New Approach to the Accuracy Description of Unbalanced Bridge Circuits with the Example of Pt Sensor Resistance Bridges

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Abstract:

After short introduction transfer coefficients of the unloaded four arms bridge of arbitrary variable arm resistances, supplied by current or voltage source, are given in Table 1. Their error propagation formulas are find and two rationalized forms of accuracy measures, i.e. related to the initial bridge sensitivities and of double component form as sum of zero error and increment error of the bridge transfer coefficients are introduced. Both forms of transfer coefficient measures of commonly used bridge - of similar initial arm resistances in balance and different variants of their jointed increments, are given in Table 3. As the example limited errors of some resistance bridges with platinum Pt100 industrial sensors of class A and B are calculated Table 4 and analyzed. Presented approach is discussed and found as the universal solution for all bridges and also for any other circuits used for parametric sensors.

Keywords: *resistance bridge, sensor, measures of accuracy, error, uncertainty of measurements.*

1. Introduction

This paper is based on earlier author proposals given in papers [1], [6] - [8]. As it has been pointed there, the generalized accuracy description of the 4R bridge of arbitrary variable arm resistances was not existing in the literature. Some considerations of the bridge accuracy with sensors of very small increments only have been found in [9], [10]. Generalized description is urgently needed mainly for:

- initial conditioning circuits of analogue signals from broadly variable immittance sensor sets,
- identification of the changes of several internal parameters of the equivalent circuit of the object working as twoport X, when it is measured from its terminals for testing, monitoring and diagnostic purposes.

Near the bridge balance state, application of relative errors or uncertainties is useless, as they are rising to $\pm\infty$. In [1], [6] this obstacle was bypassed by relating the absolute value of any bridge accuracy measure to the initial sensitivity of the current to voltage or voltage-to-voltage bridge transfer function. These sensitivities are valuable reference parameters, as they do not change within the range of the bridge imbalance. In paper [7] the new double component approach to describing the bridge accuracy is developed. It has the form of sum of initial stage and of bridge imbalance accuracy measures. Such double component method of describing accuracy is commonly used for the broad range instruments, e.g. digital voltmeters. Relation of each component to accuracy measures of all bridge arm resistances have been developed. As the example formulas of accuracy measures of two bridges used for industrial Pt sensors will be presented and their limited errors calculated.

2. Basic formulas of bridge transfer functions

Four resistance (4R) bridge circuit with terminals ABCD working as passive X type twoport of variable internal resistances is shown in Fig 1.

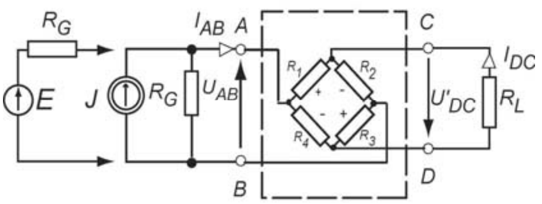


Fig. 1. Four arms bridge as the unloaded twoport of type X with the voltage or current supply source branch.

In measurements the ideal supply of the bridge by current $I_{AB} \to J = \text{const.}$, if $R_G \to \infty$ or by voltage $U_{AB} = E = \text{const.}$; when $R_G = 0$; is commonly used. Also the output is unloaded, i.e.: $R_L \to \infty$, $U'_{DC} \to U^{\infty}_{DC}$. For single variable measurements it is enough to know changes of one bridge terminal parameter and the output circuit voltage U_{DC} is mostly used. With notations of Fig. 1 formulas (1), (2) of U^{∞}_{DC} are given in Table 1 [2],

Table 1. Open circuit bridge voltage and its transfer functions.

a) current supply	b) voltage supply
$U'_{DC} \to U^{\infty}_{DC} = I_{AB}\, r_{21}$ (1)	$U'_{DC} \to U^{\infty}_{DC} = U_{AB}\, k_{21}$ (2)
$\boxed{r_{21} \equiv \frac{U^{\infty}_{DC}}{I_{AB}} = \frac{R_1R_3 - R_2R_4}{\sum R_i} \equiv t_0\, f(\boldsymbol{\varepsilon}_i)}$ (3)	$\boxed{k_{21} \equiv \frac{U^{\infty}_{DC}}{U_{AB}} = \frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_3+R_4)} \equiv k_0\, f_E(\boldsymbol{\varepsilon}_i)}$ (4)
where: $t_0 \equiv \frac{mnR_{10}}{(1+m)(1+n)}$; $f(\boldsymbol{\varepsilon}_i) = \frac{\Delta L(\boldsymbol{\varepsilon}_i)}{1+\varepsilon_{\Sigma R}}$; $\boldsymbol{\varepsilon}_i = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4]^T$; $\Delta L(\boldsymbol{\varepsilon}_i) = \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 + \varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4$; $\varepsilon_{\Sigma R} = \frac{\varepsilon_1 + m\varepsilon_2 + n(\varepsilon_4 + m\varepsilon_3)}{(1+m)(1+n)}$	$k_0 \equiv \frac{m}{(1+m)^2}$, n is arbitrary; $f_E(\boldsymbol{\varepsilon}_i) \equiv \frac{\Delta L(\boldsymbol{\varepsilon}_i)}{(1+\varepsilon_{12})(1+\varepsilon_{34})}$; $\varepsilon_{12} \equiv \frac{\varepsilon_1 + m\varepsilon_2}{1+m}$; $\varepsilon_{43} \equiv \frac{\varepsilon_4 + m\varepsilon_3}{1+m}$

where:

I_{AB}, U_{AB} - current or voltage on bridge supply terminals $A\,B$,

$R_i \equiv R_{i0} + \Delta R_i \equiv R_{i0}\,(1+\varepsilon_i)$ - arm resistance of initial value R_{i0} and absolute ΔR_i and relative ε_i increments,

r_{21}, k_{21} - transfer functions of the bridge of open-circuited output, i.e.: current to voltage (transfer resistance) and ratio of two voltages.

$$t_0 = \frac{R_{10}R_{30}}{\sum R_{i0}},\ k_0 = \frac{R_{10}R_{30}}{(R_{10}+R_{20})(R_{30}+R_{40})}$$

- initial bridge open circuit sensitivities of r_{21} and of k_{21},

$\sum R_i \equiv \sum_{i=1}^{4} R_i = (1+\varepsilon_{\Sigma R}) \sum R_{i0}$ - sum of bridge arm resistances; $\varepsilon_{\Sigma R}(\varepsilon_i)$, $\sum R_{i0}$ - its increment and initial value,

$f(\varepsilon_i)$, $f_E(\varepsilon_i)$ - normalized bridge imbalance function of r_{21} and of k_{21},

$\Delta L(\varepsilon_i)$, $\varepsilon_{\Sigma R}(\varepsilon_i)$ - increments of the function $f(\varepsilon_i)$ numerator and of the denominator.

