NUKLEONIKA 2004;49(Supplement 2):S41–S44

PROCEEDINGS

# Pion entropy and phase space density in RHIC collisions

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**Abstract** We present the source-averaged phase space density <f>, the entropy per particle $S_\pi/N_\pi$, and the total pion entropy per participant $(dS_\pi/dy)/N_p$ extracted at midrapidity in six momentum intervals and seven centrality intervals, for pions produced in $\sqrt{s_{NN}} = 130$ GeV Au+Au collisions at RHIC. These are inferred from measured $\pi^-$ momentum spectra and HBT correlation analysis of pion data from the STAR detector. The source-averaged pion phase space density <f> reaches a plateau with centrality and is significantly larger than that observed for $\sqrt{s_{NN}} = 17$ GeV Pb+Pb central collisions at the CERN SPS. The entropy per particle $S_\pi/N_\pi$ is relatively low, approaching the Landau limit. The total pion entropy per participant $(dS_\pi/dy)/N_p$ is about 6.5 ± 0.1, independent of centrality, suggesting that there is no per-participant increase in initial entropy production with increasing collision centrality.

**Key words** entropy • pion • phase space density

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Received: 4 February 2004

## Introduction

A fundamental quantum-statistical quantity probed by ultra-relativistic heavy ion collisions is the pion six-dimensional phase space density $f(r,p)$, a Lorentz-invariant scalar that characterizes the local number of pions at freeze-out contained in a six-dimensional phase space cell of volume $h^3$. The source-averaged phase space density <f> $\equiv \int_{-\infty}^{\infty} f^2 d\vec{r} / \int_{-\infty}^{\infty} f d\vec{r}$ can be extracted from experimental observables, is readily calculated in many theoretical models, sets the scale for multiparticle correlations, and can be directly related to the entropy of the system. Previous publications have reported estimates of the source-averaged pion phase space density <f> for heavy ion collisions at the Brookhaven AGS [1, 5] and the CERN SPS [4, 7].

The system entropy $S$ is perhaps even more fundamental. Entropy production is expected to occur primarily in the initial stages of a nucleus-nucleus collision and to grow only slightly as the system subsequently expands and cools. The entropy density of a quark-gluon plasma (QGP) is estimated to be an order of magnitude larger than that of a hadronic gas under similar conditions, so entropy production in an initial QGP phase might be significantly larger than that in an initial hadronic phase. Thus, the system entropy at kinetic freeze-out, studied as a function of centrality, can provide clues to initial conditions and can place strong constraints on initial collision dynamics. Pal and Pratt [8] have recently estimated the entropy of central RHIC Au+Au collisions, but the system entropy has not previously been studied as a function of centrality.

As first suggested by Bertsch [2, 3], for azimuthally-averaged sources, <f> at a given vector momentum $p$ can be written as:

$$\left\langle f\left(\vec{p}\right)\right\rangle = \left[\frac{1}{E_\pi}\right]\left[\frac{d^2N}{2\pi m_T dm_T dy}\right]\left[\frac{1}{\sqrt{\lambda}}\right]V_p \tag{1}$$

where

$$V_p \equiv \int_{-\infty}^{\infty} (C_2(\vec{q}) - 1)\, d\vec{q} \cong \frac{\lambda(\hbar\sqrt{\pi})^3}{R_O R_S R_L}\ .$$

Here $E_\pi$, the pion total energy, is a Jacobian for Lorentz invariance, the second bracket is the pion spectrum, and the third bracket is a correction for pion purity. The final factor $V_p$ is the momentum volume of the HBT correlation "bump". The quantity $C_2(q)$ is the Coulomb-corrected two-particle momentum correlation function of $q$, the vector momentum difference of correlated pion pairs. The last expression for $V_p$ assumes that the source is Gaussian and gives $V_p$ at midrapidity with azimuthal averaging in terms of the usual Bertsch-Pratt Hanbury-Brown-Twiss (HBT) parameters $R_O$, $R_S$, $R_L$, and $\lambda$.

