Data Fusion in Measurements of Angular Position

Received 24th September 2009; accepted 3rd December 2009.

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Abstract:

The paper describes a possibility of increasing accuracy of measurements of angular position with application of data fusion. Two cases of determining the angular position are considered: accelerometer-based measurements of tilt and measurements of angular position with application of incremental rotation-to-pulse sensor (coupled with an original measuring system described in the paper). Application of the proposed data fusion ensures in both cases decrease of the uncertainty of the related measurement of ca. 40%.

Keywords: *data fusion, rotation-to-pulse angle sensor, tilt.*

1. Introduction

The paper refers to two specific cases of measuring angular position. The first is tilt sensing realized by means of an accelerometer (what recently has become very popular due to application of inexpensive MEMS acceleration sensors), while the second is an angle measurement by means of an incremental rotation-to-pulse sensor. In both cases we usually encounter a redundancy of the measurement data, which can be used for increasing accuracy of the considered measurements employing appropriate data fusion.

According to one of many definitions, data fusion is a way of integrating into one whole data generated by various sources, or merging data generated by a single source, yet related to different features of an object or a phenomenon, and separated from a signal generated by a single sensor [1]. In the considered cases, we deal with a single sensor generating two or three output signals that can be merged in various ways.

As for the dynamics of the considered measurements, accelerometers sensing tilt operate under static or quasistatic conditions, as far as the existing accelerations are concerned [2]. This is connected with low frequencies of performing such measurements, whereas in the case of incremental rotation-to-pulse sensors the conditions can be dynamic. However, because of the foreseen application, the proposed measuring system based on an incremental sensor is designed for operation at a very low rotational speed, what does not result in rigorous requirements with regard to operation speed of the electronic circuits as well as the software processing the output signals from the sensor.

2. Determining the tilt

The case of determining a tilt angle with application of accelerometers is presented in Fig. 1, where:

g_x, g_y, g_z - components of the gravitational acceleration indicated by accelerometers with sensitive axes arranged as the Cartesian axes x, y, z,

g_{xz}, g_{yz} - geometric sums of pairs of respective component accelerations,

g - gravitational acceleration,

φ - arbitrarily oriented tilt angle,

α - pitch,

β - roll.

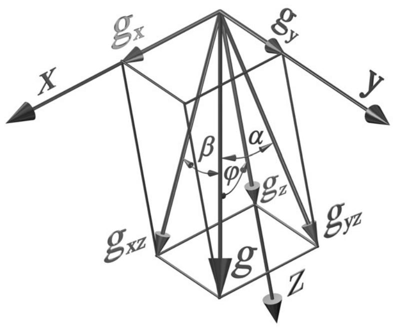

Fig. 1. Components of the gravitational acceleration against the tilt angles.

The basic dependencies used for calculating the angu-lar position represented by two component angles α and β can be found e.g. in [2]-[4]. However, application of these dependencies results in a low accuracy of deter-mining the tilt angles.

An original method that makes it possible to determine with a higher accuracy an angular position (represented by the component angles α and β), employing appropriate data fusion, has been proposed in [5]. The idea of the measurement is based on computing these angles as a weighted average (having variable weight coefficients) of three different signals generated by the applied accelerometers:

$$\alpha = w_1 \arcsin\frac{g_x}{g} + w_2 \arccos\frac{\sqrt{g_y^2 + g_z^2}}{g} \tag{1}$$

$$\beta = w_3 \arcsin\frac{g_y}{g} + w_4 \arccos\frac{\sqrt{g_x^2 + g_z^2}}{g} \tag{2}$$

where:

w_1, w_2, w_3, w_4 - weight coefficients.

In order to determine the weight coefficients, the following dependencies have been accepted that guarantee to obtain a minimal value of the combined standard uncertainty of the component tilt angles [5]:

$$w_1 \approx \cos^2\alpha \tag{3}$$
$$w_2 \approx \sin^2\alpha \tag{4}$$
$$w_3 \approx \cos^2\beta \tag{5}$$
$$w_4 \approx \sin^2\beta \tag{6}$$

