FUZZY MODEL OF DECISION MAKING PROCESS

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Abstract

The decision making process is one of the most important and complicated human activity. The process could be met during the analysis of different domains. Unfortunately, the algorithms of decision making are defined only for some kinds of the problems. In more complex and complicated cases the optimal and universal algorithm of decision making doesn't exists. All that reasons limits the possibility of decision making process computerisation. From the other hand the experts of the problem domain makes optimal or suboptimal decision. The quality of the decisions is high enough to drive sometimes very complicated systems with acceptable efficiency. So, it is possible to solve the problem of decision making using the human mind. But the human mind is limited for the number of input data point of view. Additionally the quality of decision depends on the experts' experience, knowledge and mood. Therefore the computerised system to support the decision making process is very wanted. The first step of the computerisation is the considered problem model creation. In the paper the fuzzy model of decision making process is presented. It enables to model the approximate character of the process. Implementation of the model makes the computerisation of the considered issue possible.

Keywords: *fuzzy logic, fuzzy modelling , fuzzy sets, decision making process, multi-objective analysis*

ROZMYTY MODEL PROCESU PODEJMOWANIA DECYZJI

Streszczenie

Z problemem podejmowania decyzji można spotkać się w wielu dziedzinach ludzkiej aktywności. Podjęcie optymalnej lub suboptymalnej decyzji leży u podstaw wszelkiej działalności zarówno naukowej jak i przemysłowej. Niestety algorytmy pozwalające na automatyzację procesu podejmowania decyzji opracowane są wyłącznie dla niewielkiej grupy problemów charakteryzujących się względna prostotą. Dla zagadnień bardziej złożonych ogólny algorytm podejmowania decyzji nie istnieje. Z tego powodu automatyzacja i komputeryzacja procesu podejmowania decyzji napotyka znaczące trudności. Należy jednak zauważyć, że eksperci z dziedziny problemu są w stanie podjąć decyzję nawet w przypadkach systemów złożonych. Podejmowane decyzje mimo, że wielokrotnie nieoptymalne, pozwalają na rozwiązywanie rzeczywistych problemów z wystarczającą efektywnością. Niestety decyzje podejmowane przez ekspertów posiadają pewne ograniczenia właściwe ludzkiemu rozumowaniu i percepcji ludzkich zmysłów. Dodatkowo jakość podejmowanych decyzji zależy od doświadczenia, wiedzy i dyspozycji eksperta. W celu uniezależnienia procesu podejmowania decyzji od zmiennych czynników ludzkich przydatnym byłoby stworzenie komputerowego systemu wspomagającego analizowany proces. Pierwszym i podstawowy krokiem do stworzenia takiego systemu jest przyjęcie modelu procesu podejmowania decyzji. W opracowaniu zaprezentowano model rozmyty rozważanego procesu. Dzięki zastosowaniu modelu tej klasy możliwe stało się uwzględnienie przybliżonego charakteru rozumowania przeprowadzanego w trakcie procesu. Zastosowanie stworzonego modelu umożliwiło opracowanie oprogramowania wspomagającego proces podejmowania decyzji.

***Słowa kluczowe**: logika rozmyta, modelowanie rozmyte, zbiory rozmyte, proces podejmowania decyzji, analiza wielokryterialna*

1. Introduction

The decision making process can be met at many different domains of human activity. It applies to technical, medical, economic, planning, management and other problems. Experts of the problem domain realise the decision making process. Such way is limited by the human mind possibilities to process too many data. [1]. Every field of knowledge can be described by the systems theory. If we understand the set of mutually correlated elements and relations between them as a system [2], we can say that there is a relation in direct proportion between quantity and complexity of the system and on the other side the complication of decision process in discussed system.

We can notice the growing complication of the real systems nowadays [3]. That is why computerisation of the discussed problem becomes necessary. The computerised systems for supporting the decision process can particular improve the decision quality in case of many input data and limited time for taking the optimal or suboptimal decision.