Figure 1(a) shows the corrected momentum volume $V_p/\sqrt{\lambda}$, as obtained from Gaussian fits to HBT correlation functions, for seven centrality bins: 0–5% of $\sigma_{tot}$ (circles), 5–10% (+), 10–20% (triangles), 20–30% (diamonds), 30–40% (5-stars), 40–50% (6-stars), and 50–80% (7-stars). Lines through the curves are a global Sinyukov-inspired fit [6] that assumes the power-law dependence $V_p/\sqrt{\lambda} \sim m_T^{3\alpha}$. Figure 1(b) shows the $\pi^-$ spectrum, not corrected for long lived decays or secondary production and fitted with a global effective-temperature Bose-Einstein distribution in the same centrality intervals.

## The pion phase space density <f>

The corrected momentum volume and pion spectra from Fig. 1 are combined to give the source-averaged charged-pion phase-space density <f>. Figure 2 shows <f> as points at the bin-average momenta of the HBT analyses, using the spectrum global fit to provide interpolated spectrum values at the same momenta. We note that the four most central bins (0–40%) have very similar phase space densities, suggesting the action of some limiting process. The open crosses are <f> values for CERN experiment NA49 [5] for Pb+Pb collisions at $\sqrt{s_{NN}}$ = 17 GeV and

