prototype of a new nonholonomic gear, which can transmit velocities to many passive joints. More complex and detailed scheme of such a gear has been introduced in [5]. In this paper only basic scheme is presented, see Fig. 2.

The basic components of the gear presented in Fig. 2 are a ball and three wheels – an input wheel IW and two output wheels OW_1 and OW_2. The velocity constraints of the ball are only due to the point contact with the wheels. The input wheel IW is mounted in the first joint, the output wheels are mounted in the next joint. The wheel IW rotates around the fixed axis α_I with an angular velocity $\dot{\rho} = \eta_2$,

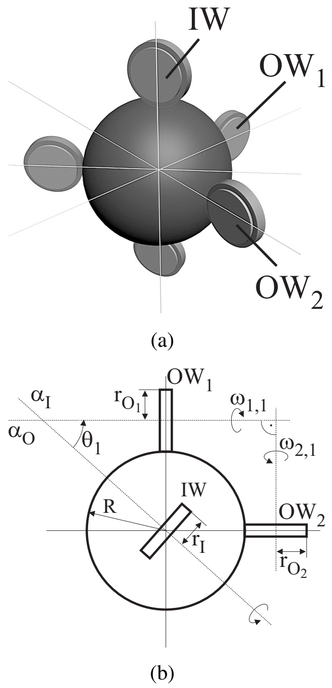

Fig. 2. Nonholonomic gear: a) schematic of the the gear, b) the gear seen from above.

which plays the role of a control input. The rotating input wheel IW makes the ball rotate. The wheel OW_1 rotates around an axis α_O, which forms with the axis of the input wheel the joint angle θ_1.

The nonholonomic constraints in the gear appear by the assumption of rolling contact without slippage between the elements of the gear (balls and wheels) in the robot joints. The nonholonomic constraints (the kinematics) of 3-pendulum can be expressed as the following relationship

$$\dot{q}_r = G_2(q_r)\eta,$$

or, in more detail,

$$\begin{aligned} \dot{q}_r &= \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a_2\sin\theta_1 \\ 0 & a_3\sin\theta_2\cos\theta_1 \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \\ &= G_2\eta, \end{aligned} \tag{4}$$

where θ_i is the position of the i-joint in the 3-pendulum, and positive coefficients a_2 and a_3 depend on gear ratios between output and input wheels in the second and third joint.

Angular velocity of the first joint $\dot{\theta}_1 = \eta_1$ and angular velocity of the input wheel IW in the first joint $\dot{\rho} = \eta_2$ play the role of control inputs to the nonholonomic constraints given by equation (4).

2.3. Dynamics of doubly nonholonomic mobile manipulator

Because of the nonholonomy of constraints (3) and (4) appearing in the motion of doubly nonholonomic mobile manipulator, to obtain the dynamic model of the mobile manipulator, the d'Alembert Principle has to be used

$$Q(q)\ddot{q}+C(q,\dot{q})\dot{q}+D(q) = A_{11}(q_m)\lambda_1+A_{21}(q_r)\lambda_2+B\tau, \tag{5}$$

where:
$Q(q)$ – inertia matrix of the mobile manipulator,
$C(q,\dot{q})$ – matrix of centrifugal and Coriolis forces acting on the mobile manipulator,
$D(q)$ – vector of gravity,
A_{ii} – matrix of nonholonomic constraints for the ith subsystem,
λ_i – vector of Lagrange multipliers for the ith subsystem,
B – input matrix,
τ – vector of controls.
Matrices $A_{11}(q_m)$ and $A_{21}(q_r)$ are Pfaff's matrices for the platform and the manipulator respectively, whereas B is the so-called input matrix

$$A_{11} = \begin{bmatrix} A_1^T(q_m) \\ 0 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} 0 \\ A_2^T(q_r) \end{bmatrix},$$

