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Single particle spectra from information theory point of view

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Abstract It is demonstrated how to obtain the least biased description of a single particle spectra measured in all multiparticle production processes by using an information theory approach (known also as MaxEnt approach). The case of e^+e^- annihilation in hadrons process is discussed in more detail as an example. Comparison between the MaxEnt approach and a simple dynamical model based on the cascade process is presented as well.

Key words information theory • high energy collisions • MaxEnt

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Information theory is nowadays widely used in all branches of sciences to provide the least biased, most probable description of data for which we have only a limited amount of information [3]. The information is defined by means of information entropy, which, in turn, can assume extensive (Shannon) or nonextensive (Tsallis) form, depending on circumstances (see [3, 4, 6, 8] for details and further references). In our case, where we are interested in single particle rapidity distributions, dN/dy, maximization of such entropy (known as MaxEnt procedure [3, 4, 8]) with constraint given by the energy conservation leads to the following form of the probability distribution to find one of the N produced particles of transverse mass $\mu_T = \sqrt{m^2 + <p_T>^2}$ in the longitudinal phase space described by rapidity y such that energy of this particle is $E = \mu_T \cdot \cosh y$ and its longitudinal momentum is $p_L = \mu_T \cdot \sinh y$:

$$p(y) = \frac{1}{N}\frac{dN}{dy} = \frac{1}{Z_q}\exp_q\left(-\beta_q \cdot \mu_T \cdot \cosh y\right). \tag{1}$$

This form is identical with that used in statistical models, but now Z_q and β_q are no longer free parameters to be fitted when comparing with experimental data, but instead are given by the normalization condition and energy conservation constraint[1],

$$\begin{cases} \int\limits_{-Y_m}^{Y_m} dy p(y) = 1 & \text{and} \\ \int\limits_{-Y_m}^{Y_m} dy \mu_T \cdot \cosh y \cdot \left[p(y)\right]^q = \dfrac{\kappa_q \cdot W}{N} & \end{cases} \tag{2}$$

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[1] Here $\exp_q(x/\Lambda) = \left[1+(1-q)x/\Lambda\right]^{1/(1-q)} \xrightarrow{q\to 1} \exp(x/\Lambda)$.