We can often meet the automatic decision making systems in the area of the industrial process control and operation processes of the exploitation systems [4]. But this decision-making is limited to the control range means as conscious and purposeful influence on industrial process control to obtain the supposed effects [5]. In order to create universal decision - making support system, problem analysis has been curry out and the decision-making process model has been created.

2. Sharp model of decision making process

The first step of computerised expert system creation is the decision-making process description. The decision-making could be defined as a mental process, where the decision maker analyses input information, limitations, criteria and its weights based on his experience, knowledge and intuition. Chosen decision is the result of the process (Fig. 1).

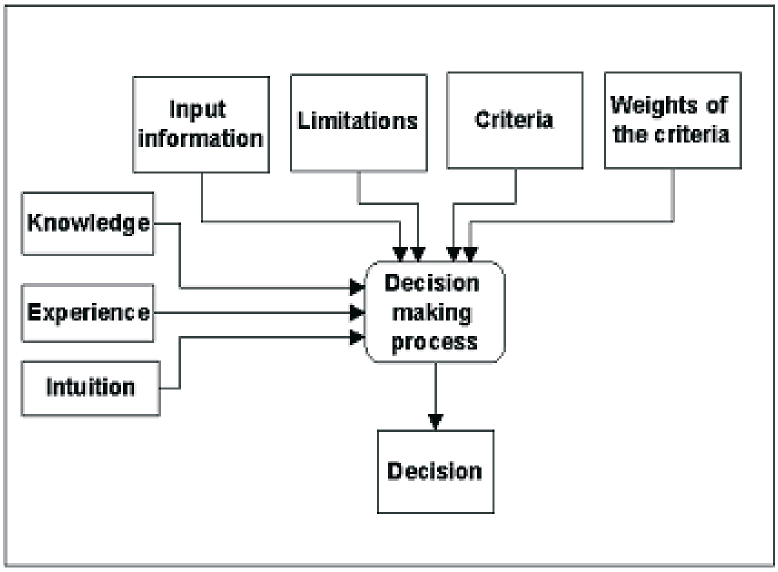

Fig. 1. The decision making process schema

In fact during the decision making process the expert of the domain assess the variants of the decision according to his knowledge, experience and intuition taking into consideration input information, limitations and criteria. So the diagram of the considered process could be transformed. On the base of knowledge, experience, intuition, limitations and input criteria the set of criteria and their weights could be formulated (Fig. 2.). The input information defines variants of solution. So, the considered problem is the multi-objective issue defined as assessment of finite set of variants from the finite set of criteria point of view [6]. Each criterion could be described by equation or inequality. Therefore the set of equations or inequalities is received. In particular cases the set could have more than one solution or could have not any one. In such cases it is not possible to receive an unambiguous solution [7].

