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How to model BEC numerically?

Oleg V. Utyuzh,
Grzegorz Wilk,
Zbigniew Włodarczyk

Abstract A new method for numerical modelling of Bose-Einstein correlations observed in all kinds of multiparticle production processes is proposed.

Key words correlations • BEC • HBT

O. V. Utyuzh✉, G. Wilk
Nuclear Theory Department,
The Andrzej Sołtan Institute for Nuclear Studies,
69 Hoża Str., 00-681 Warsaw, Poland,
Tel.: +48 22/ 553 22 28, Fax: +48 22/ 621 60 85,
e-mail: utyuzh@fuw.edu.pl

Z. Włodarczyk
Institute of Physics,
Świętokrzyska Academy,
15 Świętokrzyska Str., 25-405 Kielce, Poland

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The quantum mechanical in its origin Bose-Einstein correlations (BEC) between identical bosons are our basic source of the knowledge on the space-time characteristics of the hadronizing objects formed in high-energy collisions [4]. Despite the long history of the BEC study the question of their numerical modelling of some Monte-Carlo (MC) event generators remains still open. This is because MC event generators are (by construction) probabilistic in nature. One is therefore, usually using some methods to enhancing certain final configurations of momenta of observed secondaries to enhance the observed amount of pairs of identical bosons with small momenta differences [4]. Here, we would like to advocate a different approach using observation made [5] that symmetrization of the amplitudes of system of identical bosons leads to geometrical distribution of the like-particles in the phase-space cell they occupy (cf. Table 1). There are two possible ways to implement this idea numerically. The first one (see [2, 3] for details) is a kind of afterburner, which can be used with any MC event generator providing us with distribution of particles. The idea is to use the energy-momentum distribution as given by this MC, but to allocate a new charges to all particles (conserving the number of +/– and neutrals as given by MC) in such a way as to effectively put some particles to the same cell (in which they are distributed according to geometrical distribution). Here, we would like to go a step further and present a MC event generator that produces particles according to the Bose-Einstein statistics. This can be done in the following way (cf. Fig. 1). Particles (pions in our case) are selected from the pool of energy W according to a thermal distribution $f(E) = \exp(-E/T)$ and allocated to a phase-space cell given by its 4-momentum p (we call it elementary emitting cell (EEC)). New particles of the same charge and energy are then added to this cell according to $P = P_0 \exp(-E/T)$ weights until the first failure.[1] The process continues with formation of new EEC's until the initial energy W is used up. At the end, one corrects for energy-momentum and charge conservation. In this way: (i) dis-

[1] It should be stressed that such a choice results immediately in Bose-Einstein character of the cell occupancy, $\langle n \rangle = P/(1-P) = 1/[\exp(E/T)/P_0 - 1] = 1/[\exp(E+\mu)/T - 1]$ where $\mu = \ln P_0$.

Table 1. Boltzmann *vs*. BEC.

Boltzmann		BEC
$\Psi_N = \Pi_i \psi_i(x_i)$	SYMMETRIZATION	$\Psi_N = \frac{1}{N!}\sum_{p\{i,j\}} \Pi_i \psi_i(x_j)$
POISSONIAN		GEOMETRICAL
$P_{\text{Boltzmann}}(N) = \frac{\nu^N}{N!}e^{-\nu}$	$\times N!$	$P_{\text{BEC}}(N) = (1-\nu)\nu^N$