Output voltage U_{DC} may change its sign for some set of arm resistances. If transfer function $r_{21} = 0$ or $k_{21} = 0$, the bridge is in balance and in (3) and (4) its conditions of both supply cases are the same: $R_1R_3 = R_2R_4$.

The balance of the bridge can occurs for many different combinations of R_i, but the basic balance state is defined for all $\varepsilon_i = 0$, i.e. when:

$$R_{10}R_{30} = R_{20}R_{40} \tag{5}$$

Formulas of the bridge terminal parameters are simplified by referencing all resistances to their initial values in the balance, i.e. $R_i = R_{i0}(1+\varepsilon_i)$ and referencing initial resistances R_{i0} to one of the first arm, i.e.: $R_{20} \equiv mR_{10}$, $R_{40} \equiv nR_{10}$ and from (3) $R_{30} = mnR_{10}$. Bridge transfer functions can also be normalized, as is shown in Table 1.

3. Accuracy description of broadly variable resistances

The accuracy of measurements depends in complicated way on structure of the instrumentation circuit, values and accuracy of its elements and on various environmental influences of natural conditions and of the neighboring equipment. Two type of problems have been met in practice:

- description of circuits and measurement equipment by instantaneous and limited values of systematic and random errors, absolute or related ones, as well by statistical measures of that errors,
- estimation of the measurement result uncertainty, mainly by methods recommended by guide GUM.

Measures of accuracy (errors, uncertainty) of the single value of circuit parameter are expressed by numbers, of variable parameter - by functions of its values. In both cases they depend on equivalent scheme of the circuit, on environmental and parameters of instrumentation used or have to be use in the experiment.

The measures of broadly variable resistance R_i, e.g. of the stress or of the temperature sensor, could be expressed by two components: for its initial value and for its increment as it is shown by formulas of Table 2.

Instantaneous absolute error Δ_i and its relative value δ_{Ri} referenced to R_i are given by formulas (6) and (7), **relative limited error** $|\delta_{Ri}|$ of the poorest case of values and signs of $|\delta_{i0}|$ and $|\Delta\varepsilon_i|$ or $|\delta\varepsilon_i|$ - by (8), and **standard statistical measure** $\bar{\delta}_{Ri}$ for random errors or uncertainties - by (9).

Table 2. Two-component formulas of the sensor resistance accuracy measures.

Sensor resistance R_i	$R_i = R_{i0}(1+\varepsilon_i)$ if $\varepsilon_i > -1$
Absolute error $\Delta_i \equiv R_i - R_{i\,\mathrm{nominal}}$	$\Delta_i = \Delta_{i0}(1+\varepsilon_i) + R_{i0}\Delta_{\varepsilon i}$ (6)
Relative error $\delta_{Ri} \equiv \frac{\Delta_i}{R_i}$	$\delta_{Ri} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1+\varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}$ (7) where: $\delta_{i0} = \frac{\Delta_{i0}}{R_{i0}}$, $\delta_{\varepsilon i} = \frac{\Delta_{\varepsilon i}}{\varepsilon_i}$ - relative errors of: initial value and of resistance increment
Relative limited error $\lvert\delta_{Ri}\rvert \equiv \frac{\lvert\Delta_i\rvert}{R_i}$	$\lvert\delta_{Ri}\rvert = \lvert\delta_{i0}\rvert + \frac{\lvert\Delta_{\varepsilon i}\rvert}{1+\varepsilon_i} = \lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert$ (8)
Statistical measure standard deviation of δ_{Ri} (for random error or uncertainty) $\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i}$	$\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i} = \sqrt{\bar{\delta}_{i0}^2 + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^2 + 2k_i \frac{1}{1+\varepsilon_i}\bar{\delta}_{i0}\bar{\Delta}_{\varepsilon i}}$ (9) where: $\bar{\delta}_{i0}$, $\bar{\delta}_{\varepsilon i}$ – standard measures of initial value R_{i0} and of relative increment ε of resistance R_i, $k_i \subset (-1 \ldots 0 \ldots +1)$ – correlation coefficient
Particular cases of correlation: full: $k_i = \pm 1$	$\bar{\delta}_{Ri} = \left\lvert \bar{\delta}_{i0} \pm \left(\frac{1}{1+\varepsilon_i}\right)\bar{\Delta}_{\varepsilon i} \right\rvert$ (9a)
Particular cases of correlation: no $k_i = 0$	$\bar{\delta}_{Ri} = \sqrt{\bar{\delta}_{i0}^2 + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^2}$ (9b)

If errors of increment and of initial value of resistance are statistically independent then correlation coefficient $k_i = 0$, but if they are strictly related each to the other then $k_i = \pm 1$. Exact k_i value can only be find experimentally. From (8) follows that borders of the worse cases $\pm|\delta_{Ri}|$ of possible values of δ_{Ri} nonlinearly dependent on ε_i even if $|\delta_{i0}|$ and $|\Delta\varepsilon_i|$ or $|\delta\varepsilon_i|$ are constant [1], [6] - [8].

Distribution of the initial values and relative increments ε_i of the sensors' set resistances depends on their data obtained in the production process. Its actual values also depend on influences of the environmental conditions.

4. Description of the accuracy of bridge transfer functions

Instantaneous values of measurement errors of bridge transfer functions r_{21} and k_{21} result from the total differential of analytical equations (3) and (4) from Table 1.

After ordering all components of δ_{Ri} **absolute error of transfer function** r_{21} is:

$$\Delta_{r21} = R_1\frac{R_3 - r_{21}}{\Sigma R_i}\delta_{R1} - R_2\frac{R_4 + r_{21}}{\Sigma R_i}\delta_{R2} + R_1\frac{R_1 - r_{21}}{\Sigma R_i}\delta_{R3} - R_4\frac{R_2 + r_{21}}{\Sigma R_i}\delta_{R4} = \sum_{i=1}^{4} w_{Ri}\,\delta_{Ri} \tag{10}$$

where: $w_{Ri} \equiv R_i\dfrac{(-1)^{i+1}R_j - r_{21}}{\sum R_i}$ - weight coefficients of error δ_{Ri} components

- subscript $i = 1,2,3,4$ when $j = 3,4,1,2$;
- multiplier $(-1)^{i+1} = +1$ if i is 1, 3 or -1 if i is 2, 4.