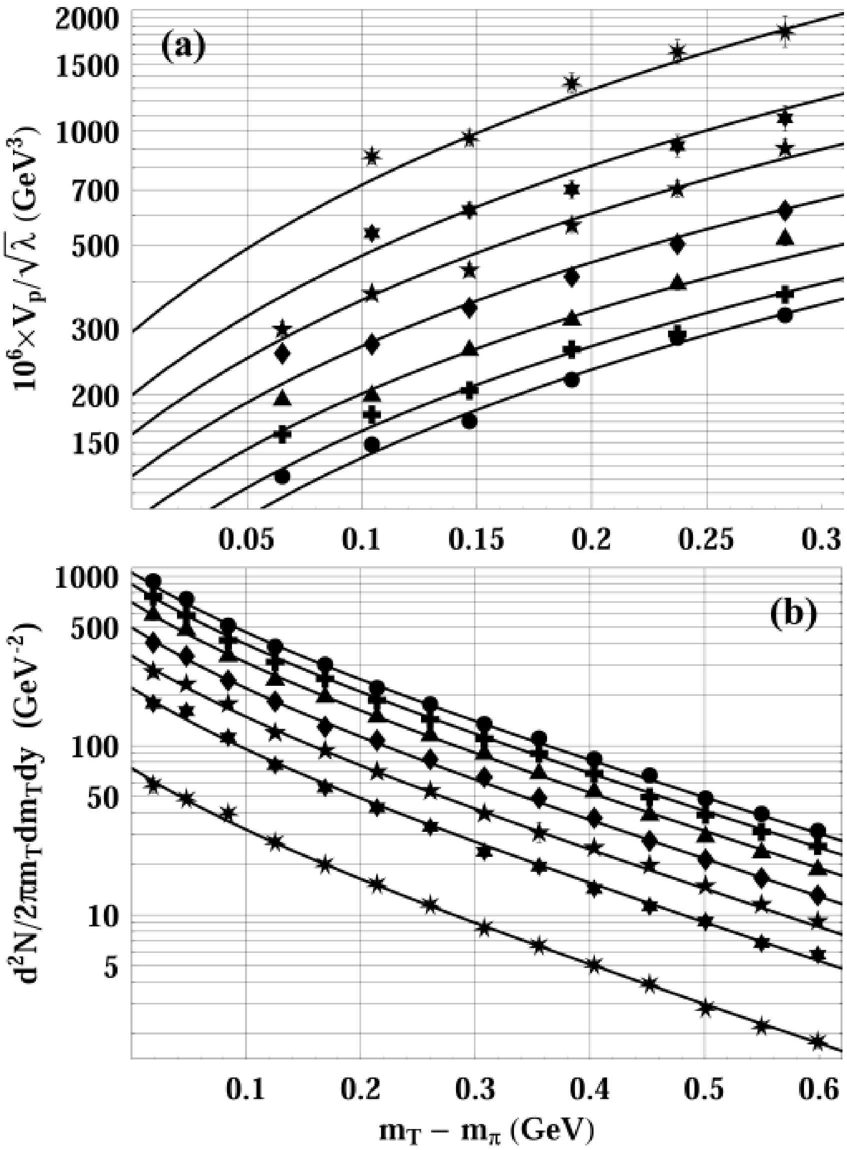


**Fig. 1.** Relative transverse mass plots of (a) HBT corrected momentum volume $V_p/\sqrt{\lambda}$ and (b) negative pion spectra for identified $\pi^-$ in seven centrality intervals (see text) for $\sqrt{s_{NN}}$ = 130 GeV Au+Au collisions at RHIC. Spectra are not corrected for secondary or long-lived resonance decay production. The lines represent fits to the points (see text). Error bars, where visible, are statistical only, were propagated using covariance matrices, and are usually smaller that the point sizes.

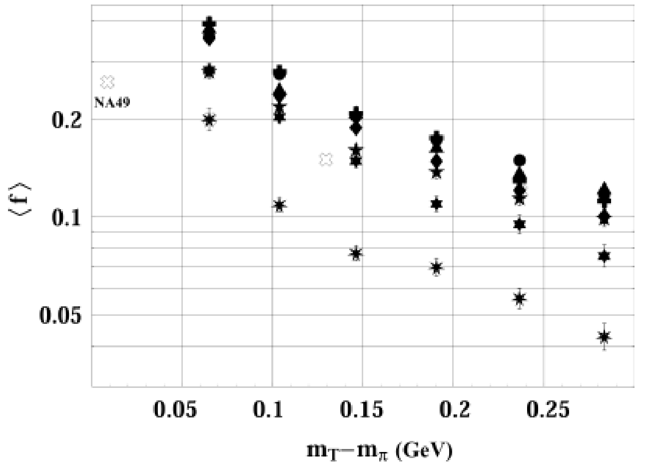


**Fig. 2.** Source-averaged $\pi^-$ phase space density <f> for $\sqrt{s_{NN}}$ = 130 GeV Au+Au collisions at seven values of centrality (see text for symbol definitions) calculated with $V_p/\sqrt{\lambda}$ in 6 momentum bins combined with interpolated spectrum values. Open cross symbols are <f> values taken from Ref. [3] using NA49 results for $\sqrt{s_{NN}}$ = 17 GeV midrapidity Pb+Pb collisions at centrality 0–4% of $\sigma_{tot}$.

centrality 0–4% of $\sigma_{tot}$. We see that the pion phase space density from NA49 is exceeded by that at RHIC by almost a factor of 2 and has clearly not reached some "universal" limiting value [4].

## The entropy per pion $S_\pi/N_\pi$

From the statistical mechanics of bosons, the local phase space density $f(r,p) \equiv f$ is related to the average entropy per particle $S/N$ by:

$$(2) \qquad S/N \equiv \int_{-\infty}^{\infty} S_6\, d\vec{r}\, d\vec{p} \Big/ \int_{-\infty}^{\infty} \mathrm{f}\, d\vec{r}\, d\vec{p}$$

where

$$S_6 \equiv -\mathrm{f}\log\mathrm{f} + (\mathrm{f}+1)\log(\mathrm{f}+1)$$
$$\cong -\mathrm{f}\log\mathrm{f} + \mathrm{f} + {}^1\!/_2\,\mathrm{f}^2 - {}^1\!/_6\,\mathrm{f}^3 + {}^5\!/_{96}\,\mathrm{f}^4 + \ldots$$

The last expression above is a truncated series approximation that reproduces the exact expression to ±0.16% for $0 < f < 1$. We note that while the numerator of the $S/N$ expression tends to grow with increasing $f$, the denominator grows faster, so that an increase in $f$ produces a decrease in $S/N$.