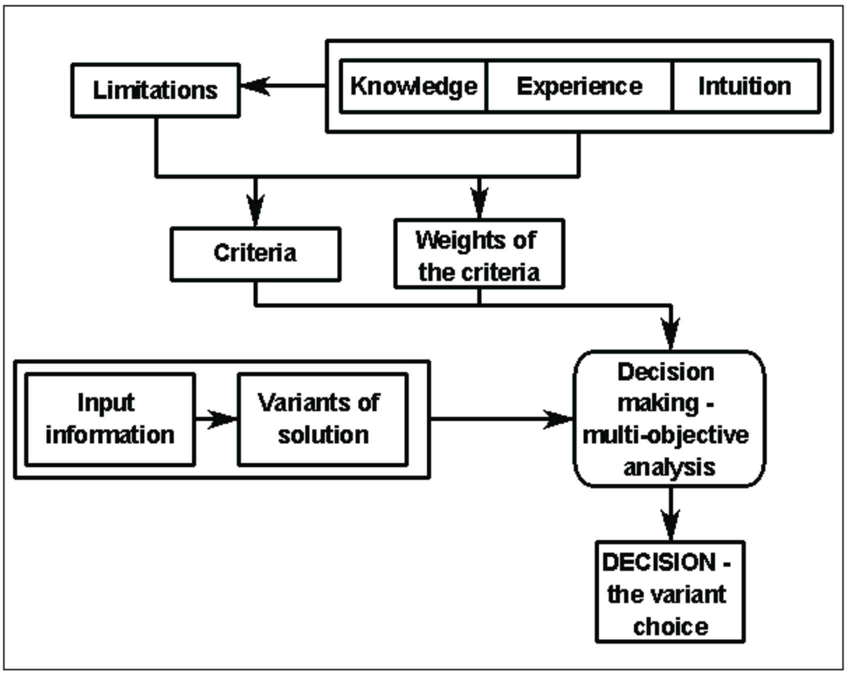

Fig. 2. Decision making as the multi-objective analysis issue

To solve this problem the decision making process was described using the SMART method. According to the implemented method, the domain of each criterion was defined on base of the criterion argument extent. The domain was divided into six intervals. The size of interval increases according to the geometric series as a distance from the optimum value. The quotient of the series equals to 2 [8].

Different types of the criteria were expressed by different functions. The criterion where the optimal value is the smallest one and the most important differences in level of criterion fulfilment are around the minimum point of the criterion domain was expressed by the function (1). The criterion where the optimal value is the biggest one was expressed by the function (2). For the function values corresponding to the arguments from the formula (3) for the criterion (1), and formula (4) for the criterion (2) the scale from 4 to 10 was introduced according the formula (5).

$$\upsilon = \log_2\left(\frac{P_\upsilon - P_{\min}}{P_{\max} - P_{\min}} \cdot 64\right), \tag{1}$$

where:

υ - value of criterion function,
P_υ - argument of criterion function,
P_{min} - minimal value of the criterion function argument,
P_{max} - maximal value of the criterion function argument.

$$\upsilon = \log_2\left(\frac{P_{\max} - P_\upsilon}{P_{\max} - P_{\min}} \cdot 64\right), \tag{2}$$

$$P_\upsilon = P_{\min} + \left(P_{\max} - P_{\min}\right) \cdot \frac{2^\upsilon}{64}, \upsilon = 0,1,......,6, \tag{3}$$

$$P_\upsilon = P_{\max} - \left(P_{\max} - P_{\min}\right) \cdot \frac{2^\upsilon}{64}, \upsilon = 0,1,......,6, \tag{4}$$

$$g = 10 - \upsilon, \tag{5}$$

where:

g - the level of the criterion fulfilment.

For the (1) criterion with desirable the biggest value [9] a scale from 4 to 10 was introduced according the formula (6).

$$g = 4 + \upsilon. \tag{6}$$

Weights of particular criterions were specified by AHP method (Analytic Hierarchy Process) [10]. It allows specifying the expert knowledge in a natural for human mind way. The method is based on the system of particular criterion comparison between each other. The scale for assigning linguistic comparisons to their values is specified (Tab. 1).

Tab. 1. Relative preference scale used to compare the criteria

Variants comparison	Preference of variant	Value
a is much more significant than *b*	Strong preference of *a*	6
a is more significant than *b*	Preference of *a*	4
a is little more significant than *b*	Weak preference of *a*	2
a is as significant as *b*	No preference	0
a is little less significant than *b*	Weak preference of *b*	-2
a is less significant than *b*	Preference of *b*	-4
a much less significant than *b*	Strong preference of *b*	-6

Based on assigned preferential values q_{ab} additional variable is calculated according to formula (7). q_{wjk} value symbolises the appearance preferential values q_{ab} between criterion *wj* and *k* numbers.

$$h_{wj} = \frac{1}{n_w} \sum_{k=1}^{n_w} q_{wjk}, \tag{7}$$

where:

q_{ab} - number value of preference.

This variable is determining the importance grade of particular criteria according to the (8):

$$c_{ki} = \frac{\left(\sqrt{2}\right)^{h_{ki}}}{\sum_{ki} \left(\sqrt{2}\right)^{h_{ki}}}, \tag{8}$$

where:

c_k - criterion weight,
h - criterion's importance grade.

The total grade of the variant has been calculated based on formula (9):

$$s_{ki} = \sum_{i=1}^{ki} c_i g_i, \tag{9}$$

where:

s_{ki} - complete variant grade.