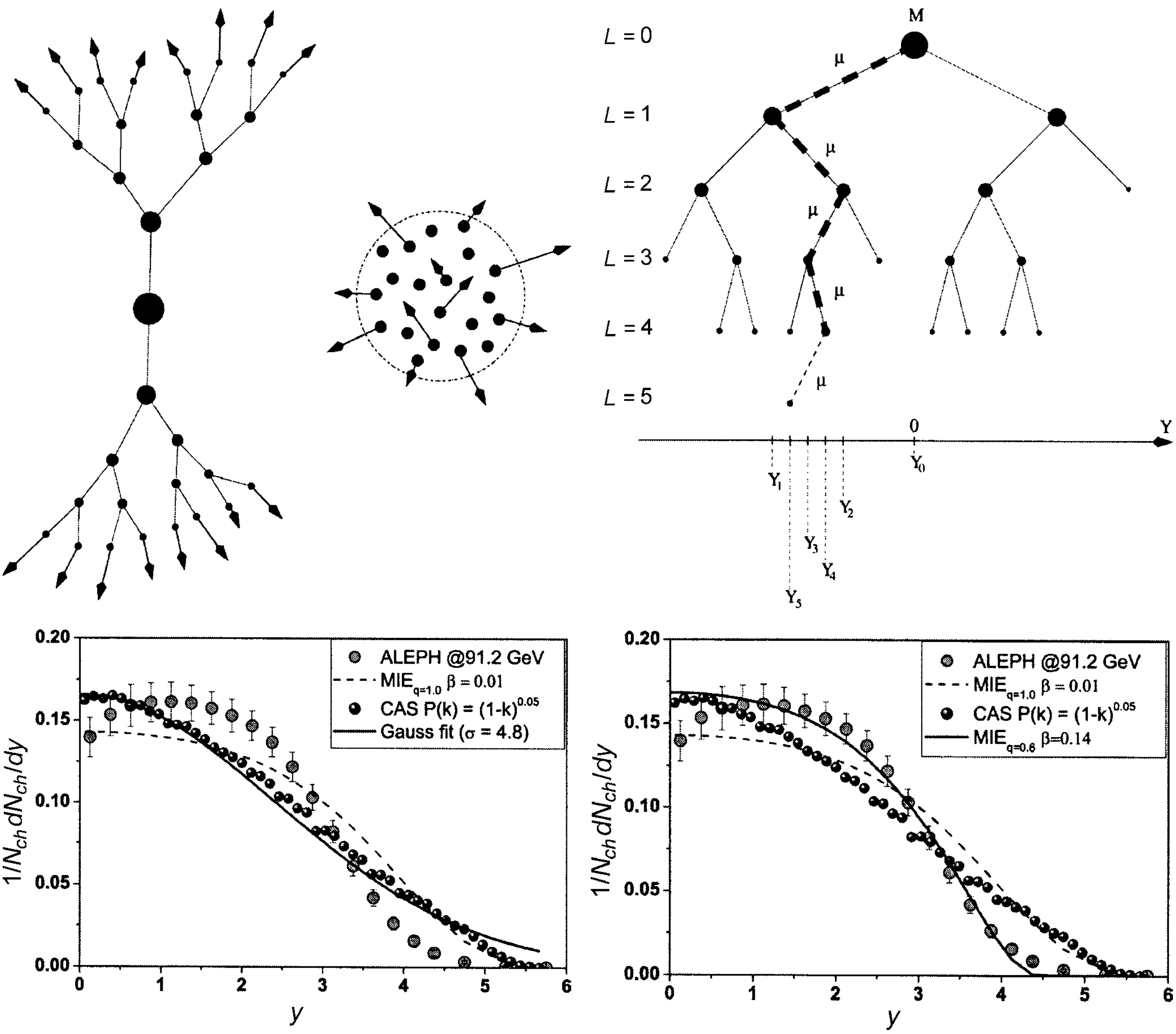

Fig. 1. Upper-left panel: visualization of differences between sequential cascade hadronization (CAS) [7] and the instantaneous one described by MaxEnt [8]. Upper-right panel: closer look at the way dN/dy arises in CAS (resembling random walk in y-space). Lower panels: results of calculations of dN/dy using CAS and MaxEnt and compared with e^+e^- annihilation data by ALEPH [1]. Left panel – comparison with CAS model giving the same multiplicity [7] (it can be extremely well approximated by Gaussian fit) and with MaxEnt for $K_q = 1$ and $q = 1$ (it does not fit data). Right panel – Gaussian fit has been replaced by MaxEnt fit with total inelasticity $K_q = 1$ and with varying q; for $q = 0.6$ we obtain quite good agreement with data.

(where $\pm Y_m$ are the maximal rapidities available in the rest frame of a hadronizing source, see [4, 8] for details). As is demonstrated in [4, 5] with $q > 1$ (responsible for dynamical fluctuations, see [9, 10] and q-inelasticity κ_q (connected to true inelasticity K_q defining a fraction of the original available energy W used for the production of secondaries, $K_q = \kappa_q/(3 - 2q)$, cf. [4]) as the only parameters we can describe all data for pp and $p\bar{p}$ collisions as well as recent data for nuclear collisions. Here, we show in Fig. 1 that we can also fit fairly well e^+e^- annihilation data for which, by definition, $K_q = 1$ (the whole energy must be used for the production, there are no leading particles) and q remains the only free parameter. It turns out that in this case $q < 1$.[2] This fact indicates (see [10]) that in e^+e^- processes (where $K_q = 1$) one cannot obtain equilibrium state because the corresponding temperature parameter depends now on energy: $T = 1/\beta_q = T_0 - (1 - q)E$ and system is highly influenced by the conservation of energy constraint. In Fig. 1 we have also shown attempts to fit the same data using the well defined sequential decay (cascading) hadronization developed in [7] instead of instantaneous hadronization.[3] Cascade here is defined by the sequence of decay processes, $M_l \rightarrow [M^{(1)}_{l+1} = k_1 M_l] + [M^{(2)}_{l+1} = k_1 M_l]$ (proceeding until $M_l > 2\mu_T$) in which all dynamics is summarily described

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[2] To be contrasted with $q > 1$ values needed to describe the p_T distributions in the same processes instead [2].

[3] Cascade could be regarded as a viable alternative because of the *a priori* cascade character of the quark→gluon and gluon→quark-antiquark processes preceding hadronization expected to be present in e^+e^- annihilations [1].

by distribution $P(k)$ of decay parameters $k_i \in (0,1)$ (such that in each vertex $k_1 + k_2 < 1$, see [7] for details). It is easy to realize that in this case dN/dy is given by a kind of random walk in rapidity space, which, in turn, results in its Gaussian-like shape (cf. Fig. 1). However, one cannot find any reasonable values of decay parameters k_i to fit data with such distribution, whereas MaxEnt in its nonextensive version can do it with $q = 6$. The remaining small discrepancies for small and large rapidities indicate therefore a need for some additional information to be incorporated here (see, for example, discussion in [5]). When introduced, it could then be again checked against experimental data. In this way, we could always obtain good description of data with only a minimal number of assumptions, i.e., with minimal information content [3, 8]. This is the main advantage of the method presented here and in our opinion it deserves further detailed investigations by using a wide spectrum of the available multi-particle production data of all kinds.

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