Computation of Positive Realizations of SISO Singular Hybrid Linear Systems

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Abstract:

The realization problem for 2D positive singular linear hybrid systems is formulated, as well as a method based on the state variable diagram for finding a positive realization of a given improper transfer function is proposed. Sufficient conditions for the existence of a positive realization of a given improper transfer function are established. A procedure for computation of a positive realization is proposed and illustrated by a numerical example.

Keywords: *hybrid linear systems, singular, positive, realization.*

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behaviour can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of the state of the art in positive systems theory has been given in the monographs [2, 9]. The realization problem for positive discrete-time and continues-time systems without and with delays was considered in [1, 2, 9-13]. The reachability, controllability and minimum energy control of positive linear discrete-time systems with delays have been considered in [3]. The relative controllability of stationary hybrid systems has been investigated in [20] and the observability of linear differential-algebraic systems with delays has been considered in [21]. A new class of positive 2D hybrid linear system has been introduced in [14]. The realization problem for this class of systems has been considered in [6, 15] and for class of positive hybrid systems with delays in [5, 7].

The main purpose of this paper is to present a new method for computation of positive realizations for 2D single-input single-output singular hybrid linear systems using the state variable diagram method. Sufficient conditions for the existence of a positive realization of a given improper transfer function will be established and a procedure for computation of positive realizations will be proposed. Considerations will be illustrated by numerical example.

2. Preliminaries and formulation of the problem

Consider a SISO singular hybrid system described by the equations

$$\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1(t,i) \\ x_2(t,i+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t,i) \tag{1a}$$

$$y(t,i) = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} x_1(t,i) \\ x_2(t,i) \end{bmatrix}, \quad t \in R_+ = [0,+\infty], \quad i \in Z_+ \tag{1b}$$

where $\dot{x}_1(t,i) = \frac{\partial x_1(t,i)}{\partial t}$, $x_1(t,i) \in R^{n_1}$, $x_2(t,i) \in R^{n_2}$ are the state vectors $u(t,i) \in R^m$, $y(t,i) \in R^p$ are input and output vectors and $E, A \in R^{(n_1+n_2)\times(n_1+n_2)}$, $B \in R^{(n_1+n_2)\times m}$, $C \in R^{p\times(n_1+n_2)}$ are real matrices.

It is assumed that $\det\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} = 0$

and $\det\left[\begin{bmatrix} E_1 s & 0 \\ 0 & E_2 z \end{bmatrix} - \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\right] \neq 0$.

Boundary conditions for (1a) have the form

$$x_1(0,i) = x_1(i),\ i \in Z_+ \text{ and } x_2(t,0) = x_2(t),\ t \in R_+ \tag{2}$$

Note that the hybrid system (1) has the similar structure as Roesser model [5, 14, 16].

Let $R_+^{n\times m}$ is the set of $n\times m$ real matrices with non-negative entries and $R_+^n = R_+^{n\times 1}$.

Definition 1. The hybrid system (1) is called internally positive if $x_1(t,i) \in R_+^{n_1}$, $x_2(t,i) \in R_+^{n_2}$ and $y(t,i) \in R_+^p$, $t \in R_+$, $i \in Z_+$ for arbitrary boundary conditions $x_1(i) \in R_+^{n_1}$, $i \in Z_+$, $x_2(t) \in R_+^{n_2}$, $t \in R_+$ and all inputs $u(t,i) \in R_+^m$, $t \in R_+$, $i \in Z_+$.

Transfer matrix of the system (1) is given by the formula

$$T(s,z) = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} E_1 s - A_{11} & -A_{12} \\ -A_{21} & E_2 z - A_{22} \end{bmatrix}^{-1}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{3}$$

Definition 2. The matrices $E_1, E_2, A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, C_1, C_2$ are called the positive realization of the improper transfer matrix, if they satisfy the equality (3). A realization is called minimal if the matrices A_{11} and A_{22} have minimal dimensions among all positive realizations of $T(s,z)$.

The realization problem can be stated as follow.

With a given proper rational transfer function $T(s,z)$, find its positive realization E, A, B, C.