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}.$$

Submatrices B_1 and B_2 describe which coordinates of subsystems are directly driven by actuators. Equations of constraints (3) and (4) can be rewritten as the kinematics in the block form

$$\dot{q} = \begin{pmatrix} \dot{q}_m \\ \dot{q}_r \end{pmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{pmatrix} u \\ \eta \end{pmatrix} = G\zeta, \qquad (6)$$

$$\zeta = \begin{pmatrix} u \\ \eta \end{pmatrix},$$

where ζ is the vector of auxiliary velocities for both subsystems of mobile manipulator. After substituting the equation (6) into the dynamics (5) we get

$$Q^*\dot{\zeta} + C^*\zeta + D^* = B^*\tau \qquad (7)$$

with elements defined in the following way

$$Q^* = G^T QG, \quad C^* = G^T(Q\dot{G} + CG),$$

$$D^* = G^T D, \quad B^* = G^T B.$$

Equation (7) describes the dynamics of the doubly nonholonomic mobile manipulator expressed in the auxiliary coordinates.

3. Control problem statement

In this paper we are looking for a control algorithm preserving the proper cooperation between the mobile platform and its onboard manipulating arm. We assume that the desired task for the mobile manipulator can be decomposed into two independent parts for both subsystems:

- the end-effector of the robotic arm has to go to the desired point in the configuration space,
- the task of the platform is to go asymptotically to the desired trajectory $q_{md}(t)$ lying on the plane (only for checking the influence of the dynamics on behavior of manipulator's joints) or to go to the desired configuration in such a way that the position tracking errors go to 0 (main task).

A goal of the paper is to address the following control problem for the doubly nonholonomic mobile manipulators:

> Design a control law $\tau^T = (\tau_m^T, \tau_r^T)$ such that a mobile manipulator with fully known dynamics will realize tasks defined separately for each subsystem, and tracking errors will converge asymptotically to zero.

For developing the control law for the mobile manipulator, it is important to note that a complete model of the nonholonomic system has a structure of two cascaded equations: kinematics (nonholonomic constraints) and dynamics. For this reason the structure of the controller has to be divided into two independent parts connected in the cascaded way. Therefore the backstepping-like procedure for the design of the control law should be evoked [3]:

1) kinematic controller $\zeta_r(t)$ – represents an embedded control input, which ensures the realizability of the tasks defined for both groups of nonholonomic constraints. The kinematic controller can be treated as a solution to the kinematics (6), if the dynamics were not present. Such a controller generates a 'velocity profile', which has to be executed in practice. The kinematic control algorithm can be designed with or without feedback from the output.
2) dynamic controller τ – as a consequence of the cascaded structure of the model, the system's auxiliary velocities ζ cannot be commanded directly, as it is assumed in the design of the kinematic control, but they must be realized as the output of the dynamics (7) driven by τ. The dynamic input τ intends to regulate the real velocities ζ toward the reference control ζ_r and, therefore, attempts to provide control input necessary to achieve the desired tasks for both subsystems.

4. Algorithms of point stabilization for nonholonomic manipulator

As it was mentioned earlier, we take into considerations a manipulating arm equipped in special nonholonomic gears designed by Nakamura, Chung and Sørdalen. From the control point of view, it is very convenient to transform the kinematics of the nonholonomic manipulator into some generic form, namely the so-called "chained form" [4]. Such an approach makes it possible to use all the existing control laws developed for the chained systems.

The kinematics of the 3-pendulum, given earlier by equation (4), should be expressed now as follows

$$\begin{aligned} \dot{\theta}_1 &= \eta_1, \\ \dot{\theta}_2 &= a_2 \sin\theta_1 \eta_2, \\ \dot{\theta}_3 &= a_3 \sin\theta_2 \cos\theta_1 \eta_2, \\ \dot{\phi} &= \cos\theta_1 \cos\theta_2 \eta_2, \end{aligned} \qquad (8)$$

where ϕ is the orientation of wheel OW in the second joint of the 3-pendulum. It is mentioned in [5] that it is inconvenient to transform the kinematics of the nonholonomic manipulator into the chained form if the variable ϕ is not added to the set of state variables of the manipulator. Transformation coordinates $z = h(\phi, q_r)$ and feedback signal $\nu = F(\phi, q_r)$ introduced in [5] are local (they are valid only for angles $\theta_i \in (-\frac{\pi}{2}, \frac{\pi}{2}), i = 1, 2$) and they can have the following form

$$\begin{aligned} z_1 &= \phi, \\ z_2 &= a_2 a_3 \tfrac{\tan\theta_1}{\cos^3\theta_2}, \\ z_3 &= a_3 \tan\theta_2, \\ z_4 &= \theta_3, \end{aligned} \tag{9}$$

with new defined inputs to the system

$$\begin{aligned} \nu_1 &= \cos\theta_1 \cos\theta_2 \eta_2, \\ \nu_2 &= a_2 a_3 \left(\tfrac{\eta_1}{\cos^2\theta_1 \cos^3\theta_2} + 3a_2 \tfrac{\sin^2\theta_1 \sin\theta_2 \eta_2}{\cos\theta_1 \cos^4\theta_2} \right). \end{aligned}$$