If resistances are expressed as $R_i = R_{i0}(1+\varepsilon_i)$, $R_j = R_{j0}(1+\varepsilon_j)$ formula (10) is

$$\Delta_{r21} = \frac{t_0}{1+\varepsilon_{\Sigma R}}\sum_{i,j}\left[(-1)^{i+1}(1+\varepsilon_j) - \frac{r_{12}}{R_{j0}}\right](1+\varepsilon_i)\delta_{Ri} \tag{10a}$$

Absolute error of transfer function k_{21} has other forms, i.e.:

$$\Delta_{k21} = \frac{R_1 R_2}{(R_1+R_2)^2}(\delta_{R1} - \delta_{R2}) + \frac{R_3 R_4}{(R_3+R_4)^2}(\delta_{R3} - \delta_{R4}) \tag{11}$$

or

$$\Delta_{k21} = k_0\left[\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{(1+\varepsilon_{12})^2}(\delta_{R1} - \delta_{R2}) + \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{(1+\varepsilon_{34})^2}(\delta_{R3} - \delta_{R4})\right] \tag{11a}$$

From (10) and (11) one could see that if errors δ_{Ri} of the neighbouring bridge arms, e.g.: 1, 2 or 1, 4 have the same sign they partly compensate each other.

If errors δ_{Ri} of resistances R_i are expressed, as in (7), by their initial errors δ_{i0} and incremental errors $\delta_{i\varepsilon}$, then

$$\Delta_{r21} = \sum w_{Ri}\left(\delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right) \tag{12}$$

$$\text{where: } w_{Ri} = \frac{t_0}{1+\varepsilon_{\Sigma R}}[(-1)^{i+1}(1+\varepsilon_j) - \frac{R_{i0}}{R_{j0}}\Delta L(\boldsymbol{\varepsilon}_\mathbf{i})](1+\varepsilon_i) \tag{12a}$$

If arm resistance R_i is constant, $\varepsilon_i = 0$, $\delta_{Ri} = \delta_{i0}$, but weight coefficient w_{Ri} of its component in Δ_{r21} still depends on other arm increments $\boldsymbol{\varepsilon}_\mathbf{j}$. In initial balance state, i.e. when all arm increments $\boldsymbol{\varepsilon}_\mathbf{i} = 0$, the nominal transfer function $r_{21}(0) \equiv r_{210} = 0$, but real resistances R_i have some initial errors δ_{i0} and usually $\Delta_{r210} \neq 0$, $\Delta_{k210} \neq 0$.

$$\Delta_{r210} = t_0\delta_{210} \tag{13a}$$

$$\Delta_{k210} = k_0\delta_{210} \tag{13b}$$

where: $\delta_{210} = \delta_{10} - \delta_{20} + \delta_{30} - \delta_{40}$

Relative errors are preferable in measurement practice, but it is not possible to use them for transfer functions near the bridge balance as the ratio of absolute error $\Delta_{r21} \rightarrow \Delta_{r210} \neq 0$ and the nominal $r_{21} \rightarrow r_{210} = 0$ (or for the voltage supplied bridge of Δ_{k21} and $k_{21} \rightarrow k_{210} = 0$) is rising to $\pm\infty$. Then other possibilities should be applied. There are two possible ways to describe accuracy of the bridge transfer function r_{21} (or k_{21}) in the form of one or of two related components:

- absolute error of the bridge transfer function may be referenced to initial sensitivity factor t_0 of r_{21} (or to k_0 of k_{21}) or to the range of transfer function $r_{21\max} - r_{21\min}$ (or $k_{21\max} - k_{21\min}$);

- initial error Δ_{r210} have to be subtracted from Δ_{r21} and then accuracy could be described by two separate terms: for zero and for transfer function increment, as it is common for digital instrumentation.

In the **first type method** it is preferable if error Δ_{r21} is referenced to the initial sensitivity factor t_0 as constant for each bridge, then to full range of r_{21} as it could be change. Such relative error δ_{r21} could be presented as sum:

$$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \delta_{210} + \delta_{r21\varepsilon}(\varepsilon_i) \qquad (14)$$

where: $\delta_{210} = \delta_{10} - \delta_{210} + \delta_{30} - \delta_{40}$ - initial (or zero) relative error of $r_{21}=0$;
$\delta_{r21\varepsilon}(\varepsilon_i)$ relative error of normalized imbalance function $f(\varepsilon_i)$ when $r_{21}\neq 0$, also referenced to t_0.

Error δ_{210} is similar for any mode of the supply source equivalent circuit of the bridge as twoport.

Zero of the bridge may be corrected on different ways: by adjustment of the bridge resistances, by the opposite voltage on output or by the digital correction of converted output signal. In such cases from (14) it is

$$\delta_{r21\varepsilon} = \frac{1}{t_0}\sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i+1}\right]\delta_{i0} + \frac{1}{t_0}\sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \qquad (15)$$

From (15) follows that related to t_0 error $\delta_{r21\varepsilon}$ of r_{21} increment depends not only on increment errors $\delta_{\varepsilon i}$ of resistances R_i but also on their initial errors $\delta_{i0} \neq 0$ even when initial error of the whole bridge $\delta_{210}=0$, because after (12a) weight coefficients of δ_{i0} in (15) depends on ε_i. The component of particular error δ_{i0} despairs only when $\delta_{i0}=0$. Functions of $\Delta_{\varepsilon i}$ or $\delta_{\varepsilon i}$ may be approximated for some ε_i intervals by constant values.

In the **second type method** absolute error of transfer function r_{21} (12) after subtracting its initial value is

$$\Delta_{r21} - \Delta_{r210} = \sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i-1}\right]\delta_{i0} + \sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \qquad (16)$$

And after referenced it to r_{21}, and substitution w_{Ri} from (12a)

$$\delta_{r21r} \equiv \frac{\Delta_{r21} - \Delta_{r210}}{r_{21}} = \frac{\delta_{r21} - \delta_{210}}{f(\varepsilon_i)} = \sum_{i=1}^{4} w'_{r\,i0}\,\delta_{i0} + \sum_{i=1}^{4} w'_{r\,\varepsilon i}\,\delta_{\varepsilon i} \qquad (17)$$

where: $$w'_{r\,i0} = (-1)^{i-1}\frac{\varepsilon_i + \varepsilon_j + \varepsilon_i\varepsilon_j}{\Delta L(\boldsymbol{\varepsilon}_i)} - t_0\frac{1+\varepsilon_i}{R_{j0}};$$

$$w'_{r\,\varepsilon i} = \left[\frac{(-1)^{i-1}(1+\varepsilon_j)}{\Delta L(\boldsymbol{\varepsilon}_i)} - \frac{t_0}{R_{j0}}\right]\varepsilon_i \qquad (17a,b)$$

Weight coefficients (17a,b) are finite for any value of r_{21} including $r_{21}=0$ because if all $\varepsilon_i \to 0$ also $\Delta L \to 0$.

Error δ_{r21r} is equivalent to error $\delta_{\varepsilon i}$ of the resistance R_i increment ε_i in formula (7).