The initial values of the component angles α and β that occur in the formulae (3) - (6) can be calculated in few ways: using the basic arc sine or arc cosine equations, which appear in formulae (1) - (2) (choice of appropriate equation is determined by value of angles α and β [2], [5], [6]) or using an arc tangent equation [4]. Additionally, an iterative method can be applied here, where in a successive step of computations the aforementioned weight coefficients are calculated once more, yet using values of angles α_1 and β_1 determined in the previous step according to formulae (1) - (2). In such case we employ recurrent formulae (7) - (8), and owing to that, the final values of angles α and β can be determined with a higher accuracy [6]:

$$\alpha_{k+1} \approx \cos^2\alpha_k \cdot arc\sin\frac{g_x}{g} + \sin^2\alpha_k \cdot arc\cos\frac{\sqrt{g_y^2+g_z^2}}{g} \tag{7}$$

$$\beta_{k+1} \approx \cos^2\beta_k \cdot arc\sin\frac{g_y}{g} + \sin^2\beta_k \cdot arc\cos\frac{\sqrt{g_x^2+g_z^2}}{g} \tag{8}$$

However, in the case of the last method, when formulae (3) - (6) contain values of angles α and β determined in a previous iterative step according to (1) and (2), experimental studies indicated that no significant increase of the accuracy related to determination of tilt angles has been obtained in the following iterations. Additionally, application of the recurrent formulae may considerably limit dynamics of the sensor operation.

3. Measuring the angular position with incremental sensor

Various types of incremental sensors are used for measuring all sorts of physical quantities. A principle of their operation is described in numerous works, e.g. in [7]. In order to determine an angular displacement of a graduated wheel, there are usually used two separate signals in space quadrature, generated by two detectors. The phase shift of $\pi/2$ results from the geometric configuration of the detectors and corresponds to a geometric angle of $\theta/4$ (where θ is the pitch angle between adjacent markers on the wheel). Such solution makes it possible to distinguish the sense of rotation and to double the resolution of the sensor [7] (in the case when the two analogue output signals are directly converted into logical ones), which normally equals the number of markers created on the graduated wheel. Course of one period of the signals is depicted in Fig. 2 (assuming that their offsets are of 0, and their amplitudes of 1; electric phase angle γ is expressed in degrees arc).

High accuracy systems of this type, manufactured by such companies as Renishaw or Heidenhain, feature a subdivision resolution owing to interpolation of the analogue signals generated between adjacent markers on the graduated wheel (same as in Fig. 2). For instance, a measuring system by Jenoptik Carl Zeiss with an incremental sensor IDW 2/16384 with 16,384 markers (what corresponds to a physical resolution of ca. 1.3 minute arc) that cooperates with a standard AE 101 counter unit, employs an 8-bit interpolation of the amplitude of the incremental signal, what makes it possible to obtain the accuracy and resolution of 1 or even 0.5 second arc (i.e. the measurement resolution is increased almost 160-times) [8].

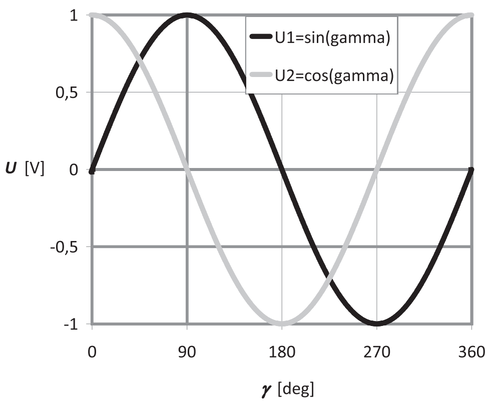

Fig. 2. The measuring signals of the incremental sensor.

While realizing the subdivision, one can alternately use the sine signal generated by the first detector within the range of the phase angle of $-\pi/4 \div \pi/4$, and then the cosine signal generated by the second detector within the range of $1/4\ \pi \div 3/4\ \pi$ (and respectively within the whole period), as it has been proposed in an analogous case of measuring tilt angles [2]. Such approach is also a kind of data fusion. Owing to this procedure, accuracy of determining the angular position within one period significantly increases, because the input signals are not used within the angular range where they feature a considerable nonlinearity, which results in a small sensitivity of the measurement [2]. The same result can be obtained while measuring a difference between instantaneous amplitude of the sine and cosine signal [7].