The end estimation value comprised from 4 to 10 range. It allows the linguistic interpretation (Tab. 2). This fact is used in case of variants estimated usually in a qualitative way. Simultaneously, the linguistic interpretation of the results of the analysis allows the consideration of the variant total grade inaccuracy in automatic way [11].

SMART method is very useful in case of considered decision-making process because it enables to build coherent assessment system made from the criteria defined for completely different domains.

Tab. 1. Interpretation of complete variant estimation value

Complete variant estimation	Interpretation
10	Ideal
9	Very good
8	Good
7	Sufficient
6	Acceptable
5	Bad
4	Very bad

3. Fuzzy extension of created model

Unfortunately, during the studies, presented above method prove insufficient. The problem is related to the approximate characteristic of the decision making process.

The criteria of the decision making process could be divided into two groups. The first group consists of the criteria defined for the domain described by the measured values. The second one consists of criterion defined for parameters estimated in a discrete way. This discrete estimation is made by determination the degree of meet the criteria (criteria for particular attributes). In case when attributes values are estimated based on measurement, received value is defined with the measuring equipment accuracy [12].

It is not possible to define this value precisely. It is only possible to define its range.

$$X_o(t) \in \left\langle X_p(t) - \delta_u, X_p(t) + \delta_u) \right\rangle, \tag{10}$$

where

X_0 - analytical value,
X_P - measured value,
δ_u - measuring equipment error.

This value has also the measuring method error in case of indirect measurement. This fact impact in taking the tolerance interval that has to be considered during the analysis of received results. That is why the representation of considered tolerance range is taken as a fuzzy set.

Presented above approximation was modelled by the fuzzy sets implementation [13]. In case of the measurements with the insensitive zone the tolerance interval was modelled by the Π type fuzzy set. In the remaining cases of measurements the tolerance interval was modelled in form of the Λ type fuzzy set. As a modal value of the fuzzy set the measured value was admitted. The support of the fuzzy set was equal to the sharp interval of the considered tolerance.

The similar situation is in case of the criterion defined for the domain described by the linguistic values. The value is estimated by scale implementation. The result of the estimation is of course approximated. The level of approximation depends on the distance between the grades of the scale. In this case the inaccuracy was modelled using the Λ type fuzzy sets. The modal value of the fuzzy set was equal to the chosen grade and the extent of the fuzzy set support was equal to the distance between the discreet values of the assessment scale.

The second place where the inaccuracy appears is the fulfilment level of criterion assessment. To model this kind of inaccuracy the criteria was expressed in form of the fuzzy sets. The fuzzy set member function for criterion where the optimal value is as big as possible (MAXINV type of criterion) was determined according to equation (3) and equation (6). The shape of the member function is presented on (Fig. 3).