From (13a) and (17) is:
for current to voltage transfer function
$$\Delta_{r21} = t_0\,\delta_{210} + r_{21}\,\delta_{r21r} \qquad (18a)$$
and similarly for voltage transfer function k_{21}
$$\Delta_{k21} = k_0\,\delta_{210} + k_{21}\,\delta_{k21k} \qquad (18b)$$

where: $t_0\,\delta_{210} = \Delta_{r210}$, $k_0\,\delta_{210} = \Delta_{k210}$ - absolute errors of initial value r_{21} or k_{21} e.g. $r_{210}=0$ or $k_{210}=0$,
δ_{r21r}, δ_{k21k} - related errors of increments r_{21} - r_{210} or k_{21} - k_{210} from the initial stage.

Two component accuracy equation of k_{21} transfer function was funded by the same way as for r_{21}.

Actual values of instantaneous errors of r_{21} or k_{21} could be calculated only if signs and values of errors of all resistances are known. In reality it happens very rare. More frequently are used their limited systematic errors (of the worst case) and statistical standard deviation measures. Formulas of these accuracy measures of r_{21} or k_{21} could be obtained after transformation of error formulas (10) - (18a,b). All these accuracy measures are possible to find in one component or two component forms. One-component formulas for arbitrary and main particular cases of 4R bridge are given in Tables in [1], [6], [7].

The two-component method of the bridge transfer function r_{21} accuracy representation, separately for its initial value (e.g. equal to zero) and for increment is similar like unified one used for digital instruments and of the broad range sensor transmitters. It is especially valuable if zero of the measurement track is set handily or automatically. Absolute measures could be transformed also by the sensor set linear or nonlinear function to the units of any particular measurand, e.g. in the case of platinum sensors - to °C [6], [8].

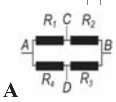
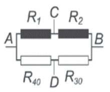
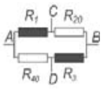
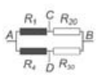
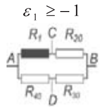
5. Description of accuracy measures of particular 4R bridges mostly used for sensors

In the measurement practice the mostly used for sensors are four-arm bridges of resistances equal in the balance state. Formulas of accuracy measures for transfer functions r_{21} and k_{21} of these particular resistance bridges are much simpler then general ones [6]. They are presented in Table 3 with assumption that all correlation coefficients $k_{ij}=0$. Formulas of k_{21} and its errors given are mainly for comparison as current supply is preferable one for resistance sensors. The transfer function r_{21} or k_{21} of the bridge including differential sensors of four or two increments $\pm\,\varepsilon$ and transfer function r_{21} of two equal ε in opposite arms may be linear but their measures depend on ε by different way each other. From Table 3 it is possible to compare the accuracy measure formulas of the current or voltage supplied 4R bridges of few element and single element sensors. For example formulas of accuracy measures of similarly variable two opposite arm resistances are simpler then for the single variable arm. Form of the limited error $|\delta_{r21r}|$ is similar for four linear A-D bridges and $|\delta_{k21k}|$ only for two of them: A and B.

6. Tolerances of industrial Pt sensors

The good example of the broadly variable resistance sensors is platinum sensors Pt 100 of A and B classes commonly used in industrial temperature measurements. Tolerated differences from their nominal characteristic are given in standard EN 60751÷A2 1997. They are expressed in °C or as permissible resistance values in [ohm] - see $|\Delta|$ of classes A and B in Fig. 2. Characteristic of class A sensors is determined up to 650°C and for less accurate class

Table 3. Accuracy measures of the most common open-circuit 4R bridges of similar all arm initial resistances $4R_{10}$.