4. Data fusion in incremental measurements

In order to obtain a further increase of the accuracy related to measurements of angular position, it has been decided to employ the same principle as in tilt measurements, described in section 2. It consists in a simultaneous, instead of an alternate as in the previous case, processing of both signals. Just as in the case of determining tilt angles, fusion of the information contained in both output signals is realized by calculating value of the phase angle γ on the basis of a weighted average with variable weight coefficients. Analogously to formulae (1) - (2), this angle can be determined as follows:

$$\gamma = w_1\, arc\sin\frac{U_1}{U_3} + w_2\, arc\cos\frac{U_2}{U_4} \tag{9}$$

where:
w_1, w_2 - weight coefficients,
U_1 - analogue voltage signal from the first detector,
U_2 - analogue voltage signal from the second detector,
U_3 - amplitude of the signal from the first detector,
U_4 - amplitude of the signal from the second detector.

Formula (7) is true when the weight coefficients meet the following self-evident condition:

$$w_1 + w_2 = 1 \tag{10}$$

Just as before, it is crucial to find such dependencies for determining the weight coefficients, being a function of phase angle γ, that ensure obtaining a minimal value of the combined standard uncertainty of angle γ, which can be expressed as [9]:

$$u_c(\gamma) = \sqrt{\left(\frac{\partial\gamma}{\partial U_1}u(U_1)\right)^2 + \left(\frac{\partial\gamma}{\partial U_2}u(U_2)\right)^2 + \left(\frac{\partial\gamma}{\partial w_1}u(w_1)\right)^2 + \left(\frac{\partial\gamma}{\partial w_2}u(w_2)\right)^2} \tag{11}$$

where:

$u(U_1)$ - uncertainty of determining the voltage signal from the first detector,

$u(U_2)$ - uncertainty of determining the voltage signal from the second detector,

$u(w_1)$ - uncertainty of determining the first weight coefficient,

$u(w_2)$ - uncertainty of determining the second weight coefficient.

As it has been supported by an analysis of experimental results related to tilt measurements, in order to simplify formula (11) a certain assumption may be accepted while determining the respective partial derivatives of the weight coefficients. It is that the coefficients can be regarded as having a constant value, even though in fact they are functions of the measured angle. Under such assumption, combination of formula (9) with the simplified dependency (11) yields the following equation:

$$u_c(\gamma) \approx \sqrt{\left(\frac{w_1}{\sqrt{1-U_1^2}}u(U_1)\right)^2 + \left(\frac{w_2}{\sqrt{1-U_2^2}}u(U_2)\right)^2} \tag{12}$$

Still another simplification may be accepted here: uncertainties of determining the voltage signals from both detectors as well as their amplitudes are equal. It is fully justified in the cases when both detectors have an identical structure. So, let us assume:

$$u(U_1) = u(U_2) = u(U) \tag{13}$$

$$U_3 = U_4 = U_0 \tag{14}$$

While expressing the voltages from the detectors as functions of phase angle γ, formula (12) can be transformed as follows:

$$u_c(\gamma) \approx \frac{u(U)}{U_0}\sqrt{\left(\frac{w_1}{\cos\gamma}\right)^2 + \left(\frac{w_2}{\sin\gamma}\right)^2} \tag{15}$$

Taking into account formula (10), equation (15) can be rearranged as:

$$u_c(\gamma) \approx \frac{u(U)}{U_0}\sqrt{\frac{w_1^2 - 2w_1\cos^2\gamma + \cos^2\gamma}{\sin^2\gamma\cos^2\gamma}} \tag{16}$$

Function (16) reaches its minimum, when its derivative with respect to w_1 equals zero, i.e.:

$$u_c'(\gamma) \approx \frac{u(U)}{2U_0\sin\gamma\cos\gamma} \cdot \frac{2w_1 - 2\cos^2\gamma}{\sqrt{w_1^2 - 2w_1\cos^2\gamma + \cos^2\gamma}} = 0 \tag{17}$$

what boils down to the following equation:

$$2w_1 - 2\cos^2\gamma = 0 \tag{18}$$

Formula (18) yields dependencies identical compare to their counterparts (3) - (6) related to measurements of tilt angles:

$$w_1 \approx \cos^2\gamma \tag{19}$$
$$w_2 \approx \sin^2\gamma \tag{20}$$

Courses of weight coefficients w_1 (black curve) and w_2 (gray curve) are presented in Fig. 3. The graphs indicate that for three characteristic values of phase angle γ: 0, 1/4π and 1/2π, values of the coefficients will be respectively: (1, 0); (0.5, 0.5); (0, 1).