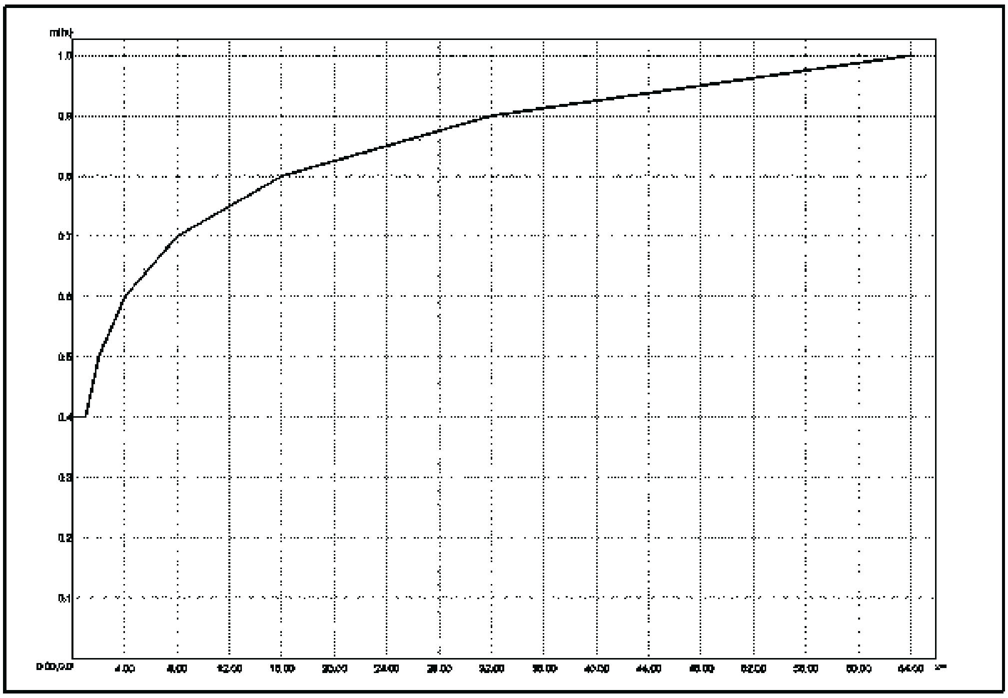

Fig. 3. Fuzzy set for criterion where the optimal value is as big as possible

The fuzzy set member function for criterion where the optimal value is the biggest one (MAXSIMP type of criterion) was determined according to equation (4) and equation (6). The shape of the member function is presented on (Fig. 4).

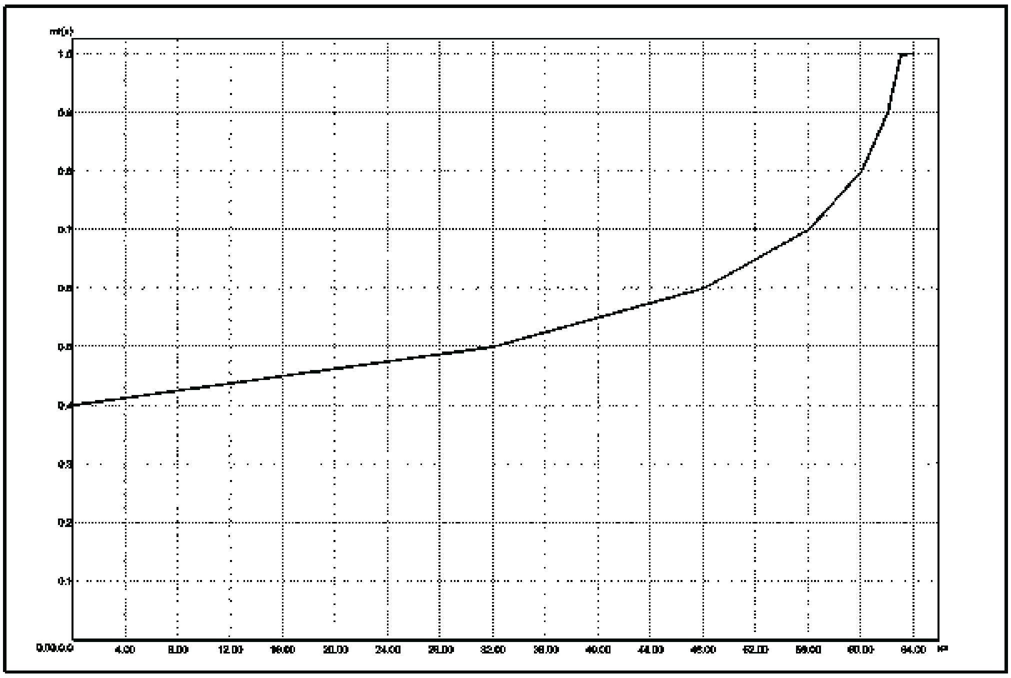

Fig. 4. Fuzzy set for criterion where the optimal value is the biggest one

The fuzzy set member function for criterion where the optimal value is the smallest one (MINSIMP type of criterion) was determined according to equation (3) and equation (5). The shape of the member function is presented on (Fig. 5).