To evaluate this expression we assume that $f(r,p) = \langle f\rangle 2^{3/2} \exp[-x^2/2R_x(p)^2 - y^2/2R_y(p)^2 - z^2/2R_z(p)^2]$, that $R_x(p)R_y(p)R_z(p) = R^3_{eff}$, and that $R_{eff} = \hbar c\sqrt{\pi}\,[V_p/\sqrt{\lambda}]^{-1/3} = R_0/m_T^{\alpha}$. In other words, we spatially integrate over a source with the radial shape and momentum dependence provided by HBT. Performing these spatial integrals gives:

$$(3) \quad S/N \equiv \int_{-\infty}^{\infty} S_3 m_T^{-3\alpha} d\vec{p} \Big/ \int_{-\infty}^{\infty} \langle \mathrm{f}\rangle m_T^{-3\alpha} d\vec{p}$$

$$\cong \int_{-\infty}^{\infty} S_3 m_T^{1-3\alpha} p_T\, dp_T \Big/ \int_{-\infty}^{\infty} \langle \mathrm{f}\rangle m_T^{1-3\alpha} p_T\, dp_T$$

where

$$S_3 \equiv -\langle f\rangle\log\langle f\rangle + \frac{5-\log 8}{2}\langle f\rangle + {}^1\!/_2\langle f\rangle^2 - \frac{4}{9\sqrt{3}}\langle f\rangle^3 + \frac{5}{24\sqrt{2}}\langle f\rangle^4 + \ldots$$

The second $S/N$ expression above assumes boost invariance, azimuthal averaging, and implicitly has integrated over $f$ and $y$, thereby reducing the integrations to one momentum dimension. It also implicitly makes the assumption that the phase space density $f$ and the entropy per particle $S/N$, as averaged over the spatial homogeneity volume that is accessed by HBT interferometry, are the same as those in all other regions of the source.

We use the fits shown in Fig. 1 to perform the $S/N$ integral and obtain the entropy per particle $S_\pi/N_\pi$ for each centrality bin. These are shown in Fig. 3. The dashed lines show $S_\pi/N_\pi$ calculated with the phase space density values shifted up and down by the estimated systematic error in the extraction of $V_p/\sqrt{\lambda}$ from the HBT analysis. The horizontal lines are static thermal estimates with zero pion chemical potential of the entropy per particle for a pion Bose gas and for the Landau limit, a massless Bose gas. If we assume a pion freeze-out temperature of about 120 MeV, the observed $S_\pi/N_\pi$ for central collisions would imply a pion chemical potential of about $\mu_\pi$ = 41 MeV.

The total pion entropy $dS_\pi/dy$, which is the entropy carried by all freeze-out pions, can be obtained by multiplying $S_\pi/N_\pi$ by the total pion number in a given rapidity interval $dN_\pi/dy$, as calculated by integrating the pion spectrum over momentum, correcting with $\sqrt{\lambda}$ to count only freeze-out pions, and multiplying by 3 to include all pion charge-species. Figure 4 shows $(dS_\pi/dy)/N_p$, the total pion entropy per participant within $|y| < 0.5$. Here the number of participants $N_p$ is estimated from Glauber calculations. We see that for all collisions from peripheral to central, each participant contributes about 6.5 units of entropy to the collision. This result appears to be inconsistent with any change in reaction mechanism with centrality that affects entropy production. We note that Ref. [8] used PHENIX data to estimate the total pion entropy of central Au+Au