4.1. Astolfi control algorithm

The control law introduced by Astolfi in [1] belongs to the algorithms, which realize the point stabilization of nonholonomic system using discontinuous static feedback from the state. This algorithm is dedicated to the chained systems with only two control inputs, similar to other algorithms, e.g. [6]. The chained form is a special generic form, which many mechanical systems could be transformed to. Into such form can be transformed two classes of single wheeled mobile platforms, namely $(2,0)$ (unicycle) and $(1,1)$ (kinematic car) and e.g. mobile platforms with trailers.

The equations of the chained system can be expressed as a driftless control system as follows

$$\dot{x} = g_1(x)u_1 + g_2 u_2, \tag{10}$$

with vector fields

$$g_1 = \begin{pmatrix} 1 \\ 0 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}, \qquad g_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Let us assume, that the control signal u_1 is constant, and denote it as $u_1 = \tilde{k}$. Now the system containing only the state variables $x_2, \dots, x_n$ can be rewritten in the matrix form

$$\begin{pmatrix} \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \vdots \\ \dot{x}_n \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ \tilde{k} & 0 & 0 & \dots & 0 & 0 \\ 0 & \tilde{k} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \tilde{k} & 0 \end{bmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} u_2. \tag{11}$$

We consider the following signal as the control u_2

$$u_2 = \tilde{p}_2 x_2 + \tilde{p}_3 x_3 + \ldots + \tilde{p}_{n-1} x_{n-1} + \tilde{p}_n x_n. \tag{12}$$

Then subsystem (11) can be expressed as a linear system

$$\dot{x}_{ob} = A\, x_{ob},$$

where

$$x_{ob} = (x_2\; x_3\; \dots\; x_n)^T,$$

and matrix has a form

$$A = \begin{bmatrix} \tilde{p}_2 & \tilde{p}_3 & \tilde{p}_4 & \dots & \tilde{p}_{n-1} & \tilde{p}_n \\ \tilde{k} & 0 & 0 & \dots & 0 & 0 \\ 0 & \tilde{k} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \tilde{k} & 0 \end{bmatrix}. \tag{13}$$

System (11) is a controllable linear system, which can be stabilized by using linear feedback control given by equation (12). The closed-loop system (11)-(12) must be asymptotically stable. It means that coefficients $\tilde{p}_i$ have to be selected in such the way that all eigenvalues of the matrix A have negative real part. In other words, the characteristic polynomial of the A matrix, i.e.

$$\begin{aligned} p_A(\lambda) = {} & \lambda^{n-1} - \tilde{p}_2\, \lambda^{n-2} - \tilde{p}_3\, \tilde{k}\, \lambda^{n-3} - \ldots \\ & - \tilde{p}_{n-1}\, (\tilde{k})^{n-3}\, \lambda - \tilde{p}_n\, (\tilde{k})^{n-2} \end{aligned}$$

must be Hurwitz polynomial. If an absolute value of $\tilde{k}$ decreases, the eigenvalues of A matrix are constant for properly scaled $\tilde{p}_i$ coefficients.

