# ADHESION INFLUENCE ON THE OIL VELOCITY AND FRICTION FORCES IN HYPERBOLIC MICROBEARING GAP

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***Abstract***

*In this paper are presented the influences of the adhesion forces occurring in hyperbolic micro contact on the oil velocity and friction forces in hyper bolic micro bearing gap. Here are elaborated the dependences between adhesive forces and oil dynamic viscosity in super t hin boundary layer and in fluences of adhesive forces on the oil velocities and friction forces in micro- and nano-scale arising between two cooperating hyperbolic surfaces in micro-bearings.*

*Oil dynamic viscosity is a sum of classical viscosity and viscosity caused by adhesion and cohesion forces.*

*In hyperbolic micro-bearing the influence of adhesive forces on friction forces is visible not only in friction forces caused by the hydrodynamic pressure but also in friction forces caused by the oil flow velocity.*

*The pressure distributions and capacity val ues in machine hyperbolic slide micro-bearings are determined in the lubrication region which is defined on the micro-bearing surfaces.*

*Numerical calculations are performed in Mathcad 14 Professional Program implemented by virtue of difference method.*

*In this paper are derived the formulas for velocity components, pressure distributions, friction forces and friction coefficients in hyperbolic coordinates and at the same time the adhesion forces are considered.*

*Up to now the influence of adhesion forces on oil velocity and friction force changes in hyperbolic micro-bearing gap were not considered in analytical way. Present paper elaborates the preliminary assumptions of hydrodynamic theory of lubrication for hyperbolic micro-bearing in the case if during the lubrication the adhesion forces are taken into account.*

***Keywords:*** *adhesion forces, hyperbolic micro-bearing, oil velocity, friction forces, hydrodynamic pressure*

## 1. Introduction

In recent papers relating to the problems of adhesion forces the slide bearing with the hyperbolic journal are not considered [11-14]. Up to now the scientific papers are investigated adhesion forces problem in experimental way taking into account roughness of surfaces [15], nano-contact [2], ferrofluids as lubricant [7], protein coated surfaces [5], hydrate particles [1], biological surfaces [9], charged particles [6], elastic-plastic micro-contact [4], pharmaceutical particle [3], [10], polymer-polymer micro-bearings [8]. After authors' knowledge, up to now the influence of adhesion on the oil velocity changes in hyperbolic micro-bearing gap and on the friction forces were not considered in analytical way. Such considerations are very important in exploitation problems concerning the lubrication of hyperbolic micro-bearing journals. We assume that the adhesion forces can to change the dynamic oil viscosity in super thin hyperbolic micro-bearing gap. Taking into account hyperbolic coordinates ($\varphi$, $y_h$, $\zeta_h$) in circumferential, radial and longitudinal directions, then the changes of the dynamic oil viscosity are described by the following formula:

$$\eta_T(\varphi, y_h, \zeta_h) = \eta(\varphi, \zeta_h) + \eta_{adh}(\varphi, y_h, \zeta_h). \tag{1}$$

where dimensional values are as follows:
$\eta_T$ - total oil dynamic viscosity,
$\eta$ - classical oil dynamic viscosity,
$\eta_{adh}$ - oil dynamic viscosity caused by the adhesion forces.

Molecules of the oil can be adsorbed on a cooperating hyperbolic micro-bearing surfaces and create high elasticity layer. Adsorption and adhesion changes the dynamic oil viscosity if gap height is smaller than 1 micrometer [1].

The groove and ridge geometry located on the hyperbolic surface are presented here. Figure 1 shows that the grooves on the hyperbolic journal surfaces can be situated in circumferential or longitudinal directions [2]. Groove location affects the dynamic performance of HDD spindle system. The hyperbolic micro-bearing lubrication is characterized by the dynamic viscosity changes in thin gap- height direction.