No	Bridge $4R_{10}$ parameters	Errors of R_i	Related accuracy measures of bridge transfer functions r_{21} k_{21}: a) related to initial sensitivities t_0 or k_0	b of increments $r_{21}-r_{210}$ and $k_{21}-k_{210}$
1	**Arbitrary** $\varepsilon_1, \ldots \varepsilon_4$ $R_i=R_{10}(1+\varepsilon_i)$ $\boxed{r_{21}=\frac{R_{10}}{4}\cdot\frac{\Delta L(\varepsilon_i)}{1+\frac14\Sigma\varepsilon_i}}$	arbitrary	$\delta_{r21}\equiv\frac{\Delta_{r21}}{t_0}=\sum_{i=1}^4 w'_{Ri}\delta_{Ri}=\sum_{i=1}^4(1+\varepsilon_i)\left[\frac{(-1)^{i-1}(1+\varepsilon_j)}{1+\frac14\Sigma\varepsilon_i}-\frac{\Delta L(\varepsilon_i)}{4(1+\frac14\Sigma\varepsilon_i)^2}\right]\delta_{Ri}$ where: $t_0=\frac{R_{10}}{4}$, $w'_{Ri}=\frac{1}{t_0}w_{Ri}$ if $i=1, 2, 3, 4$ then $j=3, 4, 1, 2$	$\delta_{r21r}\equiv\frac{\Delta_{r21}-\Delta_{r210}}{r_{21}}=\frac{1+\frac14\Sigma\varepsilon_i}{\Delta L(\varepsilon_i)}\sum_{i=1}^4\left[\left(w'_{Ri}-(-1)^{i-1}\right)\delta_{i0}+w'_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right]$ $\boxed{\delta_{r21}\equiv\delta_{210}+f(\varepsilon_i)\delta_{r21r}}$, $\boxed{\delta_{k21}\equiv\delta_{210}+f_E(\varepsilon_i)\delta_{k21k}}$
* 2	$k_{21}=\frac14\cdot\frac{\Delta L(\varepsilon_i)}{\left(1+\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left(1+\frac{\varepsilon_3+\varepsilon_4}{2}\right)}$	* measures of k_{12} - on grey	$\delta_{k21}=\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{(1+\frac12\varepsilon_1+\frac12\varepsilon_2)^2}(\delta_{R1}-\delta_{R2})+\frac{(1+\varepsilon_3)(1+\varepsilon_4)}{(1+\frac12\varepsilon_3+\frac12\varepsilon_4)^2}(\delta_{R3}-\delta_{R4})=\sum_{i=1}^4 w'_{ki}\delta_{Ri}$	$\delta_{k21k}=\frac{1}{f_E(\varepsilon_i)}\sum_{i=1}^4\left[\left(w'_{ki}-(-1)^{i-1}\right)\delta_{i0}+w'_{ki}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}\right]$
3	**Jointed** $R_1,\ldots R_4$ $\varepsilon_i=\pm\varepsilon$ $\lvert\varepsilon\rvert\le 1$	arbitrary	$\delta_{r21}=(1+\varepsilon)(\delta_{10}+\delta_{30})-(1-\varepsilon)(\delta_{20}+\delta_{40})+\varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$	$\delta_{r21r}=\frac14(\delta_{10}+\delta_{30}+\delta_{20}+\delta_{40}+\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$
4			$\lvert\delta_{r21}\rvert=(1+\varepsilon)(\lvert\delta_{10}\rvert+\lvert\delta_{30}\rvert)+(1-\varepsilon)(\lvert\delta_{20}\rvert+\lvert\delta_{40}\rvert)+\lvert\varepsilon\rvert\sum_1^4\lvert\delta_{\varepsilon i}\rvert$	$\lvert\delta_{r21r}\rvert\equiv\frac{\lvert\Delta_{r21}-\Delta_{r210}\rvert}{\lvert r_{21}\rvert}=\frac14\left(\lvert\delta_{210}\rvert+\sum_1^4\lvert\delta_{\varepsilon i}\rvert\right)$
5	A	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon i}\rvert=\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert=4(\lvert\delta_0\rvert+\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert)$	$\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$
6	$R_1=R_3=R_{10}(1+\varepsilon)$ $R_2=R_4=R_{10}(1-\varepsilon)$	$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=2\sqrt{(1+\varepsilon^2)\bar\delta_0^2+\varepsilon^2\bar\delta_\varepsilon^2}$ correlation coefficient $k_{ij}=0$	$\bar\delta_{r21r}\equiv\frac{\overline{\Delta_{r21}-\Delta_{r210}}}{\lvert r_{21}\rvert}=0{,}5\sqrt{\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
7	$\boxed{r_{21}=0{,}25R_{10}\cdot4\varepsilon}$	arbitrary	$\delta_{k21}=(1-\varepsilon^2)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40})+\varepsilon(1-\varepsilon)(\delta_{\varepsilon1}+\delta_{\varepsilon3})-\varepsilon(1+\varepsilon)(\delta_{\varepsilon2}+\delta_{\varepsilon4})$	$\delta_{k21k}=-\frac14[\varepsilon(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40})+(1-\varepsilon)(\delta_{\varepsilon1}+\delta_{\varepsilon3})-(1+\varepsilon)(\delta_{\varepsilon2}+\delta_{\varepsilon4})]$
8	$k_{21}=0{,}25\cdot4\varepsilon$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon i}\rvert=\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{k21}\rvert=4[(1-\varepsilon^2)\lvert\delta_0\rvert+\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert]$	$\lvert\delta_{k21k}\rvert=\lvert\varepsilon\rvert\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$
9		$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{k21}=2\sqrt{(1-\varepsilon^2)^2\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{k21k}=0{,}5\sqrt{\varepsilon^2\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
10	**Jointed** R_1, R_2 $\varepsilon_1=\varepsilon=-\varepsilon_2$	arbitrary	$\delta_{r21}=(1-0{,}5\varepsilon)[(1+\varepsilon)\delta_{10}+\varepsilon\delta_{\varepsilon1}-\delta_{40}]-(1+0{,}5\varepsilon)[(1-\varepsilon)\delta_{20}+\varepsilon\delta_{\varepsilon2}-\delta_{30}]$	$\delta_{r21r}=\frac14\lvert(1-\varepsilon)\delta_{10}+(1+\varepsilon)\delta_{20}+\delta_{30}+\delta_{40}\rvert+\delta_{\varepsilon1}(0{,}5-\varepsilon)-\delta_{\varepsilon2}(0{,}5+\varepsilon)$
11	$\lvert\varepsilon\rvert\le 1$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert=\lvert\delta_{\varepsilon2}\rvert\equiv\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert=4(1-0{,}25\varepsilon^2)\lvert\delta_0\rvert+2\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$ if $\lvert\varepsilon\rvert\le0{,}5$; $\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+2\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert$ if $\lvert\varepsilon\rvert\ge0{,}5$
12		$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon1}=\bar\delta_{\varepsilon2}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=2\sqrt{[1+(1-\varepsilon^2)^2]\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r}=2\sqrt{(1+0{,}5\varepsilon^2)\bar\delta_0^2+(0{,}25+0{,}5\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
13	B $R_1=R_{10}(1+\varepsilon)$	arbitrary	$\delta_{k21}=(1-\varepsilon^2)(\delta_{10}-\delta_{20})+\varepsilon(1-\varepsilon)\delta_{\varepsilon1}+\varepsilon(1+\varepsilon)\delta_{\varepsilon2}+\delta_{30}-\delta_{40}$	$\delta_{k21k}=-0{,}5\varepsilon(\delta_{10}-\delta_{20})+0{,}5(1-\varepsilon)\delta_{\varepsilon1}+0{,}5(1+\varepsilon)\delta_{\varepsilon2}$
14	$R_2=R_{10}(1-\varepsilon)$ $R_3=R_4=R_{10}$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert=\lvert\delta_{\varepsilon2}\rvert\equiv\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{k21}\rvert=4(1-0{,}5\varepsilon^2)\lvert\delta_0\rvert+2\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{k21k}\rvert=2\lvert\varepsilon\rvert\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$
15	$\boxed{r_{21}=\frac{R_{10}}{4}2\varepsilon}$ $k_{21}=\frac14\cdot2\varepsilon$	$\bar\delta_{i0}\equiv\bar\delta_0$ $\bar\delta_{\varepsilon1}=\bar\delta_{\varepsilon2}\equiv\bar\delta_\varepsilon$	$\bar\delta_{k21}=\sqrt2\sqrt{[1+(1-\varepsilon^2)^2]\bar\delta_0^2+\varepsilon^2(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r}=\frac{1}{\sqrt2}\sqrt{\varepsilon^2\bar\delta_0^2+(1+\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