3. Problem solution

The essence of proposed method for solving of the realization problem for positive 2D hybrid systems will be presented on the following improper transfer function

$$T(s,z)=\frac{b_{22}s^2z^2+b_{21}s^2z+b_{12}sz^2+b_{20}s^2+b_{02}z^2+b_{11}sz+b_{10}s+b_{01}z+b_{00}}{sz+a_{10}s+a_{01}z+a_{00}} \tag{4}$$

Multiplying the numerator and denominator of transfer function (4) by $s^{-2}z^{-2}$ we obtain

$$T(s,z)=\frac{b_{22}+b_{21}z^{-1}+b_{12}s^{-1}+b_{20}z^{-2}+b_{02}s^{-2}+b_{11}s^{-1}z^{-1}+b_{10}s^{-1}z^{-2}+b_{01}s^{-2}z^{-1}+b_{00}s^{-2}z^{-2}}{s^{-1}z^{-1}+a_{10}s^{-1}z^{-2}+a_{01}s^{-2}z^{-1}+a_{00}s^{-2}z^{-2}}=\frac{Y}{U} \tag{5}$$

Defining

$$E=\frac{U}{s^{-1}z^{-1}+a_{10}s^{-1}z^{-2}+a_{01}s^{-2}z^{-1}+a_{00}s^{-2}z^{-2}} \tag{6}$$

from (5) and (6) we obtain

$$\begin{aligned}&U-(s^{-1}z^{-1}+a_{10}s^{-1}z^{-2}+a_{01}s^{-2}z^{-1}+a_{00}s^{-2}z^{-2})E=0\\&Y=(b_{22}+b_{21}z^{-1}+b_{12}s^{-1}+b_{20}z^{-2}+b_{02}s^{-2}+b_{11}s^{-1}z^{-1}+b_{10}s^{-1}z^{-2}+b_{01}s^{-2}z^{-1}+b_{00}s^{-2}z^{-2})E\end{aligned} \tag{7}$$

Using (7) we may draw the state variable diagram shown in Fig.1

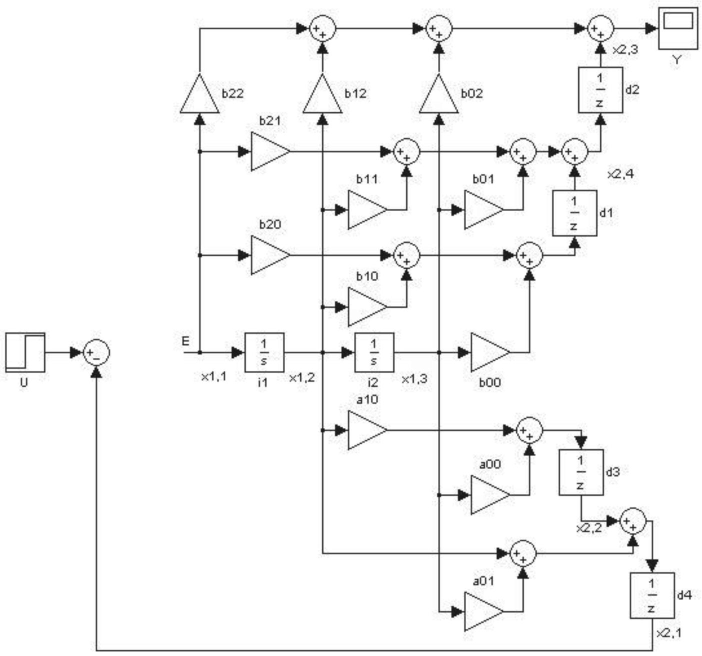

Fig. 1. State variable diagram for transfer function (4).

As a state variable we choose outputs of integrators ($x_{1,2}(t,i)$, $x_{1,3}(t,i)$) and of delay elements ($x_{2,1}(t,i)$, $x_{2,2}(t,i)$, $x_{2,3}(t,i)$, $x_{2,4}(t,i)$). We are dealing with the singular system and we have to choose one more state variable in this case $x_{1,1}(t,i)$. Using the state variable diagram (Fig.1) we can write the following state equations

$$\begin{aligned}&0=-x_{21}(t,i)+u(t,i)\\&\dot{x}_{12}(t,i)=x_{11}(t,i)\\&\dot{x}_{13}(t,i)=x_{12}(t,i)\\&x_{21}(t,i+1)=a_{01}x_{13}(t,i)+x_{12}(t,i)+x_{22}(t,i)\\&x_{22}(t,i+1)=a_{00}x_{13}(t,i)+a_{10}x_{12}(t,i)\\&x_{23}(t,i+1)=b_{01}x_{13}(t,i)+b_{11}x_{12}(t,i)+b_{21}x_{11}(t,i)+x_{24}(t,i)\\&x_{24}(t,i+1)=b_{00}x_{13}(t,i)+b_{10}x_{12}(t,i)+b_{20}x_{11}(t,i)\\&y(t,i)=b_{02}x_{13}(t,i)+b_{12}x_{12}(t,i)+b_{22}x_{11}(t,i)+x_{23}(t,i)\end{aligned} \tag{8}$$

Taking into account Definition 1 and (8), the following has been proved.