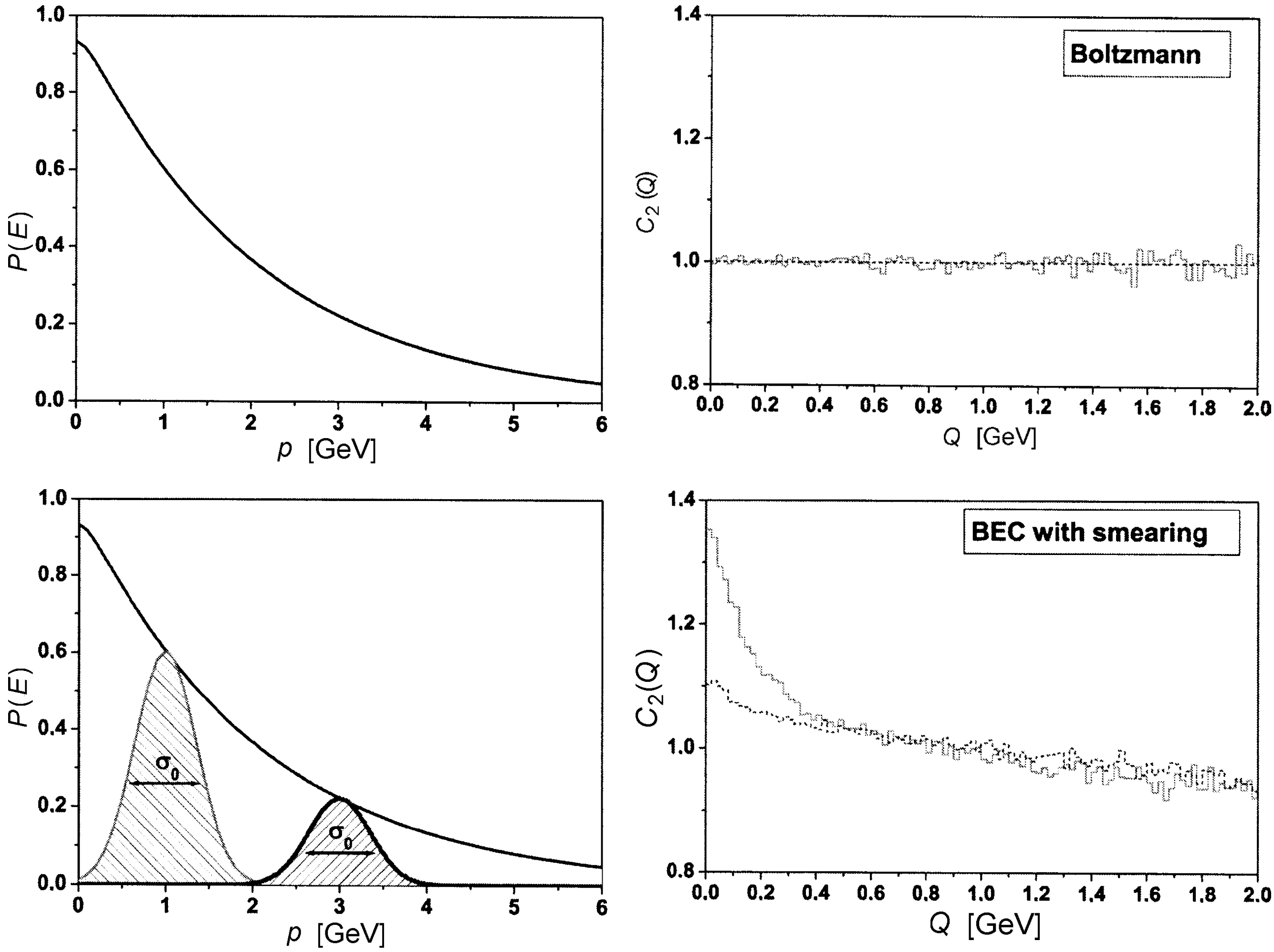

Fig. 1. The particles selected according to thermal distribution (upper-left) show no BEC (upper-right). Correlations occur only (lower-right) when one allows more particles at the same cell (lower-left). The shape of the correlation function $C_2(Q)$ is highly correlated with the spread of the cells in the momentum space.

tribution of EEC's is poissonian and (*ii*) distribution of particles (of the same charge) in a cell is geometrical. These results in a Negative Binominal (NB) form of the over all multiplicity distribution. It turns out that in order to get proper, observed structure of correlation function $C_2(Q)$ (and to be able to describe the experimental data, cf. Fig. 2) one has to allow for some smearing of momenta of like particles allocated to a given EEC (here given, for example, by a Gaussian form with width σ defining the size of average EEC).

In Fig. 2, we show some examples of results for BEC obtained this way (including a comparison with experimental data for BEC observed in e^+e^- annihilations [1]) together with the corresponding results for multiplicity distributions, distributions of the number of formed EEC's and distributions of particles in them. Although the fit to data is quite reasonably, one should treat results presented here only as a necessary first proof that such program is in principle possible to be implemented and that it is flexible to address (so far only some) experimental data.[2]

[2] Our approach is still essentially one-dimensional with transverse momenta entering only via their mean value, $<p_T>$, all produced particles are assumed to be $\pi^{(+,-,0)}$ only (no resonances are considered either), no corrections for any form of final state interaction is attempted. All these factors have to be included before one can use this method to serious description of experimental data for all kind of reactions. We plan to pursue such research in the future.