16	**Jointed** R_1, R_3	arbitrary	$\delta_{r21}=(1+\varepsilon)(\delta_{10}+\delta_{30})-\delta_{20}-\delta_{40}+\varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3})$	$\delta_{r21r}=0{,}5(\delta_{10}+\delta_{30}+\delta_{\varepsilon1}+\delta_{\varepsilon3})$
17	$\varepsilon_1=\varepsilon=\varepsilon_3$ $\varepsilon\ge-1$		$\lvert\delta_{r21}\rvert=(1+\varepsilon)(\lvert\delta_{10}\rvert+\lvert\delta_{30}\rvert)+\lvert\delta_{20}\rvert+\lvert\delta_{40}\rvert+\lvert\varepsilon\rvert(\lvert\delta_{\varepsilon1}\rvert+\lvert\delta_{\varepsilon3}\rvert)$	$\lvert\delta_{r21r}\rvert=0{,}5(\lvert\delta_{10}\rvert+\lvert\delta_{30}\rvert+\lvert\delta_{\varepsilon1}\rvert+\lvert\delta_{\varepsilon3}\rvert)$
18			$\bar\delta_{r21}=\sqrt{(1+\varepsilon)^2(\bar\delta_{10}^2+\bar\delta_{30}^2)+\bar\delta_{20}^2+\bar\delta_{40}^2+\varepsilon^2(\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2)}$ $k_{ij}=0$	$\bar\delta_{r21r}=\frac{1}{\sqrt2}\sqrt{\bar\delta_{10}^2+\bar\delta_{30}^2+\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2}$ $k_{ij}=0$
19	C $R_2=R_4=R_{10}$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert=\lvert\delta_{\varepsilon3}\rvert\equiv\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert=4(1+0{,}5\varepsilon)\lvert\delta_0\rvert+2\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$
20	$\boxed{r_{21}=\frac{R_{10}}{4}\cdot2\varepsilon,}$ $k_{21}=\frac14\frac{\varepsilon}{2+\varepsilon}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=2\sqrt{(1+\varepsilon+0{,}5\varepsilon^2)\bar\delta_0^2+0{,}5\varepsilon^2\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r}=\frac{1}{\sqrt2}\sqrt{\bar\delta_0^2+\bar\delta_\varepsilon^2}$ $k_{ij}=0$
21	**Jointed** R_1, R_4 $\varepsilon_1=\varepsilon=-\varepsilon_4$ $\varepsilon\le1$	arbitrary	$\delta_{r21}=(1-0{,}5\varepsilon)[(1+\varepsilon)\delta_{10}+\varepsilon\delta_{\varepsilon1}-\delta_{20}]-(1+0{,}5\varepsilon)[(1-\varepsilon)\delta_{40}+\varepsilon\delta_{\varepsilon4}-\delta_{30}]$	$\delta_{r21r}=\frac14[(1-\varepsilon)\delta_{10}+(1+\varepsilon)\delta_{40}+\delta_{30}+\delta_{20}+(2-\varepsilon)\delta_{\varepsilon1}-(2+\varepsilon)\delta_{\varepsilon4}]$
22		$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert=\lvert\delta_{\varepsilon3}\rvert\equiv\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21}\rvert=4(1-0{,}25\varepsilon^2)\lvert\delta_0\rvert+2\lvert\varepsilon\rvert\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+\lvert\delta_\varepsilon\rvert$
23	D $\boxed{r_{21}=\frac{R_{10}}{4}\cdot2\varepsilon,\ k_{21}=\frac14\frac{2\varepsilon}{1-0{,}25\varepsilon^2}}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=\sqrt2\sqrt{[1+0{,}25\varepsilon^2+(1-0{,}5\varepsilon^2)^2]\bar\delta_0^2+(1+0{,}25\varepsilon^2)\varepsilon^2\bar\delta_\varepsilon^2}$ $k_{ij}=0$	$\bar\delta_{r21r}=\sqrt{(1+0{,}5\varepsilon^2)\bar\delta_0^2+(2+0{,}5\varepsilon^2)\bar\delta_\varepsilon^2}$ $k_{ij}=0$
24	**Variable** R_1 only $\varepsilon_1\ge-1$	arbitrary	$\delta_{r21}=\frac{(1+\varepsilon_1)\delta_{10}+\varepsilon\delta_{\varepsilon1}+(1+0{,}5\varepsilon_1)^2\delta_{30}-(1+0{,}5\varepsilon_1)(\delta_{20}+\delta_{40})}{(1+0{,}25\varepsilon_1)^2}$	$\delta_{r21r}=\frac{(\frac12-\frac1{16}\varepsilon_1)\delta_{10}+(\frac12+\frac3{16}\varepsilon_1)\delta_{30}+\frac1{16}\varepsilon_1(\delta_{20}+\delta_{40})+\delta_{\varepsilon1}}{1+0{,}25\varepsilon_1}$
25		$\lvert\delta_{10}\rvert\ne\lvert\delta_{20}\rvert=\lvert\delta_{30}\rvert=$ $=\lvert\delta_{40}\rvert\equiv\lvert\delta_0\rvert$	$\lvert\delta_{r21}\rvert=\frac{(1+\varepsilon_1)\lvert\delta_{10}\rvert+\lvert\varepsilon_1\rvert\lvert\delta_{\varepsilon1}\rvert+(3+2\varepsilon_1+\frac14\varepsilon_1^2)\lvert\delta_0\rvert}{(1+0{,}25\varepsilon_1)^2}$	$\lvert\delta_{r21r}\rvert=\frac{\frac12\lvert1-\frac18\varepsilon_1\rvert\lvert\delta_{10}\rvert+\frac12(1+\frac58\varepsilon_1)\lvert\delta_0\rvert+\lvert\delta_{\varepsilon1}\rvert}{1+0{,}25\varepsilon_1}$
26	E $R_1=R_{10}(1+\varepsilon_1)$ $R_2=R_3=R_4=R_{10}$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert\ne0$	$\lvert\delta_{r21}\rvert=4\lvert\delta_0\rvert+\frac{\lvert\varepsilon_1\rvert}{(1+0{,}25\varepsilon_1)^2}(\lvert\delta_0\rvert+\lvert\delta_{\varepsilon1}\rvert)$	$\lvert\delta_{r21r}\rvert=\lvert\delta_0\rvert+\frac{\lvert\delta_{\varepsilon1}\rvert}{1+0{,}25\varepsilon_1}$ if $\varepsilon_1\le8$; $\lvert\delta_{r21r}\rvert=\frac{0{,}375\varepsilon_1\lvert\delta_0\rvert+\lvert\delta_{\varepsilon1}\rvert}{1+0{,}25\varepsilon_1}$ if $\varepsilon_1\ge8$
27	$\boxed{r_{21}=\frac{R_{10}}{4}\frac{\varepsilon_1}{1+0{,}25\varepsilon_1}}$	$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=\frac{\sqrt{[2\varepsilon_1(1+0{,}5\varepsilon_1)+(1+(1+0{,}5\varepsilon_1)^2)^2]\bar\delta_0^2+\varepsilon^2\bar\delta_{\varepsilon1}^2}}{(1+0{,}25\varepsilon_1)^2}$ $k_{ij}=0$	$\bar\delta_{r21r}=\frac{\sqrt{(1+\frac12\varepsilon_1+\frac3{16}\varepsilon_1^2)\bar\delta_0^2+4\bar\delta_{\varepsilon1}^2}}{2(1+0{,}25\varepsilon_1)}$ $k_{ij}=0$
28		arbitrary	$\delta_{k21}\equiv\frac{\Delta_{k21}}{k_0}=\frac{(1+\varepsilon_1)(\delta_{10}-\delta_{20})+\varepsilon_1\delta_{\varepsilon1}}{(1+0{,}5\varepsilon_1)^2}+\delta_{30}-\delta_{40}$	$\delta_{k21k}=\frac{-0.25\varepsilon_1(\delta_{10}-\delta_{20})+\delta_{\varepsilon1}}{1+0{,}5\varepsilon_1}$
29	$k_{21}=\frac14\frac{\varepsilon_1}{1+0{,}5\varepsilon_1}$	$\lvert\delta_{i0}\rvert=\lvert\delta_0\rvert$ $\lvert\delta_{\varepsilon1}\rvert=\lvert\delta_{\varepsilon3}\rvert\equiv\lvert\delta_\varepsilon\rvert$	$\lvert\delta_{k21}\rvert=\frac{(1+\varepsilon_1)\lvert\delta_{210}\rvert+\lvert\varepsilon_1\rvert\lvert\delta_{\varepsilon1}\rvert+0{,}25\varepsilon_1^2(\lvert\delta_{30}\rvert+\lvert\delta_{40}\rvert)}{(1+0{,}5\varepsilon_1)^2}$	$\lvert\delta_{k21k}\rvert=\frac{0{,}25}{1+0{,}5\varepsilon_1}\lvert\varepsilon_1\rvert(\lvert\delta_{10}\rvert+\lvert\delta_{20}\rvert)+\lvert\delta_{\varepsilon1}\rvert$
30		$\bar\delta_{i0}\equiv\bar\delta_0$, $\bar\delta_{\varepsilon i}\equiv\bar\delta_\varepsilon$	$\bar\delta_{r21}=\frac{\sqrt{(1+\varepsilon_1)^2(\bar\delta_{10}^2+\bar\delta_{20}^2)+(1+0{,}5\varepsilon_1)^4(\bar\delta_{30}^2+\bar\delta_{40}^2)+\varepsilon_1^2\bar\delta_{\varepsilon1}^2}}{(1+0{,}5\varepsilon_1)^2}$ $k_{ij}=0$	$\bar\delta_{k21k}=\frac{\sqrt{0{,}25^2\varepsilon_1^2(\bar\delta_{10}+\bar\delta_{20})+\delta_{\varepsilon1}^2}}{1+0{,}5\varepsilon_1}$ $k_{ij}=0$
0	**Balance accuracy**	**actual error**	$\delta_{210}=\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}$ **limited error** $\lvert\delta_{210}\rvert_m=\sum\lvert\delta_{i0}\rvert$	**mean square measure,** $k_{ij}=0$ $\bar\delta_{210}=\sqrt{\sum\bar\delta_{i0}^2}$