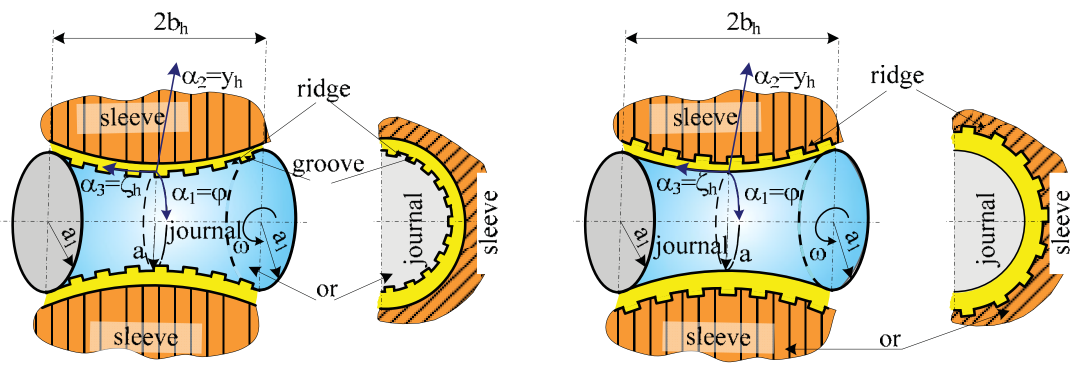

*Fig. 1. The view of hyperbolic journal surfaces with ridges and grooves*

## 2. Pressure distributions in hyperbolic micro-bearings gaps

For the hyperbolic micro-bearing we assume following hyperbolic co-ordinates: $\alpha_1=\varphi$, $\alpha_2=y_h$, $\alpha_3=\zeta_h$. Mentioned coordinates are presented in Fig. 1. For hyperbolic journal we have: $a_1$ – the largest radius of the hyperbolic journal, a – the smallest radius of the hyperbolic journal, $2b_h$ – the bearing length (see Fig. 1). From the system of conservation of momentum and continuity equation after thin boundary layer simplifications and boundary conditions in the hyperbolic coordinates ($\varphi$, $y_h$, $\zeta_h$) we obtain the oil velocity components in hyperbolic gap. Flow is generated by journal rotation and the sleeve is motionless. Dimensional lubricant velocity components $v_\varphi$, $v_y$, $v_\zeta$ in $\varphi$,$y_h$,$\zeta_h$ directions, respectively, have the following form [6]:

$$v_\varphi(\varphi, y_h, \zeta_h, t) = \frac{1}{h_\varphi}\left(\frac{\partial p}{\partial \varphi} + \frac{\partial p_{adh}}{\partial \varphi}\right)A_\eta + (1 - A_s)\omega h_\varphi, \tag{2}$$

$$v_\zeta(\varphi, y_h, \zeta_h, t) = \frac{1}{h_\zeta}\left(\frac{\partial p}{\partial \zeta_h} + \frac{\partial p_{adh}}{\partial \zeta_h}\right)A_\eta, \tag{3}$$

$$v_y(\varphi, y_h, \zeta_h, t) = -\int_0^{y_h} \frac{1}{h_\varphi}\frac{\partial v_\varphi}{\partial \varphi}dy_h - \int_0^{y_h} \frac{1}{h_\varphi h_\zeta}\frac{\partial (h_\varphi v_\zeta)}{\partial \varphi}dy_h, \tag{4}$$

$$A_s(\varphi, y_h, \zeta_h, t) \equiv \frac{\int_0^{y_h} \frac{1}{\eta + \eta_{adh}} dy_h}{\int_0^{y_h} \frac{1}{\eta + \eta_{adh}} dy_h}, \tag{5}$$

$$A_\eta(\varphi, y_h, \zeta_h, t) \equiv \int_0^{y_h} \frac{y_h}{\eta + \eta_{adh}} dy_h - A_s(\varphi, y_h, \zeta_h, t) \int_0^{\varepsilon_T} \frac{y_h}{\eta + \eta_{adh}} dy_h, \tag{6}$$

where dimensional values are as follows:

$0 \le y_h \le \varepsilon_T$, $0 \le \varphi < 2\pi\theta_1$, $0 \le \theta_1 < 1$, $-b_h \le \zeta_h \le b_h$

$\eta = \eta(\varphi, \zeta_h)$ - liquid dynamic viscosity without adhesion influences,

$\varepsilon_T(\varphi, \zeta_h, t)$ - gap height,

$p(\varphi, \zeta_h, t)$ - pressure without influences of adhesion forces,

$\eta_{adh} = \eta_{adh}(\varphi, y_h, \zeta_h)$ - oil dynamic viscosity changes caused by the adhesion forces,

$p_{adh}$ - changes of pressure caused by the adhesion,

$\omega$ - angular velocity of the journal,

$t$ - time.

For the hyperbolic shapes of micro-bearing journals we have following Lame coefficients:

$$h_\varphi = a \cos^{-2}(\Lambda_{h1}\zeta_{h1}), \quad h_\zeta = \sqrt{1 + 4(\Lambda_{h1}/L_{h1})^2 \tan^2(\Lambda_{h1}\zeta_{h1})} \cos^{-2}(\Lambda_{h1}\zeta_{h1}), \tag{7}$$

$$\Lambda_{h1} \equiv \sqrt{\frac{a_1 - a}{a}}, \quad L_{h1} \equiv \frac{b_h}{a}, \quad \zeta_{h1} = \zeta_h / b_h, \tag{8}$$

where dimensional values $a$, $a_1$, $b_h$ are defined before. Imposing boundary condition $v_y=0$ for $y_h=\varepsilon_T$ on the radial velocity component (4), than the dimensional unknown pressure function $p(\varphi,\zeta_h,t) + p_{adh}(\varphi,\zeta_h,t)$ satisfies the modified Reynolds equations in the following form curvilinear coordinates [6]:

$$\frac{\partial}{\partial\varphi}\left[\left(\frac{\partial p}{\partial\varphi} + \frac{\partial p_{adh}}{\partial\varphi}\right)\int_0^{\varepsilon_T} A_\eta dy_h\right] + \frac{h_\varphi}{h_\zeta}\frac{\partial}{\partial\zeta_h}\left[\frac{h_\varphi}{h_\zeta}\left(\frac{\partial p}{\partial\zeta_h} + \frac{\partial p_{adh}}{\partial\zeta_h}\right)\int_0^{\varepsilon_T} A_\eta dy_h\right] =$$

$$= \omega h_\varphi^2 \frac{\partial}{\partial\varphi}\left[\int_0^{\varepsilon_T} A_s dy_h - \varepsilon_T\right] + h_\varphi^2 \frac{\partial \varepsilon_T}{\partial t}. \tag{9}$$

### 3. Friction forces in curvilinear micro-bearing gap

This section presents the friction forces calculation in curvilinear micro-bearing gaps. The components of friction forces in curvilinear $\varphi$, $\zeta_h$ directions occurring in micro-bearing gaps, have the following forms:

$$F_{R\varphi} = \iint_\Omega \left[(\eta + \eta_{adh})\frac{\partial v_\varphi}{\partial y_h}\right]_{y_h=\varepsilon_T} h_\varphi h_\zeta d\varphi d\zeta_h, \quad F_{R\zeta} = \iint_\Omega \left[(\eta + \eta_{adh})\frac{\partial v_\zeta}{\partial y_h}\right]_{y_h=\varepsilon_T} h_\varphi h_\zeta d\varphi d\zeta_h, \tag{10}$$

where:

$0 \le y_h \le \varepsilon_T$, $0 \le \varphi < 2\pi\theta_1$, $0 \le \theta_1 < 1$, $-b_h \le \zeta_h \le b_h$

$\eta = \eta(\varphi, \zeta_h)$ - dimensional liquid dynamic viscosity without adhesion influences,

$\eta_{adh}(\varphi, y_h, \zeta_h)$ - dimensional oil dynamic viscosity caused by adhesion forces,

$\varepsilon_T(\varphi, \zeta_h)$ - dimensional gap height,

$v_\varphi, v_\zeta$ - dimensional fluid velocity components (2), (3) in $\varphi$, $\zeta$ directions,
$t$ - time,
$\Omega$ - lubrication surface.