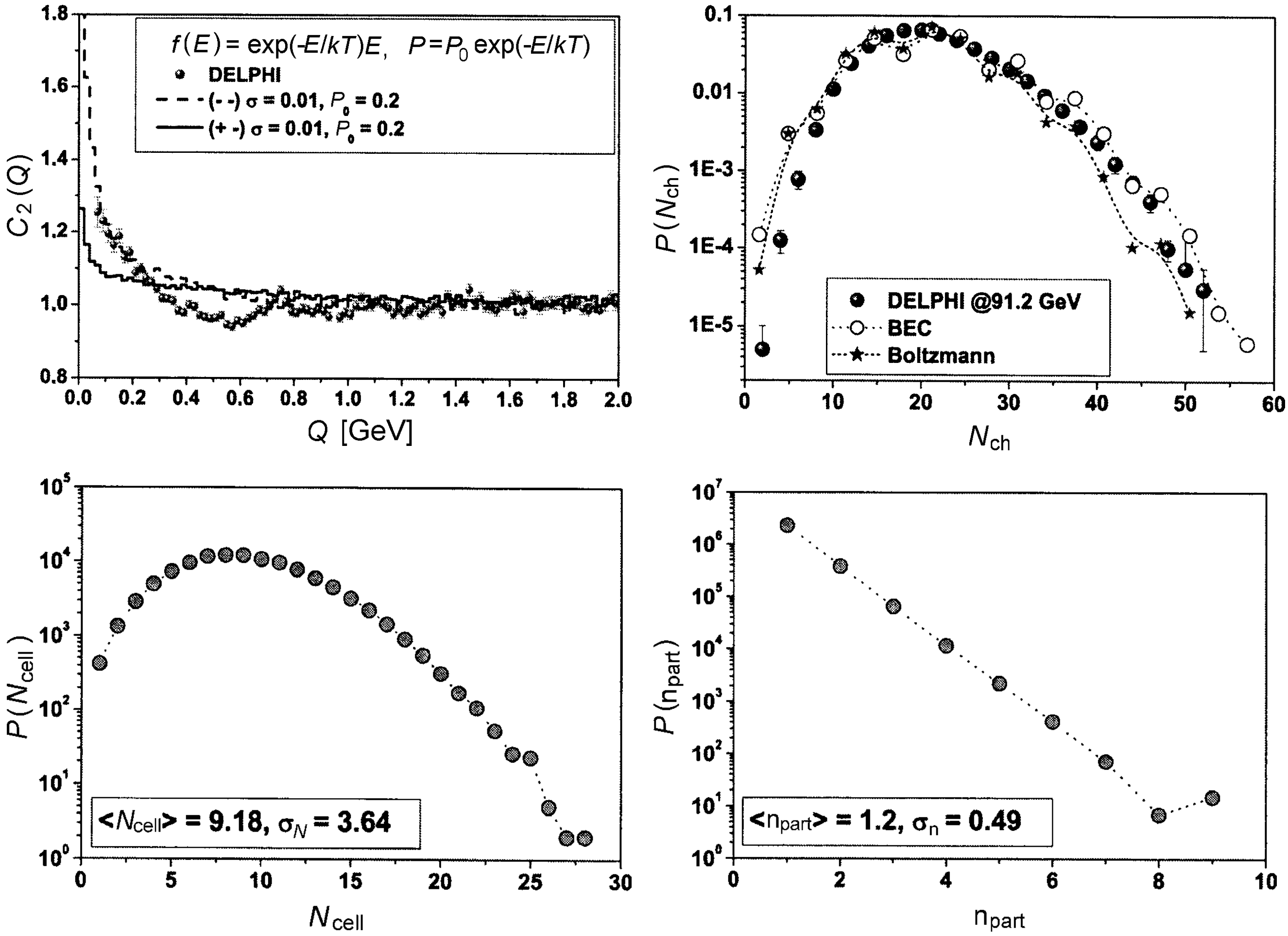

Fig. 2. Example of calculations showing that we can fit e^+e^- data [1] with some choice of parameters (upper-left) and at the same time corresponding multiplicity distribution (upper-right). Lower panels show corresponding distributions of the number of EEC's (which has poissonian distribution shape – left panel) and distribution of a number of particles in EEC (which has geometrical distribution shape – right panel).

We would like to close with the following remarks. The method presented here can be traced to first attempts of describe effect of BEC, only later came space-time descriptions which dominate at present and which use symmetrization of the corresponding wave functions with the space-time integration over some assumed source function (i.e., function describing space-time distribution of points in which finally observed secondaries are produced[3]) [4, 5]. The shape of this source function consists then of the main object of investigation. From our approach, it is obvious that this shape emerges from the necessary spread in the momenta (cf. Fig. 1) and therefore, is not so much given by the dimension of the source as by the length at which particles could still be considered as belonging to a given cell (cf. [4] where such distinction has been also introduced but on different grounds).

[3] By this we understand points after which they no more interact among themselves.

References

1. Abreu P *et al.* (for the DELPHI Collaboration) (1992) Bose-Einstein correlations in the hadronic decays of the Z^0. Phys Lett B 286:201–210
2. Utyuzh O, Wilk G, Włodarczyk Z (2001) Numerical modelling of Bose-Einstein correlations. Phys Lett B 522:273–279
3. Utyuzh O, Wilk G, Włodarczyk Z (2002) Bose-Einstein correlation as reflection of correlations of fluctuations. Acta Phys Pol B 33:2681–2693
4. Weiner R (2000) Boson interferometry in high energy physics. Phys Rep 327:249–346
5. Zalewski K (1999) Bose-Einstein correlations in multiparticle production. Nucl Phys B–Proc, Suppl 74:65–67