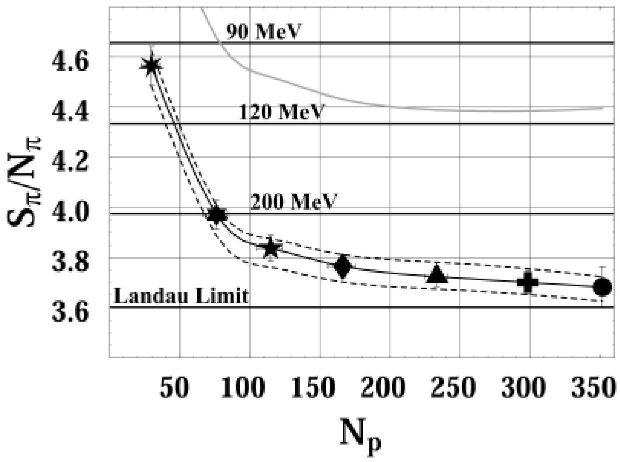


**Fig. 3.** Pion entropy per particle $S_\pi/N_\pi$ in 7 centrality intervals, calculated using fits to spectra and $V_p/\sqrt{\lambda}$. Symbols are defined in text. Dashed envelope curves indicate the systematic error arising from extraction of $V_p/\sqrt{\lambda}$ from the HBT correlation functions. Horizontal lines show static pion-mass Bose gas $S_\pi/N_\pi$ at 90 MeV, 120 MeV and 200 MeV and the Landau massless-boson $S/N$ limit.

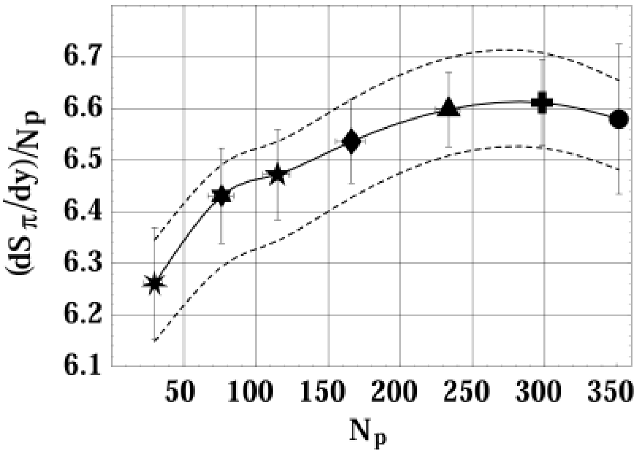


**Fig. 4.** Total pion entropy per participant $(dS_\pi/dy)/N_p$ at mid-rapidity *vs*. participant number $N_p$. Vertical error bars are statistical only, were propagated using covariance matrices, and do not include systematic uncertainties in the Glauber-model estimates of $N_p$, which are indicated by the horizontal error bars. Dashed envelope curves indicate the systematic error arising from extraction of $V_p/\sqrt{\lambda}$ from the HBT correlation functions. Note the suppressed zero; the entropy per participant is surprisingly constant.

collisions and found a value that is in good quantitative agreement with that shown by the rightmost point in Fig. 4.

## Conclusion

We find that the average pion phase space densities <f> observed fpr Au+Au collisions at RHIC are very large, rising to around 50% occupancy at low $p_T$ and high centrality. In particular, <f> is qualitatively larger than those extracted from SPS Pb+Pb collisions [4]. The average freeze-out pion entropy per particle $S_\pi/N_\pi$ is rather small, approaching the Landau limit and implying a pion chemical potential of about 41 MeV at kinetic freeze-out. This leads to a relatively low pion total entropy per participant $(dS_\pi/dy)/N_p$, which is surprisingly constant, about 6.5 ± 0.1. The absence of centrality dependence suggests that there is no per-participant change in initial entropy production with collision centrality, such as might be expected if there was qualitatively different initial dynamics in central and peripheral collisions.

We feel that the low observed entropy in this system and the apparent constancy of total pion entropy per participant should place strong constraints on reaction models. We urge more attention to the entropy implications of the various theoretical models that have been applied to the physics of RHIC collisions.

Supported in part by the Division of Nuclear Sciences of the U.S. Department of Energy.

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