Putting formulae (2), (3) and the functions $A_s$, $A_\eta$ from the formulae (5), (6) into equation (10) for hyperbolic journal, then we obtain the friction components $F_{R\varphi}$, $F_{R\zeta}$ in circumferential $\varphi$, and longitudinal $\zeta$ directions:

$$F_{R\zeta} = \iint_\Omega \frac{1}{h_\zeta}\left(\frac{\partial p}{\partial \zeta_h} + \frac{\partial p_{adh}}{\partial \zeta_h}\right)\left[\varepsilon_T(\varphi,\zeta_h) - \frac{\displaystyle\int_0^{\varepsilon_T(\varphi,\zeta_h)} \frac{y_h dy_h}{\eta(\varphi,\zeta_h)+\eta_{adh}(\varphi,y_h,\zeta_h)}}{\displaystyle\int_0^{\varepsilon_T(\varphi,\zeta_h)} \frac{dy_h}{\eta(\varphi,\zeta_h)+\eta_{adh}(\varphi,y_h,\zeta_h)}}\right] h_\varphi h_\zeta d\varphi d\zeta_h, \tag{11}$$

$$F_{R\varphi} = \iint_\Omega \left(\frac{\partial p}{\partial \varphi} + \frac{\partial p_{adh}}{\partial \varphi}\right)\left[\varepsilon_T(\varphi,\zeta_h) - \frac{\displaystyle\int_0^{\varepsilon_T(\varphi,\zeta_h)} \frac{y_h dy_h}{\eta(\varphi,\zeta_h)+\eta_{adh}(\varphi,y_h,\zeta_h)}}{\displaystyle\int_0^{\varepsilon_T(\varphi,\zeta_h)} \frac{dy_h}{\eta(\varphi,\zeta_h)+\eta_{adh}(\varphi,y_h,\zeta_h)}}\right] h_\varphi h_\zeta d\varphi d\zeta_h +$$

$$- \iint_\Omega \left[\frac{\omega h_\varphi^2 h_\zeta}{\displaystyle\int_0^{\varepsilon_T(\varphi,\zeta_h)} \frac{dy_h}{\eta(\varphi,\zeta_h)+\eta_{adh}(\varphi,y_h,\zeta_h)}}\right] d\varphi d\zeta_h, \tag{12}$$

where:
$\eta=\eta(\varphi,y_h,\zeta_h)$, $0\le y_h \le \varepsilon_T$, $0\le\varphi<2\pi\theta_1$, $0\le\theta_1<1$, $\zeta_{h1}=\zeta_h/b_p$, $-b_h\le\zeta_h\le b_h$, $\Omega(\varphi,\zeta_h)$ – lubrication surface.

## 4. Numerical calculations

The pressure distributions and capacity values in machine hyperbolic slide micro-bearings are determined in hyperbolic coordinates: ($\varphi$, $y_h$, $\zeta_h$), where the lubrication region $\Omega$, is defined by the following inequalities: $0\le\varphi\le\varphi_k$, $-b_h\le\zeta_h\le b_h$ where $2b_h$ – hyperbolic micro-bearing length.

Numerical calculations of the pressure distribution and friction forces component for adhesion influences and without adhesion forces are performed in Mathcad 14 Professional Program implemented by the difference method by virtue of the equation (9), (11), (12). Pressure distributions are presented in Fig. 2. For such pressure distributions are calculated the capacity forces (Fig. 3) and friction forces by virtue of Eq. (11), (12) and the results of calculations are presented in Fig. 4.