Now we assume, that $\tilde{k} = -kx_1$ and the form of control input u_2 is the same as previously. Then the chained system can be written as follows

$$\begin{aligned} \dot{x}_1 &= -kx_1, \\ \dot{x}_2 &= \tilde{p}_2 x_2 + \tilde{p}_3 x_3 + \ldots + \tilde{p}_{n-1} x_{n-1} + \tilde{p}_n x_n, \\ \dot{x}_3 &= -kx_1 x_2, \\ &\vdots \\ \dot{x}_n &= -kx_1 x_{n-1}. \end{aligned}$$

Now we introduce a change of coordinates (change of basis)

$$\xi = \begin{pmatrix} \xi_2 \\ \xi_3 \\ \vdots \\ \xi_{n-1} \\ \xi_n \end{pmatrix} = \begin{pmatrix} x_2(t) \\ \frac{x_3(t)}{x_1(t)} \\ \vdots \\ \frac{x_{n-1(t)}}{x_1^{n-3}(t)} \\ \frac{x_n(t)}{x_1^{n-2}(t)} \end{pmatrix}. \tag{14}$$

Jacobi matrix for such a transformation, given below,

$$J = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{x_1(t)} & 0 & \dots & 0 & 0 \\ 0 & 0 & \frac{1}{x_1^2(t)} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{1}{x_1^{n-3}(t)} & 0 \\ 0 & 0 & 0 & \dots & 0 & \frac{1}{x_1^{n-2}(t)} \end{bmatrix}$$

is always nonsingular because $x_1(t) = x_1(0)e^{-kt}$ and $x_1(0) \neq 0$ from assumption.

The time derivative of i^{th} variable ξ_i $(i = 3, 4, \dots, n)$ equals to

$$\begin{aligned} \dot{\xi}_i &= \tfrac{d}{dt}\left(\frac{x_i}{x_1^{i-2}}\right) = \frac{\dot{x}_i\, x_1^{i-1} - x_i(i-2)\, x_1^{i-3}\, \dot{x}_1}{\left(x_1^{i-2}\right)^2} \\ &= -k\frac{x_{i-1}}{x_1^{i-3}} + (i-2)k\frac{x_i}{x_1^{i-2}}. \end{aligned}$$

If coefficients $\tilde{p}_i$ have the form

$$\tilde{p}_i = \frac{p_i}{x_1^{i-2}}, \qquad \forall i = 2, \ldots, n,$$

then the system (11) can be expressed as

$$\dot{\xi} = \Lambda\, \xi,$$

with matrix Λ equal to

$$\Lambda = \begin{bmatrix} p_2 & p_3 & p_4 & \ldots & p_{n-1} & p_n \\ -k & k & 0 & \ldots & 0 & 0 \\ 0 & -k & 2k & \ldots & 0 & 0 \\ 0 & 0 & -k & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & (n-3)k & 0 \\ 0 & 0 & 0 & \ldots & -k & (n-2)k \end{bmatrix}. \qquad (15)$$

The control law for the system (10) proposed by Astolfi is given below

$$u = \begin{pmatrix} -kx_1 \\ p_2x_2 + p_3\dfrac{x_3}{x_1} + \ldots + p_{n-1}\dfrac{x_{n-1}}{x_1^{n-3}} + p_n\dfrac{x_n}{x_1^{n-2}} \end{pmatrix} \qquad (16)$$

This control algorithm is well-conditioned only if the assumption $x_1 \neq 0$ holds. The above considerations can be formulated as the following theorem.

Theorem 1 (Astolfi) *Let us consider a system given by* (10) *with initial condition $x(0)$, such that $x_1(0) \neq 0$. Then discontinuous control law*

$$u = \begin{cases} \textit{equation } (16) & \textit{for } x_1 \neq 0, \\ \begin{pmatrix} 0 & 0 \end{pmatrix}^T & \textit{for } x_1 = 0, \end{cases}$$

converts the chained system to the origin, if $k > 0$ and $\sigma(\Lambda) \subset C^-$, where $\sigma(\Lambda)$ is a spectrum of the matrix Λ (Λ is described by equation (15)*) and C^- is open complex half-plane.*

4.2. Polynomial algorithm proposed by Nakamura, Chung and Sørdalen

Kinematics of the 3-pendulum expressed as the chained system can be written in the form

$$\begin{aligned} \dot{z}_1 &= \nu_1, \\ \dot{z}_2 &= \nu_2, \\ \dot{z}_3 &= z_2\nu_1, \\ \dot{z}_4 &= z_3\nu_1. \end{aligned} \qquad (17)$$