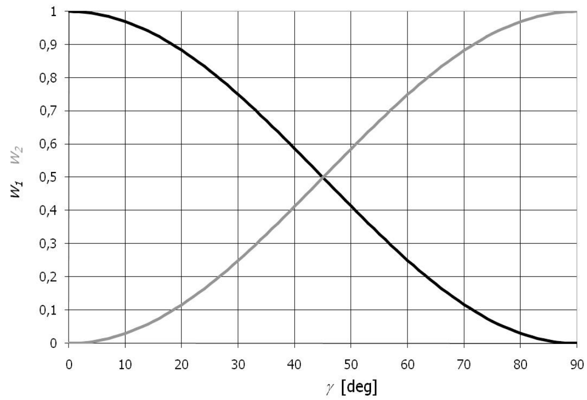

Fig. 3. Graphs of formulae (19) and (20).

When the values of coefficients w_1 and w_2 are already known, we obtain a formula for determining the phase angle γ:

$$\gamma \approx \cos^2\gamma \cdot arc\sin\frac{U_1}{U_0} + \sin^2\gamma \cdot arc\cos\frac{U_2}{U_0} \tag{21}$$

Value of angle γ determined in a previous step is to be calculated analogously as in the case of tilt angles (see section 2). Neglecting insignificant uncertainties of determining the weight coefficients, we can state that uncertainty of determining the phase angle γ is constant and equals:

$$u_c(\gamma) \approx \frac{u(U)}{U_0} \tag{22}$$

5. Measuring System with Incremental Sensor

As it has been mentioned above, the proposed idea of data fusion has been already applied in the case of tilt measurements realized by means of systems based on MEMS accelerometers [6]. It is also planned to implement the data fusion in the case of using the aforementioned incremental sensor IDW 2/16384 manufactured by Zeiss.

Due to numerous shortcomings, the standard counter AE101 has been replaced with a custom built software-controlled electronic system. Interpolation of the sine and cosine signal with subdivision accuracy is realized digitally with application of a commercial data acquisition card equipped with 16-bit A/D converters manufactured by Advantech [10] (cards with 12-bit A/D converters can be applied here as well). Theoretically, that would make it possible to increase accuracy of measuring the angular position few thousand times. It is obviously impossible because of various sources of errors that occur here. The most significant are the following:

- noises of the output signals,
- inaccuracies of the A/D converters of the data acquisition card,
- inaccuracies of the markers on the graduated wheel,
- error of the phase shift between the detectors,
- variability of the phase shift during measurements,
- different amplitudes of the output signals generated by the detectors,
- variability of amplitudes of the output signals during measurements,
- incorrect values of the offsets of the output signals due to alignment errors,
- variability of the offsets of the output signals during measurements.

As can be easily noted, it is a very significant matter to take into account the offsets, as well as amplitude magnitudes of the sine and cosine signal generated by the detectors in the incremental sensor (these values must be regarded while using formula (21)). Each sensor features an individual value of these parameters (in the case of the original counters AE 101, each of them was tuned to a single incremental sensor [8]). Therefore, it is necessary to determine the offsets and amplitude magnitudes during initialisation of the system or beforehand, for a specific sensor. Still another option, accepted in the considered measuring system, is measuring these parameters in real time and introducing possible corrections systematically. Such solution ensures obtaining the highest accuracy of the system, as it compensates for most of the errors listed above.

Schematic of the described measuring system is presented in Fig. 4. As mentioned above, it features a subdivision accuracy due to a software interpolation of the measuring signals.

A shortcoming of such system architecture is a significantly limited dynamics of its operation. In the case of measurements performed at high rotational speeds of the graduated wheel another kind of system should be built. It should be an autonomous microprocessor system processing the output signals generated by the rotation-to-pulse sensor in a real time, and transferring the computed results to a computer memory only from time to time in an off-line mode.

It is also possible to eliminate the electronic interface while applying a data acquisition module featuring a high gain for the analogue inputs (such modules are designed for measurements of temperature by means of thermocouples). However, such solution significantly limits the range of dynamic operation of the system.