If grooves length is situated in $\zeta_h$ and $\varphi$ direction then gap height of the hyperbolic micro-bearing has the following form respectively [1]:

$$\varepsilon_T(\varphi,\zeta_h,t) = \varepsilon\left[1+\lambda_h(\zeta_h,t)\cos\varphi + \varepsilon_{g1}\sum_{n=0}^{k}(-1)^n H_\eta(\varphi - 0{,}5n\varphi_T)\right],$$
$$\varepsilon_T(\varphi,\zeta_h,t) = \varepsilon\left[1+\lambda_h(\zeta_h,t)\cos\varphi + \varepsilon_{g1}\sum_{n=0}^{k}(-1)^n H_\eta(\zeta_h - 0{,}5n\zeta_T)\right], \tag{13}$$

where:
$0 \le \varphi < 2\pi, - b_h \le \zeta_h \le b_h$
$\varepsilon_{g1} \equiv \varepsilon_g/\varepsilon$, $\varepsilon_g$ - ridge height,
$\lambda_h$ - eccentricity ratio in hyperbolic micro-bearing,
$\varepsilon$ - radial clearance in hyperbolic micro-bearing,
$H_\eta$ - Heavisidea unit function.

Symbols $\varphi_T$, $\zeta_T$ denote periods of the space in grooves sequence. Such interval has value about 100 nm in $\varphi$ and $\zeta$ – directions respectively. Symbol k denotes number of ridges and equals about 1000. Oil dynamic viscosity is a sum of classical viscosity and viscosity caused by adhesion and cohesion forces accordingly with (1) and the oil dynamic viscosity changes caused by the adhesion are presented below:

$$\eta_{akh}(\varphi, y_h, \zeta_h) = \eta_o\left\{a_\eta k + \frac{b_\eta}{\varepsilon_T}\left[e^{c_\eta y_h^2 - d_\eta y_h + f_\eta}\right]\right\}, \tag{14}$$

where:
$a_\eta$, $b_\eta$, $c_\eta$, $d_\eta$, $f_\eta$ - experimental coefficients,
k - curvature (inversion of the radius of curvature),
$\eta_o$ - characteristic dimensional dynamic viscosity value.

The dynamic viscosity caused by adhesion and cohesion in super thin oil layer, attain the largest values near to the journal and sleeve surface in gap height direction.

Figure 2 shows the numerical pressure values in hyperbolic micro-bearing gap. Calculations are performed without stochastic changes for: least value of the hyperbolic journal a=0.001 m, relative radial clearance $\psi$=0.0025; radial clearance $\varepsilon=2.5\cdot10^{-6}$ m, dimensionless bearing length $L_{h1}=b_h/a=1$, oil dynamic viscosity $\eta$=0.030 Pas, curvature k=1000 $m^{-1}$, experimental coefficients $a_\eta$=0.000040 m, $b_\eta$=0.000001 m, $c_\eta$=22.222222 $m^{-2}$, $d_\eta$=33.333333 $m^{-1}$, $f_\eta$=12.500000, angular velocity $\omega$=754 $s^{-1}$, characteristic dimensional value of hydrodynamic pressure $p_o=\omega\eta R^2/\varepsilon^2$, $p_o$=3.619 MPa, relative eccentricity values $\lambda_h$=0.5; $\lambda_h$=0.6.