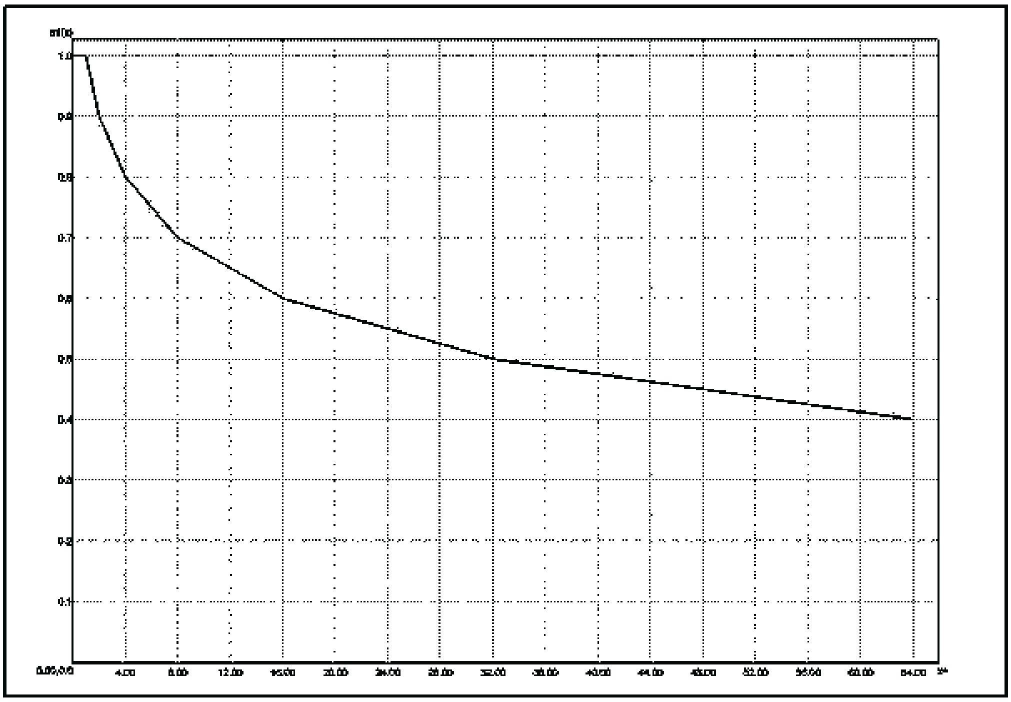

Fig. 5. Fuzzy set for criterion where the optimal value is the smallest one

The values for the criteria fulfilment level, calculated according to Equation 5 and 6, are in range from 4 to 10. The regular fuzzy sets member functions have the values from 0 to 1. To receive regular fuzzy sets for each criterion the normalisation process was carried out. Calculated values were divided by ten.

The third place of inaccuracy appearance is the criteria weights determination. The implemented method (AHP) is a subjective one, so the weights are estimated in approximated way. To model this kind of fuzziness the weights were expressed as Λ type fuzzy sets. The support of the weight fuzzy set was the sharp set in range <0,1>. The modal value of the set was equal to the estimated weight value. So the fuzzy set could be asymmetric one.

In the next steps of the multi-objective analysis the levels of the criteria fulfilment and weights of the criteria are treated as fuzzy digits. It enables to take into consideration the approximated character of the determined values. To calculate the total grade of the assessed variant the fuzzy digits should be accumulated into one value. To do it the LR notation of the triangular fuzzy digits was used [14]. The level of the criterion fulfilment is modelled in form of triangular fuzzy digit where the modal value is equal to the sharp value of the criteria fulfilment level and the support is sharp set in range <0,1>. Such triangular fuzzy digit is expressed in LR notation according to equation (11).

$$g_k \Rightarrow (x : \mu_{g_k}(x) = hgt(g_k), \min(x : x \in \sup p(g_k)), \max(x : x \in \sup p(g_k)))_{LR}, \tag{11}$$

where:

g_k - the value of the criterion fulfilment level,
x - domain of the fuzzy set,
hgt - height of the fuzzy set,
supp - support of the fuzzy set,
k - number of the criterion.