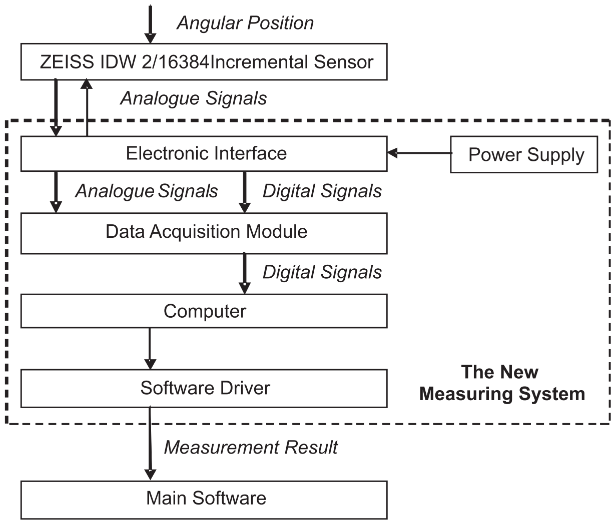

Fig. 4. Schematic of the incremental measuring system.

6. Experimental Results

As far as application of the considered data fusion is concerned, the experimental studies have been carried out so far only on a tilt sensor employing MEMS accelerometers. The following graph illustrates the obtained increase of the measurement accuracy.

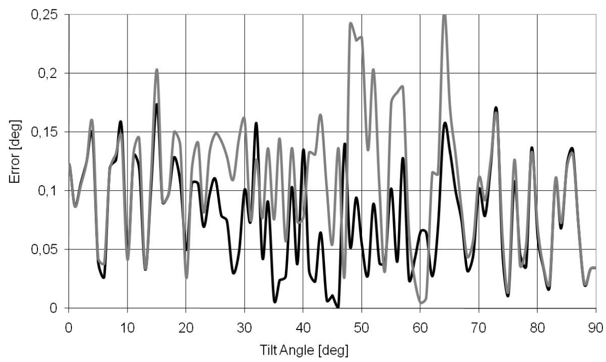

Fig. 5. Decrease of error values owing to data fusion.

The indication error expressed over axis y has been defined as an absolute value of the difference between the value of the tilt angle applied by means of the used test station and the value determined with respect to the average of respective indications of the tested sensor. The black curve is related to application of the weighted average, whereas the gray curve to application of the sine and cosine signal alternatively (as described in section 3). Increase of the measurement accuracy due to application of data fusion can be clearly observed.

7. Summary

The author demonstrated in the paper an analogy between the case of measuring component tilt angles (pitch and roll) by means of accelerometers, and quite a different case of measuring angular position by means of incremental sensors with subdivision accuracy. Owing to application of the proposed data fusion, in both cases a significant decrease of the uncertainties of determining the angular values can be obtained (at least of 40 percent with respect to the basic measuring systems where the output signals are processed individually, using arc sine or arc cosine type of formulas).

The same result can be obtained (yet in an easier way) in the case of determining tilt angles with application of an arc tangent function. A respective algorithm has been presented in [11]. The same applies to incremental sensors with subdivision accuracy, where a similar dependency can be used for calculating the phase angle realizing the subdivision [12]:

$$\gamma = \arctan \frac{U_1}{U_2} \tag{23}$$

Experimental works carried out at the Faculty of Mechatronics, Warsaw University of Technology, proved that interpolation makes it practically possible to increase the sensor accuracy even 300 times (with respect to its physical resolution resulting from the number of the markers). However, it should be noted that some manufactures realize in their incremental measuring systems a 1024-fold division [7], or even a 4096-fold one [13].

Nevertheless, it should be emphasized that application of data fusion based on a weighted average, even though more complicated than application of the arc tangent function, has this advantage over the later that it is possible to regard different uncertainties related to both output signals. Then, formulae (7) - (8) as well as (21) change, and thus relation between the phase angle and the weight coefficients takes on a form other than (3) - (6) and (19) - (20).

A case when values of uncertainties related to the output signals are different, concerns first of all triaxial MEMS accelerometers (often used in tilt measurements), since the signal related to their vertical sensitive axis is usually few times less accurate compare to the signals related to their horizontal sensitive axes. The author plans in the nearest future to carry out experimental works that are to determine how the accuracy of measuring tilt angle increases in the case of regarding the aforementioned difference and compensating for it.

Application of the measurement principle based on the weighted average has yet another advantage over the arc tangent function. It yields a unique result in the case when the measured angle is of $\pm\pi/2$, whereas while using the arc tangent function a respective angle value must be determined according to an arc cosine function, i.e. yet another data fusion must be applied.

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