Theorem 1. The singular hybrid system with improper transfer function (4) is positive if the following conditions are

satisfied:
- all coefficients of numerator of improper transfer function are non-negative,
- all coefficients of denominator of improper transfer function are non-negative.

Rewriting equations (8) in matrix form

$$\begin{bmatrix} 0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix}\begin{bmatrix} \dot{x}_{11}(t,i)\\ \dot{x}_{12}(t,i)\\ \dot{x}_{13}(t,i)\\ x_{21}(t,i+1)\\ x_{22}(t,i+1)\\ x_{23}(t,i+1)\\ x_{24}(t,i+1) \end{bmatrix}=\begin{bmatrix} 0&0&0&-1&0&0&0\\ 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&1&a_{01}&0&0&0&0\\ 0&a_{10}&a_{00}&0&0&0&0\\ b_{21}&b_{11}&b_{01}&0&0&0&1\\ b_{20}&b_{10}&b_{00}&0&0&0&0 \end{bmatrix}\begin{bmatrix} x_{11}(t,i)\\ x_{12}(t,i)\\ x_{13}(t,i)\\ x_{21}(t,i)\\ x_{22}(t,i)\\ x_{23}(t,i)\\ x_{24}(t,i) \end{bmatrix}+\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\end{bmatrix}u(t,i)$$

$$y(t,i)=\begin{bmatrix} b_{22} & b_{12} & b_{02} & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_{11}(t,i)\\ x_{12}(t,i)\\ x_{13}(t,i)\\ x_{21}(t,i)\\ x_{22}(t,i)\\ x_{23}(t,i)\\ x_{24}(t,i) \end{bmatrix} \tag{9}$$

and if conditions of Theorem 1 are met, then positive realization of improper transfer function (4) of singular hybrid system (1) has the form:

$$E=\begin{bmatrix} 0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix},\quad A=\begin{bmatrix} 0&0&0&-1&0&0&0\\ 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&1&a_{01}&0&0&0&0\\ 0&a_{10}&a_{00}&0&0&0&0\\ b_{21}&b_{11}&b_{01}&0&0&0&1\\ b_{20}&b_{10}&b_{00}&0&0&0&0 \end{bmatrix},\quad B=\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\end{bmatrix},$$

$$C=\begin{bmatrix} b_{22} & b_{12} & b_{02} & 0 & 0 & 1 & 0 \end{bmatrix} \tag{10}$$

Generalizing the considerations on any improper transfer function we obtain

$$T(s,z)=\frac{\sum_{i=0}^{q_1}\sum_{j=0}^{q_2} b_{i,j}s^i z^j}{s^{n_1}z^{n_2}+\left(\sum_{\substack{i=0\\ i+j\neq n_1+n_2}}^{n_1-1}\sum_{j=0}^{n_2-1} a_{i,j}s^i z^j\right)} \tag{11}$$

where $r_1=q_1-n_1>0$, $r_2=q_2-n_2>0$.

Multiplying the numerator and denominator of transfer function $s^{-q_1}z^{-q_2}$ we obtain

$$T(s,z)=\frac{b_{q_1,q_2}+b_{q_1-1,q_2}s^{-1}+b_{q_1,q_2-1}z^{-1}+\ldots+b_{00}s^{-q_1}z^{-q_2}}{s^{-r_1}z^{-r_2}+a_{n_1,n_2-1}s^{-r_1}z^{-(r_2+1)}+a_{n_1-1,n_2}s^{-(r_1+1)}z^{r_2}+\ldots+a_{00}s^{-q_1}z^{-q_2}}=\frac{Y}{U} \tag{12}$$