Then input signals proposed by Nakamura, Chung and Sørdalen in [5] are equal to

$$\eta_r = \begin{pmatrix} \nu_{1r} \\ \nu_{2r} \end{pmatrix} = \begin{pmatrix} b_0 \\ c_0 + c_1t + c_2t^2 \end{pmatrix} \qquad (18)$$

with coefficient b_0 equal to

$$b_0 = \frac{z_1(T) - z_1(0)}{T}.$$

Polynomial inputs are often used due to their smoothness, smoothness of obtained trajectories of the system, and simplicity of their practical realization. The control law leading the system (17) from the initial state $z(0)$ to the desired final state $z(T)$ over the bounded time horizon T, is defined by equation (18), whereas the coefficients c_i of the second input polynomial can be computed *explicite* (after integration of state equations of the chained system) as follows

$$Mc + Nz(0) = z(T) \qquad (19)$$

or, in more detail, by

$$\begin{bmatrix} T & \frac{1}{2}T^2 & \frac{1}{3}T^3 \\ b_0\frac{T^2}{2} & b_0\frac{T^3}{6} & b_0\frac{T^4}{12} \\ b_0^2\frac{T^3}{6} & b_0^2\frac{T^4}{24} & b_0^2\frac{T^5}{60} \end{bmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} +$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ b_0T & 1 & 0 \\ b_0^2\frac{T^2}{2} & b_0^2T & 1 \end{bmatrix} \begin{pmatrix} z_2(0) \\ z_3(0) \\ z_4(0) \end{pmatrix} = \begin{pmatrix} z_2(T) \\ z_3(T) \\ z_4(T) \end{pmatrix}.$$

If $z_1(0) \neq z_1(T)$, then matrix M is nonsingular, and trajectories of individual state variables (joints of nonholonomic manipulator) could be computed analytically.

It should be emphasized that the presented control algorithm realizes an open-loop control (i.e. without including any information about real state of the system and of the environment).

5. Stabilization control algorithms for nonholonomic mobile platform

There are different control schemes, which can be implemented to solve stabilization problem for nonholonomic mobile robot of $(2,0)$ class. Two discontinuous approaches are compared – Astolfi algorithm for nonholonomic systems which can be transformed into chained form and Zhang & Hirschorn algorithm dedicated to mobile robots of $(2,0)$ class.

5.1. Astolfi algorithm

Nonholonomic mobile platform can be transformed into chained form

$$\begin{aligned} \dot{z}_1 &= u_1, \\ \dot{z}_2 &= u_2, \\ \dot{z}_3 &= z_2u_1 \end{aligned}$$

by using global diffeomorphism

$$\begin{aligned} z_1 &= \theta, \\ z_2 &= -x\cos\theta - y\sin\theta, \\ z_3 &= -x\sin\theta + y\cos\theta \end{aligned}$$

and static feedback equal to

$$\begin{aligned} u_1 &= \omega, \\ u_2 &= -v - \omega z_3. \end{aligned}$$

The discontinuous control law proposed by Astolfi for such a system has the following form

$$u_r = \begin{pmatrix} u_{1r} \\ u_{2r} \end{pmatrix} = \begin{pmatrix} -kz_1 \\ p_2z_2 + p_3\frac{z_3}{z_1} \end{pmatrix}. \qquad (20)$$

5.2. Zhang & Hirschorn algorithm

The algorithm proposed by Zhang & Hirschorn in [8] ensures stabilization of the robot and smoothness of the (x, y) trajectory. It is dedicated to the wheeled mobile robots of $(2, 0)$ class. The idea of this algorithm is based on two observations: the mobile robot can achieve a pure rotation by letting $v = 0$ and $\omega \neq 0$, and for any point (x_0, y_0) in the XY-plane, there is a circle which passes both (x_0, y_0) and the origin, and is centered on the Y axis. Thus, stability can be achieved by first making a pure rotation until the orientation angle $\theta(t)$ is almost tangent to circle $\mathcal{C}$, and then driving the robot asymptotically to the origin along that circle in such a way that orientation angle is controlled to make the tangent of a trajectory approach to the tangent of the circle. Equation (21) represents the family of circles $\mathcal{C}$ in the XY-plane

$$\mathcal{C} = \{(x, y) \mid x^2 + (y - r)^2 = r^2\}. \quad (21)$$

The angle of the tangent to $\mathcal{C}$ at (x, y) can be defined as

$$\theta_d(x, y) = \begin{cases} 2\tan^{-1}\frac{y}{x}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

In Fig. 3, the example of circle $\mathcal{C}$ with marked angle $\theta_d(x, y)$ has been shown.