B - up to 850°C. Initial limited errors $|\delta_{10}|$ of both classes are 0,06% and 0,12%, respectively.

On the base of nominal characteristic of Pt 100 sensors the maximum limited error $|\delta_{R1}|_{max} \equiv |\delta| = |\delta_{10}| + |\delta_{\varepsilon i}|$ for $\varepsilon \to \infty$ of both classes is calculated as ratio of tolerances $|\Delta|$ and increments of sensor resistance [7], i.e. as

$$|\delta| = \frac{|\Delta_i(T)| - |\Delta_{i0}(0^0\mathrm{C})|}{R_i(T) - R_{i0}(0^0\mathrm{C})} = |\delta_{i0}| + |\delta_{\varepsilon i}|$$

Obtained values are given in Fig. 2. They are only slightly changing and could be approximated by the single value and related to the maximum or mean value of the temperature range of each sensor. In the full range of positive Celsius temperatures the limited error $|\delta|$ doesn't exceed 0,2% of ε for class A and $|\delta| \leq 0{,}5\%$ for class B.

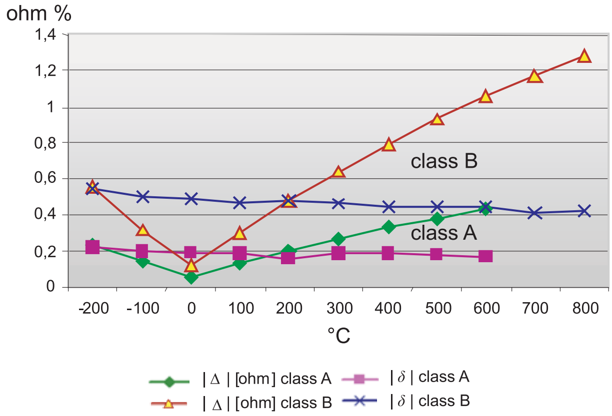


Fig. 2. Tolerances $|\Delta|$ and maximum limited relative errors $|\delta|$ of temperature sensors Pt100 type A and B evaluated from their standard [7].

7. Limited errors of the 4R bridge with single industrial Pt 100 sensors

Limited errors $|\delta_{r21}|$, $|\delta_{r21\varepsilon}| = |\delta_{r21} - \delta_{210}|$, $|\delta_{r21r}|$ of the 4R bridge transfer function r_{21} with the single industrial sensor of A or B class has be calculated from formulas of Table 3. It was assumed that limited errors $|\delta_{i0}|$ of constant bridge arms are equal and not higher that the sensor initial error $|\delta_{10}|$, balance is at 0°C, and current of supply source is stable enough or ratio of output signal and this current is measured. Maximum temperature range (0 - 600)°C is taken for calculations and for it the relative

Table 4. Limited errors of few cases of the current supplied 4R bridge with the single resistor sensor, e.g. Pt 100 type A or B.