Defining

$$U-(s^{-r_1}z^{-r_2}+a_{n_1,n_2-1}s^{-r_1}z^{-(r_2+1)}+a_{n_1-1,n_2}s^{-(r_1+1)}z^{r_2}+\ldots+a_{00}s^{-q_1}z^{-q_2})E=0$$

$$Y=(b_{q_1,q_2}+b_{q_1-1,q_2}s^{-1}+b_{q_1,q_2-1}z^{-1}+\ldots+b_{00}s^{-q_1}z^{-q_2})E \tag{13}$$

we may draw the state variable diagram shown in Fig.2

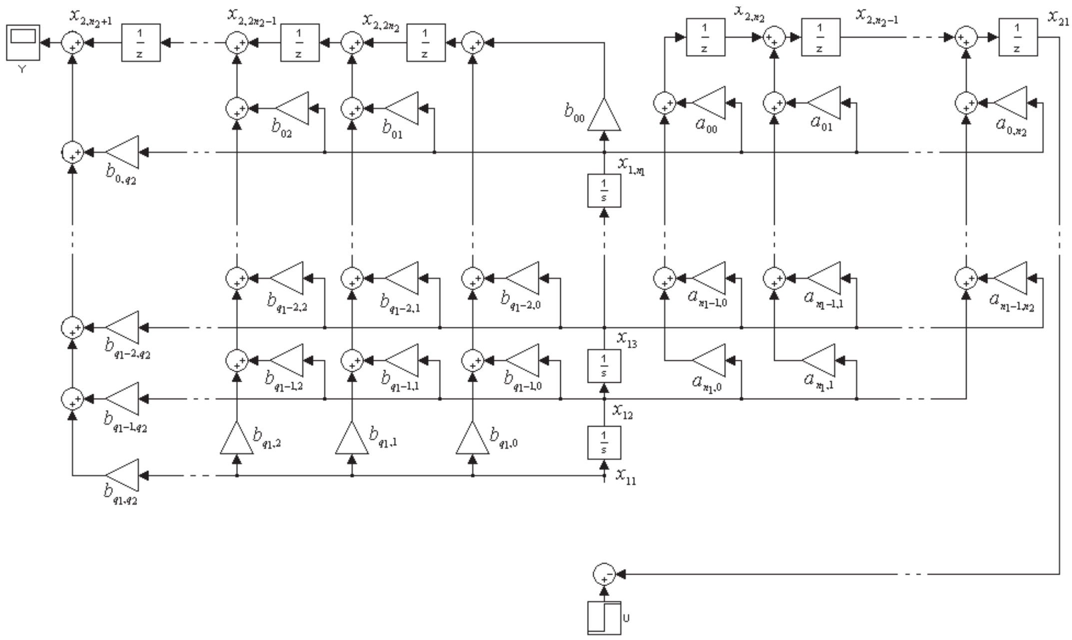

Fig. 2. State variable diagram for transfer function (11).

As a state variable we choose the outputs of integrators ($x_{1,1}(t,i),\ x_{1,2}(t,i),\ \ldots\ , x_{1,n_1+1}(t,i)$) and of delay elements ($x_{2,1}(t,i),\ x_{2,2}(t,i),\ \ldots,\ x_{2,2n_2}(t,i)$). Using state variable diagram (Fig. 2) we can write the following differential and difference equations

$$
\begin{aligned}
&0=-x_{21}(t,i)+u(t,i)\\
&\dot{x}_{12}(t,i)=x_{11}(t,i)\\
&\dot{x}_{13}(t,i)=x_{12}(t,i)\\
&\vdots\\
&\dot{x}_{1,n_1+1}(t,i)=x_{1,n_1}(t,i)\\
&x_{21}(t,i+1)=x_{12}(t,i)+a_{n_1-1,n_2}x_{13}(t,i)+\ldots+a_{0,n_2}x_{1,n_1+1}(t,i)+x_{22}(t,i)\\
&x_{22}(t,i+1)=a_{n_1,n_2-1}x_{12}(t,i)+a_{n_1-1,n_2-1}x_{13}(t,i)+\ldots+a_{0,n_2-1}x_{1,n_1+1}(t,i)+x_{23}(t,i)\\
&\vdots\\
&x_{2,n_2-1}(t,i+1)=a_{n_1,1}x_{12}(t,i)+a_{n_1-1,1}x_{13}(t,i)+\ldots+a_{01}x_{1,n_1+1}(t,i)+x_{2,n_2}(t,i)\\
&x_{2,n_2}(t,i+1)=a_{n_1,0}x_{12}(t,i)+a_{n_1-1,0}x_{13}(t,i)+\ldots+a_{00}x_{1,n_1+1}(t,i)\\
&x_{2,n_2+1}(t,i+1)=b_{q_1,q_2-1}x_{11}(t,i)+b_{q_1-1,q_2-1}x_{12}(t,i)+\ldots+b_{0,q_2-1}x_{1,n_1+1}(t,i)+x_{2,n_2+2}(t,i)\\
&\vdots\\
&x_{2,2n_2-1}(t,i+1)=b_{q_1,1}x_{11}(t,i)+b_{q_1-1,1}x_{12}(t,i)+\ldots+b_{01}x_{1,n_1+1}(t,i)+x_{2,2n_2}(t,i)\\
&x_{2,2n_2}(t,i+1)=b_{q_1,0}x_{11}(t,i)+b_{q_1-1,0}x_{12}(t,i)+\ldots+b_{00}x_{1,n_1+1}(t,i)\\
&y(t,i)=b_{q_1,q_2}x_{11}(t,i)+b_{q_1-1,q_2}x_{12}(t,i)+\ldots+b_{0,q_2}x_{1,n_1+1}(t,i)+x_{2,n_2+1}(t,i)
\end{aligned}
\tag{14}
$$