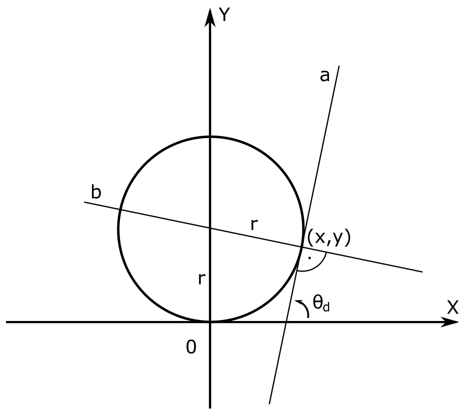

Fig. 3. The example of circle $\mathcal{C}$ and the angle $\theta_d(x, y)$.

Following discontinuous control law designed by Zhang & Hirschorn enables asymptotic stabilization of a mobile platform at the origin:

$$\omega_r(x, y, \theta) = \begin{cases} -k_1 \operatorname{sign}(\theta - \theta_d), & |\theta - \theta_d| > \epsilon, \\ -\frac{k_1}{\epsilon}(\theta - \theta_d), & |\theta - \theta_d| \leq \epsilon, \end{cases}$$

$$v_r(x, y, \theta) = \begin{cases} 0, & |\theta - \theta_d| > \epsilon, \\ -k_2(x^2 + y^2), & x \geq 0, |\theta - \theta_d| \leq \epsilon, \\ k_2(x^2 + y^2), & x < 0, |\theta - \theta_d| \leq \epsilon, \end{cases}$$

with $k_1 > 0$, $k_2 > 0$ and $\epsilon > 0$ to be sufficiently small.

Let $G = \{(x, y) : |x| \leq M, |y| \leq M\}$, where M is positive number which defines the region in which robot should stay. Practically M could be equal to $max\{|x_0|, |y_0|\}$. Imposing the condition

$$2k_2(1 + \epsilon)M \leq k_1$$

on k_1 and k_2 guarantees the requirement that once $|\theta(T) - \theta_d(T)| \leq \epsilon$ is met for some $T > 0$ then $|\theta(t) - \theta_d(t)| \leq \epsilon$ for $t > T$.

6. Dynamic control algorithm

To compare the influence of the kinematic control algorithms, we choose a simple dynamic control algorithm, namely exact linearization control algorithm

$$\tau = (B^*)^{-1}\left\{Q^*(\dot{\zeta}_r - K_m e_\zeta) + C^*\zeta + D^*\right\},$$
$$e_\zeta = \zeta - \zeta_r.$$

The above control law seems to have the least influence on the behavior of the whole object – it is due to the full compensation of Coriolis, centrifugal and gravity forces, however some disturbances may appear in discontinuity points of ζ_r and $\dot{\zeta}_r$.

7. Simulation results

Simulations have been made for the aforementioned object, namely the mobile manipulator, which consists of the nonholonomic 3-pendulum located on the mobile platform of $(2, 0)$ class (unicycle). The simulations cover two problems considered in the paper: disturbing influence of the dynamic on the solution obtained in pure kinematic algorithm, and behavior of the discontinuous control law applied to the mobile platform coming from $(2, 0)$ class.

7.1. The influence of the dynamics on behavior of the nonholonomic manipulator

The influence of the dynamic control algorithm on trajectories of the nonholonomic manipulator during convergence to the desired configuration (point stabilization problem) has been considered and simulated.

The desired configuration was equal to

$$\theta(0) = [-10°, -10°, -10°], \qquad T = 10 \text{ [s]}.$$
$$\theta(T) = [10°, 10°, 10°],$$

The trajectories of the respective joints of the 3-pendulum during convergence to the desired position using Astolfi algorithm have been presented in Fig. 4.

The regulation parameters in the Astolfi kinematic control algorithm have been chosen in such a way that all eigenvalues of the matrix Λ were equal to -2.