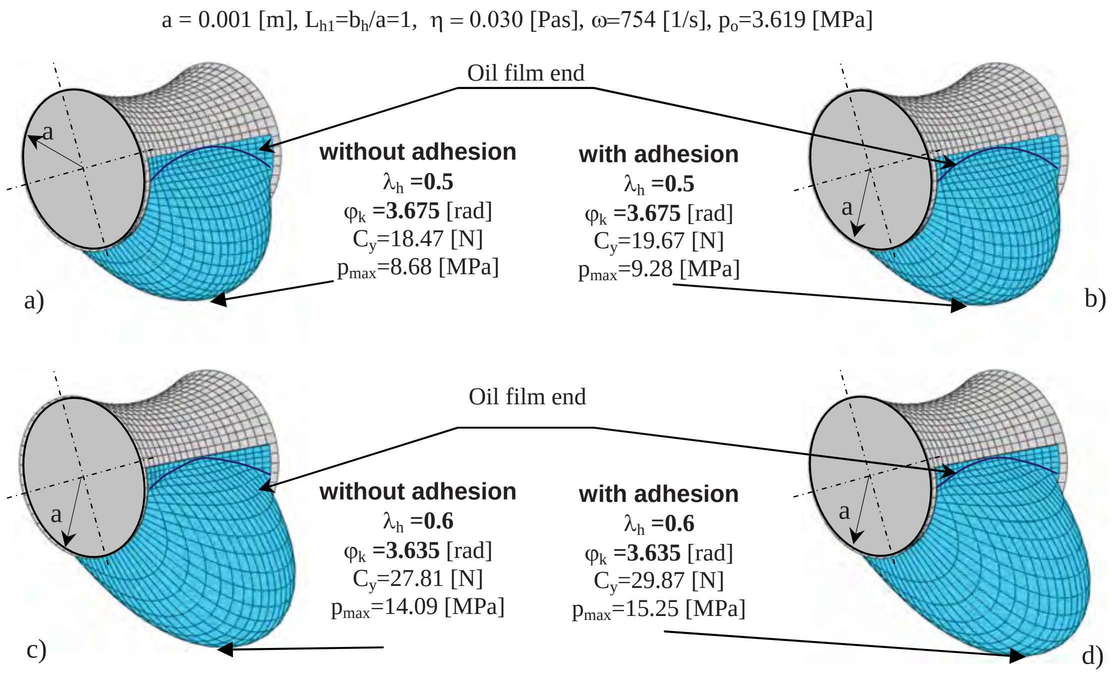


*Fig. 2. The pressure distributions in hyperbolic micro-bearings caused by the rotation in circumferential direction: a) without influences of adhesion forces on the oil viscosity, b) with viscosity changes caused by the adhesion*

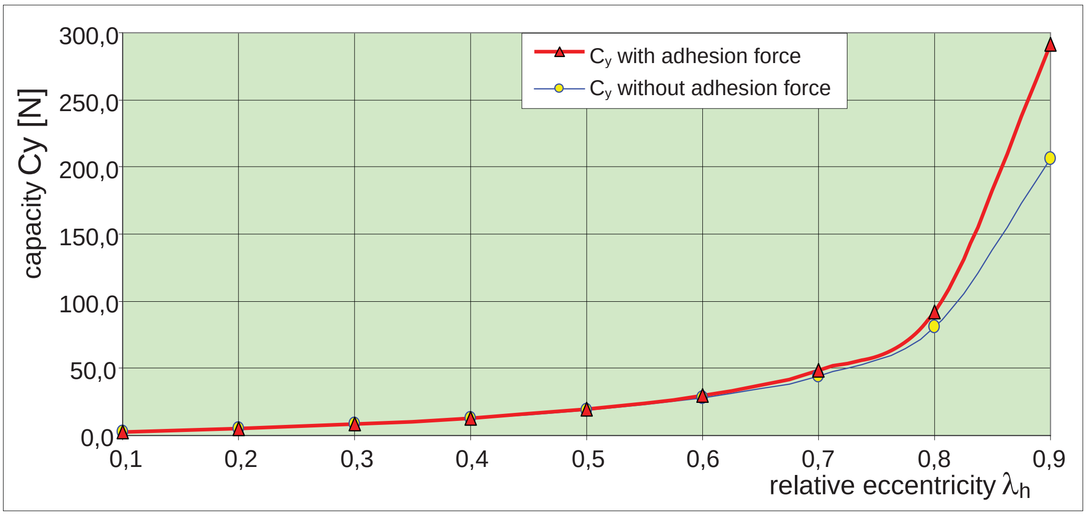


*Fig. 3. The capacity force $C_y$ with and without adhesion force influences*

If eccentricity ratio increases from $\lambda_h$=0.1 to $\lambda_h$=0.9, then the maximum value of hydrodynamic pressure increases from 0.99 MPa to 261.70 MPa and total capacity in $y_h$ direction increases from 2.45 N to 291.60 N.