Defining vectors

$$
x_1(t,i)=\begin{bmatrix}x_{1,1}(t,i)\\ \vdots\\ x_{1,n_1+1}(t,i)\end{bmatrix},\quad x_2(t,i)=\begin{bmatrix}x_{2,1}(t,i)\\ \vdots\\ x_{2,2n_2}(t,i)\end{bmatrix}
\tag{15}
$$

equations (14) we may write in the form

$$
\begin{gathered}
\begin{bmatrix}E_1 & 0\\ 0 & E_2\end{bmatrix}\begin{bmatrix}\dot{x}_1(t,i)\\ x_2(t,i+1)\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1(t,i)\\ x_2(t,i)\end{bmatrix}+\begin{bmatrix}B_1\\ B_2\end{bmatrix}u(t,i)\\
y(t,i)=\begin{bmatrix}C_1 & C_2\end{bmatrix}\begin{bmatrix}x_1(t,i)\\ x_2(t,i)\end{bmatrix}
\end{gathered}
\tag{16}
$$

where

$$E_1 = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_1} \end{bmatrix} \in R^{(n_1+1)\times(n_1+1)}, \quad E_2 = I_{2n_2},$$

$$A_{11} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \in R^{(n_1+1)\times(n_1+1)}, \quad A_{12} = \begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \in R^{(n_1+1)\times 2n_2},$$

$$A_{21} = \begin{bmatrix} A_{211} \\ A_{212} \end{bmatrix} \in R^{2n_2\times(n_1+1)}, A_{211} = [0] \in R^{(r_2-1)\times(n_1+1)},$$

$$A_{212} = \begin{bmatrix} 0 & \dots & 0 & 1 & a_{n_1-1,n_2} & \dots & a_{0,n_2} \\ 0 & \dots & 0 & a_{n_1,n_2-1} & a_{n_1-1,n_2-1} & \dots & a_{0,n_2-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & a_{n_1,0} & a_{n_1-1,0} & \dots & a_{00} \\ b_{n_1+r_1,q_2-1} & \dots & b_{n_1,q_2-1} & b_{n_1-1,q_2-1} & b_{n_1-2,q_2-1} & \dots & b_{0,q_2-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ b_{n_1+r_1,0} & \dots & b_{n_1,0} & b_{q_1-1,0} & b_{q_1-2,0} & \dots & b_{00} \end{bmatrix} \in R^{(2q_2-3r_2+1)\times(n_1+1)},$$

$$A_{22} = \begin{bmatrix} 0 & I_{n_2-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{n_2-1} \\ 0 & 0 & 0 & 0 \end{bmatrix} \in R^{2n_2\times 2n_2},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in R^{n_1+1}, \quad B_2 = [0] \in R^{2n_2}, \quad C_1 = \begin{bmatrix} b_{q_1,q_2} & \dots & b_{0,q_2} \end{bmatrix} \in R^{1\times(n_1+1)}, \quad C_2 = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix} \in R^{1\times 2n_2},$$

$$C_{21} = [0] \in R^{1\times n_2}, \quad C_{22} = [1 \quad 0 \quad \dots \quad 0] \in R^{1\times n_2} \tag{17}$$

Therefore, the following theorem has been proved.

Theorem 2. There exists a positive realization of the improper transfer function (11) if all coefficients of its numerator and denominator are nonnegative.

Remark. A characteristic feature of singular positive 2D hybrid model is that some entries of the matrix A_{12} are negative.
If the assumptions of Theorem 2 are satisfied then a positive realization can be found by the use of the following procedure.

Procedure.
Step 1. Write the transfer function $T(s, z)$ in the form (12) and the equations (13),
Step 2. Using (13) draw the state variable diagram shown in Fig. 2,
Step 3. Choose the state variables and write equations (14),
Step 4. Using (14) find the desired realization (17) of the transfer function (11).