The next algorithm proved by simulations was algorithm given by Nakamura, Chung and Sørdalen. The trajectories of respective joints of the 3-pendulum during convergence to the desired position using this algorithm have been presented in Fig. 5.

During the regulation process, the mobile platform was in persistent motion – it has tracked a circle with radius 5 m. Such a behavior is aimed to increase the influence of the dynamics on the solution of kinematic controllers, because dynamic interactions between the platform and the manipulator in the mobile manipulator are very big.

7.2. Behavior of the discontinuous control law applied to the mobile platform

The second group of simulation research is aimed to check the way the mobile platform goes to the desired configuration (point stabilization) $q_{md} = 0$. Initial configuration of the platform was equal to $(x_0, y_0, \theta_0) = (1\,\text{m}, 1\,\text{m}, 1\,\text{rad})$. The plots of the mobile platform's trajectories obtained using Astolfi and Zhang & Hirschorn algorithm have been shown in Fig. 6. The regulation parameters in the Astolfi kinematic control algorithm were

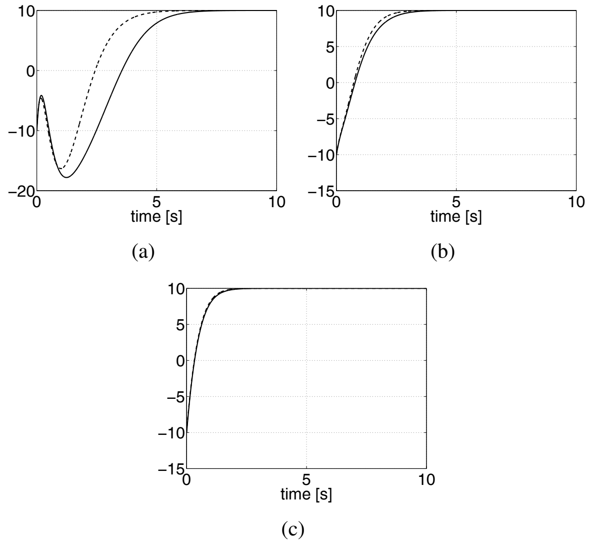

Fig. 4. Influence of the dynamic control algorithm with gain $K_m = 200$ on behavior of the system. Dotted line – hypothetical trajectories obtained by Astolfi kinematic algorithm, continuous line – real trajectories (kinematic and dynamic control) of the joints: (a) angle θ_1, (b) angle θ_2, (c) angle θ_3.

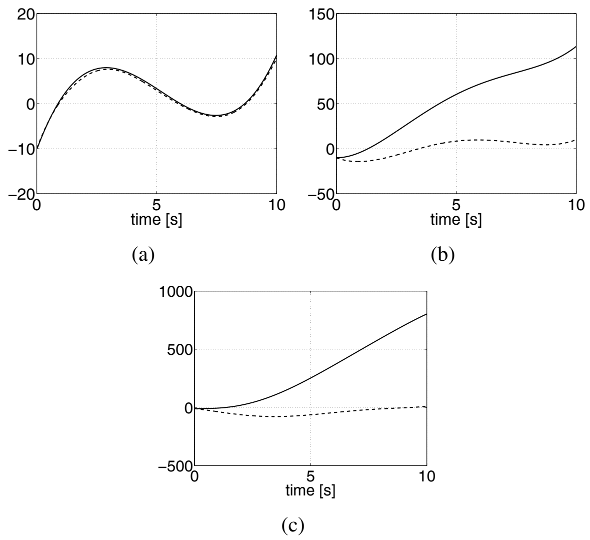

Fig. 5. Influence of the dynamic control algorithm with gain $K_m = 200$ on behavior of the system. Dotted line – hypothetical trajectories obtained by Nakamura, Chung & Sørdalen kinematic algorithm, continuous line – real trajectories (kinematic and dynamic control) of the joints: (a) angle θ_1, (b) angle θ_2, (c) angle θ_3.

chosen in such a way that all eigenvalues of the matrix Λ were equal to -2. For Zhang & Hirschorn algorithm parameters were equal to $(k_1, k_2, \epsilon) = (100, 40, 0.001)$.