In presented calculations the angular coordinate of the film end was obtained by virtue of the boundary Reynolds conditions and for eccentricities $\lambda_h$=0.1; $\lambda_h$=0.9 has the values $\varphi_k$=3.795 rad; $\varphi_k$=3.415 rad respectively.

Figures 2a and 2c show hydrodynamic pressure distributions inside hyperbolic micro-bearing gap lubricated by classical Newtonian lubricant for constant dynamic viscosity in gap height direction without changes caused by the adhesion and cohesion.

Figures 2b and 2c show hydrodynamic pressure distributions inside hyperbolic micro-bearing gap lubricated by classical Newtonian lubricant for variable dynamic in gap height direction caused by the adhesion and cohesion.

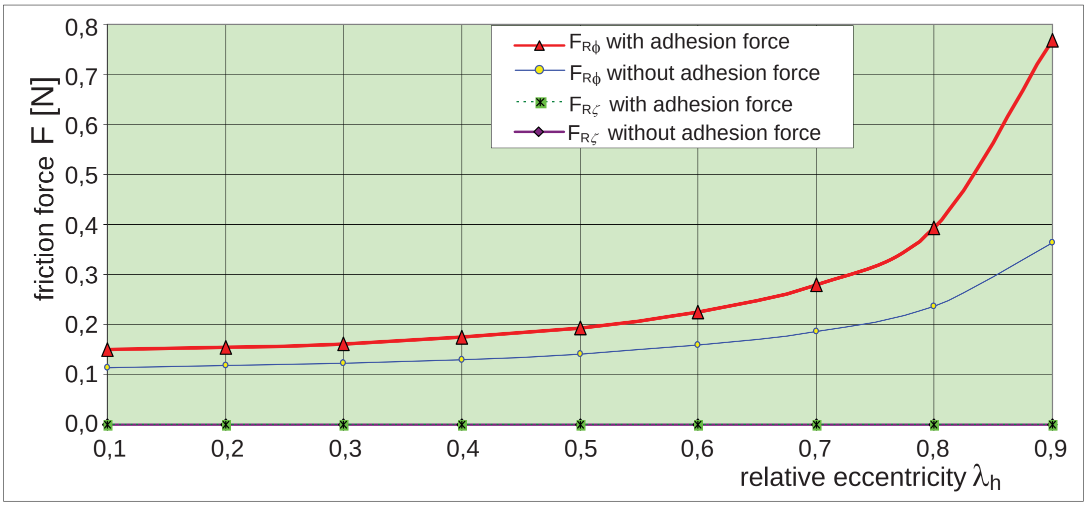


*Fig. 4. The friction force components $F_{R\phi}$, $F_{R\zeta}$ with and without adhesion force influences*

## 5. Conclusions

1) This paper presents an analytical derivation way of the oil velocity components and friction force components for hyperbolic microbearings where the oil dynamic viscosity depends on adhesion and changes in gap height direction.

2) Dynamic viscosity increases caused by the adhesion forces lead to the load carrying capacity increases for microbearing with hyperbolic journal. For eccentricities 0.1, 0.9 we obtain increases from 3 to 41 percent respectively in comparison to the load capacity values without adhesion influences.
3) Dynamic viscosity increases caused by the adhesion forces lead to the friction forces increases in circumferential direction for microbearing with hyperbolic journal. For eccentricities 0.1, 0.9 we obtain increases from 31 to 210 percent respectively in comparison to the friction values without adhesion influences.
4) Friction forces in longitudinal direction of the hyperbolic journal are negligible small.

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