4. The example

There is given improper transfer function

$$T(s,z) = \frac{sz^2 + 2z^2 + 3sz + 4s + 5z + 6}{sz + 7s + 8z + 9} \tag{18}$$

find its positive realization (17). In this case $r_1 = q_1 - n_1 = 0$ and $r_2 = q_2 - n_2 = 1$.

Using the procedure we obtain the following:
Step 1. Multiplying the numerator and denominator of transfer function (18) by $s^{-1}z^{-2}$ we obtain

$$T(s,z)=\frac{1+2s^{-1}+3z^{-1}+4z^{-2}+5s^{-1}z^{-1}+6s^{-1}z^{-2}}{z^{-1}+7z^{-2}+8s^{-1}z^{-1}+9s^{-1}z^{-2}}=\frac{Y}{U} \quad (19)$$

and

$$U-(z^{-1}+7z^{-2}+8s^{-1}z^{-1}+9s^{-1}z^{-2})E=0$$
$$Y=(1+2s^{-1}+3z^{-1}+4z^{-2}+5s^{-1}z^{-1}+6s^{-1}z^{-2})E \quad (20)$$

Step 2. In this case state variable diagram has the form shown in Fig. 3

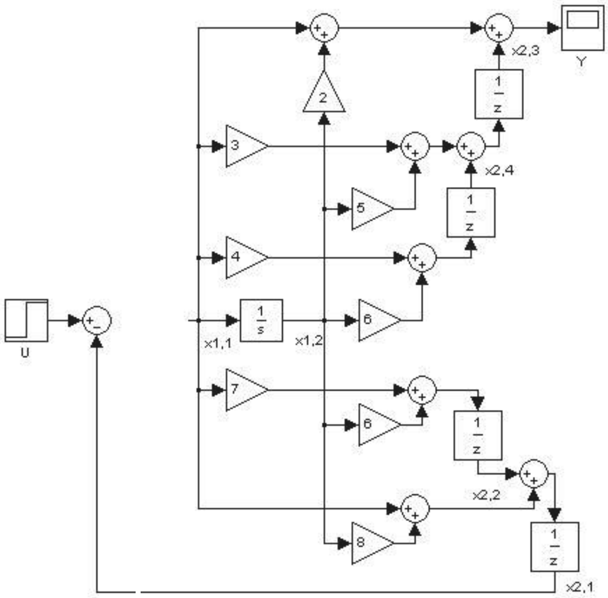

Fig. 3. State variable diagram for transfer function (18).

Step 3. Using the state variable diagram (Fig. 3) we can write the state equations

$$\begin{aligned}
&0=-x_{21}(t,i)+u(t,i)\\
&\dot{x}_{12}(t,i)=x_{11}(t,i)\\
&x_{21}(t,i+1)=x_{11}(t,i)+8x_{12}(t,i)+x_{22}(t,i)\\
&x_{22}(t,i+1)=7x_{11}(t,i)+9x_{12}(t,i)\\
&x_{23}(t,i+1)=3x_{11}(t,i)+5x_{12}(t,i)+x_{24}(t,i)\\
&x_{24}(t,i+1)=4x_{11}(t,i)+6x_{12}(t,i)\\
&y(t,i)=x_{11}(t,i)+2x_{12}(t,i)+x_{23}(t,i)
\end{aligned} \quad (21)$$

Step 4. Desired positive realization has the form

$$E=\begin{bmatrix}0&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix},\quad A=\begin{bmatrix}0&0&-1&0&0&0\\1&0&0&0&0&0\\1&8&0&1&0&0\\7&9&0&0&0&0\\3&5&0&0&0&1\\4&6&0&0&0&0\end{bmatrix},\quad B=\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix},\quad C=\begin{bmatrix}1&2&0&0&1&0\end{bmatrix} \quad (22)$$

5. Concluding remarks

A method for computation of a positive realization of a given improper transfer function of 2D hybrid linear system has been proposed. Sufficient conditions for the existence of a positive realization of a given improper transfer function have been established. A procedure for computation of a positive realization has been proposed, and illustrated by numerical example. An open problem is formulation of the necessary and sufficient conditions for the existence of solution of the positive realization problem for 2D hybrid systems in the general case. Extensions of those considerations for 2D hybrid systems described by models with structures similar to the 2D general model or the 2D second Fornasini-Marchesini model [9, 14] are also open problems.

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