Trajectories $(x(t), y(t))$ generated by Zhang & Hirschorn algorithm are smooth because they are part of circle $\mathcal{C}$ – Fig. 6b. It's a specific character of this algoritm, that the shape of $(x(t), y(t))$ trajectories is given. However $\theta(t)$ is switched so it's non-smooth, see Fig. 7b. Astolfi algorithm doesn't influence a shape of a robot's trajectory in the way Zhang & Hirschorn algorithm does. It can be observed in Fig. 6a that turns in the motion of robot

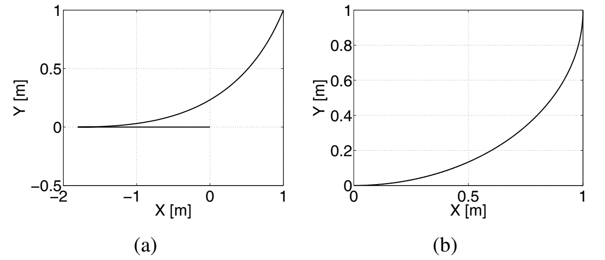

Fig. 6. Trajectories of the mobile platform during convergence to the origin: (a) – Astolfi kinematic controller for the platform, (b) – Zhang & Hirschorn kinematic controller for the platform.

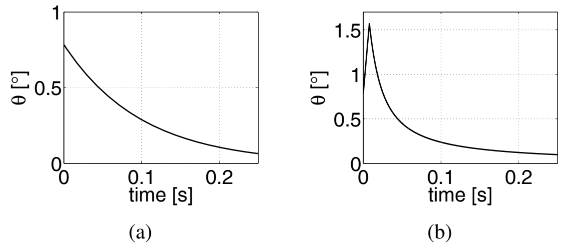

Fig. 7. Orientation of the mobile platform during convergence to the origin: (a) – Astolfi kinematic controller for the platform, (b) – Zhang & Hirschorn kinematic controller for the platform.

in XY-plane may appear but for considered mobile platform $\theta(t)$ is smooth (Fig. 7a). If we introduce the measure of smoothness as a number of turns (changes) of the state coordinates x and y, then we could conclude that Zhang & Hirschorn algorithm (no turns) is smoother that Astolfi algorithm (one turn).

8. Conclusions

The control process of a nonholonomic system needs to use two simultaneously working controllers: kinematic control algorithm (hypothetical case) solving the constraints equations and dynamic control algorithm, which realizes the kinematic solution in practice. In view of its function, the kinematic controller is often called task solver or motion planner. There are many kinematic controllers of different type, with or without feedback from the state or the robot environment, which have been presented in the literature. In this paper we have proposed to use different types of algorithms stabilizing the system in desired configuration: algorithms dedicated to the chained systems (Astolfi algorithm; polynomial algorithm given by Nakamura, Chung and Sørdalen) and an algorithm dedicated only to the mobile platform of $(2, 0)$ class.

In the simulation research, we wanted to check how useful different types of kinematic control algorithms are in practical applications. The results presented in Figure 5 show that kinematic controllers working in the open-loop are not robust on disturbances coming from dynamic level (second step in the cascade structure). Such controllers produce huge errors by generating trajectories going to the desired configurations. If the manipulator's joint is further then the position error is bigger, see Fig. 5.

In turn, for the kinematic control algorithm with closed-loop of feedback, see Fig. 4, there are differences between trajectories coming from kinematic control and kinematic and dynamic controllers working simultaneously, but the manipulator achieves the desired configuration. It means that only kinematic control laws using feedback signal should be used in practical applications. Such algorithms for point stabilization of the nonholonomic manipulator can preserve robustness on disturbances, especially generated by the dynamic level of control.

It is worth to mention that the choice of the kinematic controller for one subsystem of the doubly nonholonomic manipulator affects the behavior of the second subsystem due to big dynamical interactions between them.

During the realization of the stabilization process for nonholonomic mobile robots, trajectories of a robot in XY-plane may have different properties. Astolfi algorithm does not ensure that the trajectory is differentiable, although it is easy to be implemented for a big variety of nonholonomic systems (e.g. wheeled mobile platform, manipulator with nonholonomic gears etc.). Using Zhang & Hirschorn controller enables the state of a robot to asymptotical convergence to the target configuration, and the resulting $(x(t), y(t))$ trajectories are smooth.

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