No	Particular causes of $4R_{i0}$ bridge	Limited errors $\lvert\delta_{r21}\rvert$ (or $\lvert\delta_{r21\varepsilon}\rvert$) when: $R_{i0}=R_{10}$, and $\lvert\delta_{20}\rvert=\lvert\delta_{30}\rvert=\lvert\delta_{40}\rvert\equiv\lvert\delta_0\rvert$	Class	$\lvert\delta_{21}\rvert$, $\lvert\delta_{21\varepsilon}\rvert$, $\lvert\delta_{r21\varepsilon}\rvert$ in % for PT100: Arbitrary increments ε_1 and $\lvert\delta_0\rvert=\lvert\delta_{10}\rvert$	0-600°C, ε_1=2,137: $\lvert\delta_{r21}\rvert$	0-600°C, ε_1=2,137: $\lvert\delta_{r21\varepsilon}\rvert=0{,}72\lvert\delta_{r21}\rvert$
1	**Sensor accuracy**	$\lvert\delta_{R1}\rvert=\frac{\lvert\Delta_{R1}\rvert}{R_1}=\lvert\delta_{10}\rvert+\frac{\lvert\varepsilon_1\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon 1}\rvert$	A	$<0{,}06+\frac{\lvert\varepsilon_1\rvert}{1+\varepsilon_i}0{,}12$	0,14 (0,44°C)	0.10
			B	$<0{,}12+\frac{\lvert\varepsilon_1\rvert}{1+\varepsilon_i}0{,}34$	0,35 (1,1°C)	0,25
2	**Bridge without adjustments**	$\delta_{r21}= \frac{(1+\varepsilon_1)\lvert\delta_{10}\rvert+(3+2\varepsilon_1+\frac{1}{4}\varepsilon_1^2)\lvert\delta_0\rvert+\lvert\varepsilon_1\rvert\lvert\delta_{\varepsilon 1}\rvert}{(1+\frac{1}{4}\varepsilon_1)^2}$	A	$\frac{0{,}24+0{,}36\lvert\varepsilon_1\rvert+0{,}015\,\varepsilon_1^2}{(1+0{,}25\,\varepsilon_1)^2}$	$\lvert\delta_{r21}\rvert$=0,46	(0,33)
			B	$\frac{0{,}48+0{,}70\lvert\varepsilon_1\rvert+0{,}085\,\varepsilon_1^2}{(1+0{,}25\,\varepsilon_1)^2}$	$\lvert\delta_{r21}\rvert$=1,0	(0,72)
3	**Null setting outsight the bridge**	$\lvert\delta_{r21\varepsilon}\rvert=\lvert\delta_{r21}-\delta_{210}\rvert= \frac{\lvert\varepsilon_1\rvert\left[\frac{1}{2}\lvert 1-\frac{\varepsilon_1}{8}\rvert\lvert\delta_{10}\rvert+(\frac{1}{2}+\frac{5}{16}\varepsilon_1)\lvert\delta_0\rvert+\delta_{\varepsilon 1}\right]}{(1+\frac{1}{4}\varepsilon_1)^2}$	A	$\frac{0{,}18\lvert\varepsilon_1\rvert+0{,}0113\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$	0,143	0.103
			B	$\frac{0{,}46\lvert\varepsilon_1\rvert+0{,}023\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$	0,38	0,28
4	**Null setting in the bridge**	$\frac{\lvert\varepsilon_1\rvert\left[\frac{1}{2}\lvert\delta_{10}\rvert+(\frac{1}{2}+\frac{\varepsilon_1}{4})\lvert\delta_0\rvert+\lvert\delta_{\varepsilon 1}\rvert\right]}{(1+\frac{1}{4}\varepsilon_1)^2}$	A	$\frac{0{,}18\lvert\varepsilon_1\rvert+0{,}015\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$	0,193	0.139
			B	$\frac{0{,}46\lvert\varepsilon_1\rvert+0{,}03\varepsilon_1^2}{(1+0{,}25\varepsilon_1)^2}$	0,48	0,34
5	**Only errors of sensor increments** (setting δ_{i0})	Sensor alone: $\frac{\lvert\varepsilon_1\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon 1}\rvert$; Sensor in circuit: $\frac{\lvert\varepsilon_1\rvert}{(1+\frac{1}{4}\varepsilon_1)^2}\lvert\delta_{\varepsilon 1}\rvert$	A	$<\frac{\lvert\varepsilon_1\rvert}{(1+0{,}25\,\varepsilon_1)^2}\,0{,}12$	0,081 0,056	0,040
			B	$<\frac{\lvert\varepsilon_1\rvert}{(1+0{,}25\,\varepsilon_1)^2}\,0{,}34$	0,23 0,15	0,11
6	**negligible bridge resistance errors** $\lvert\delta_0\rvert$	$\frac{(1+\varepsilon_1)\lvert\delta_{10}\rvert+\lvert\varepsilon_1\rvert\lvert\delta_{\varepsilon 1}\rvert}{(1+\frac{1}{4}\varepsilon_1)^2}$ $(\lvert\delta_{20}\rvert=\lvert\delta_{30}\rvert=\lvert\delta_{40}\rvert\to 0)$	A	$\frac{(1+\varepsilon_1)0{,}06+\lvert\varepsilon_1\rvert 0{,}12}{(1+0{,}25\varepsilon_1)^2}$	$\lvert\delta_{21}\rvert$= 0,185	(0,133)
			B	$\frac{(1+\varepsilon_1)0{,}12+\lvert\varepsilon_1\rvert 0{,}34}{(1+0{,}25\varepsilon_1)^2}$	$\lvert\delta_{21}\rvert$= 0,47	(0,34)

increment of sensor resistance is: ε_{max}=2,137. As example numerical formulas of limited errors $|\delta_{r21}|$ or $|\delta_{k21}|$ of the class A are also estimated. Limited errors of the class B sensor bridge have been similarly estimate. For clarifying considerations the lead resistances are taken as negligible. All results are presented in Table 4.

In Table 4 five different cases of measuring circuit are considered, i.e.: bridge without any adjustments, outer and internal zero setting, negligible initial errors only of constant arms or of the sensor arm as well.

Ratio of limited errors of the bridge without adjustments and Pt sensor is 1,7 for class A and 2,9 for class B.

If errors of the bridge resistances are negligible (line 5) limited error is only slightly higher then for the sensor, but if also the initial sensors error is adjusted, then the bridge transfer function r_{21} error is even smaller then of the sensor itself ! (line 1). Results for examples 2, 3 and 4 are between 1 and 6.

For comparison: relative limited error of the output voltage of the bridge including two similar Pt 100 sensors of the class A in opposite arms, calculated from line 3 of Table 3 for the same temperature range doesn't exceed 0,51% - without null correction, and 0,39% - if is corrected. These errors are calculated for the twice higher signal that for the single sensor bridge. They are slightly higher but signal linearly depends on resistance increment equal for both sensors. For lower temperature ranges relative limited errors and uncertainties type B become higher.

8. General conclusions

Two methods of describing the accuracy measures of the arbitrary imbalanced sensor bridges are presented together and compared., i.e.:

- one component accuracy measure related to initial sensitivity of the bridge transfer functions, given before in [1], [6], [7] and
- the new double component one of separately defined measures for zero and transfer function increment [8].

The second one is similar as used for the broad range instruments, e.g. digital voltmeters. Accuracy measures of bridge arms are defined for initial resistances and for their increments. Then these methods are independent from the sensor characteristic to the measured quantity.

These methods are discussed using on few examples of 4R bridges of equal initial resistances, supplied by current or voltage source and with single, double and four element sensors.

Given formulas allow to finding accuracy of the 4R bridge or uncertainty of measurements with bridge circuits if actual or limited values of errors or standard statistical measure of their resistances and sensors are known.

Formulas of general and particular cases of the bridge may be used for computer simulation of the accuracy of various sensor bridges and measured objects of the X twoport equivalent circuit in different circumstances.

Systematic errors could be calculated as random ones for set of sensor bridges in production or in exploitation and if correlation coefficients are small obtained values should be smaller than of the worst-case limited errors.

Similar formulas as presented in this and other papers, e.g. [1], [4], [6], [7] could be formulated for any types of impedance sensor circuits as DC and AC bridges of single and double supply, active bridges linearized by feedback or multipliers, Anderson loop (developed in NASA) and impedance converters with virtual DSP processing. Given in this paper methods of the simplification of their accuracy description could be also applied in many industrial measurements.

Accuracy of current and voltage supplied strain bridges has been analyzed by M. Kreuzer [9], [10], but such a unified approach as given above to the accuracy description of unbalanced bridges and other circuits of broadly variable parameters, developed in [1], [4], [6] - [8], is not found so far in literature.

The presented method is also valuable for accuracy evaluation in testing any circuit from its terminals as two-port, which is commonly used in diagnostics and in impedance tomography. It was also used to describe the accuracy of two-parameter bridge measurements - see [3]-[5].

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