In case of the criterion weight the value is already the triangular fuzzy set which could be treated as a fuzzy digit. So, we can transform it to LR notation according to equation (12).

$$c_k \Rightarrow (x : \mu_{c_k}(x) = hgt(c_k), \min(x : x \in \sup p(c_k)), \max(x : x \in \sup p(c_k)))_{LR}, \tag{12}$$

where:

c_k - weight of the criterion.

Defined above LR fuzzy digits are the input values of the total variant grade calculation. The calculation is carried out according to equation (13).

$$s = \bullet(\underset{k}{\oplus}(g_k \otimes c_k)), \tag{13}$$

where:

s - total grade of the criterion,
$\otimes$ - T-norm operator,
$\oplus$ - S-norm operator,
$\bullet$ - defuzzification operator.

As an S-norm operator the maximum operator was used. As a T-norm operator the MIN or PROD operation could be used [15]. As a defuzzification operator the centre of gravity operator, mean of maximum operator, first of maximum operator and last of maximum operator [16] could be used. During the carried out studies the centre of gravity operator (14) implementation gives the best results. The results of the studies are analysed from the indiferention point of view. The results of the defuzzification operator implementation are assessed as good when for different variants of solution the assessment system gives different grades.

$$SC(FS) = \frac{\int x\mu_{FS}(x)dx}{\int \mu_{FS}(x)dx}, \tag{14}$$

where:

FS - fuzzy set,
μ_{FS} - member function of fuzzy set,
x - the domain of fuzzy set support.

To calculate the total grade of the analysed variant the height operator was also implemented (15):

$$S(FS) = \frac{\sum_{i=1}^{n}(x : \mu_{FS}(x) = hgt(FS_i)) \cdot hgt(FS_i)}{\sum_{i=1}^{n} hgt(FS_i)}, \tag{15}$$

where:

S - height operator,
FS_i - fuzzy set number i,
n - amount of criterion,
μ_{FS} - member function of each fuzzy set,
x - the domain of each fuzzy set support,
hgt - height of each fuzzy set.

In case of height operator implementation the accumulation process consisted only from T-norm and height defuzzification operation. After T-norm operator implementation the n fuzzy sets were received. The height operator implementation joins all fuzzy sets and determines sharp grade of the variant.

Fuzzy extension of the multi-objective analysis gives as a result the considered variant grade. The grade is interpreted the same way like the results of SMART method implementation.

4. Summary

On the base of the designed fuzzy model of the decision making process the computerised assessment system was created. The system was implemented in different domains of knowledge as an expert system to support the decision-making issues.

Presented method was implemented in the area of maintenance scheduling of power plant devices. In this case the criteria formulated in approximate way were modelled in from presented in the paper. Thanks to combine the assessment system with optimisation techniques the quality of the decisions made in the power plant maintenance system increase of about 18% [17].

The quality assessment of the transport system was the next application of the method. In this case the method enables to create the coherent system which takes into consideration the discrete and continuous criteria. Thanks to the system it is possible to assign the unique grade for each analysed exploitation strategy variant [18].

In case of sailing yacht skipper decision-making during storm conditions the method taken into consideration the fuzzy character of the criteria, their weights and the approximation of the criteria arguments values estimation. The method implementation was the base of computerised expert system that helps the skipper to make the safety decision [19].

The computerised assessment system was also implemented in case of commercial issues. It was put into practice as a choice support system in tourist agency, car showroom or estate agency. It helps to choose the best offer for individual client.

On base of theoretical considerations and practical implementations it is possible to notice that:

- the decision making process could be transform to the form of the multi - objective issue,
- known multi-objective analysis methods in most cases are insufficient because they don't consider the inaccuracy of the process,
- the fuzzy character of considered analysis exists in different steps of the process,
- to take into consideration the fuzzy character of the process the fuzzy logic elements should be implemented,
- each kind of inaccuracy needs to be modelled in different way,
- fuzzy model of decision making process enables to implement the computerised support in width range of the problems,
- the method implementation brings good result especially in case of complex systems where the decisions are made on base of the criteria formulated in approximate way.

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