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Proposition of numerical modelling of BEC

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Abstract We propose an extension of the numerical method to model effect of Bose-Einstein correlations (BEC) observed in hadronization processes which allows for calculations not only the correlation functions $C_2(Q_{\rm inv})$ (one-dimensional), but also the corresponding $C_2(Q_{x,y,z})$ (i.e., three-dimensional). The method is based on the bunching of identical bosonic particles in elementary emitting cells (EEC) in phase space in a manner leading to proper Bose-Einstein form of distribution of energy (this was enough to calculate $C_2(Q_{\rm inv})$). To obtain $C_2(Q_{x,y,z})$ as well as one has to add to it the symmetrization of the multiparticle wave function to properly correlate space-time locations of produced particles with their energy-momentum characteristics.

Key words hadronization • correlations • BEC

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Recently, we have proposed a novel numerical method to model the effect of the Bose-Einstein correlations (BEC) observed in hadronization processes, (see [3, 4] for details and for other references concerning generalities of BEC). This method allows for calculation of the so-called invariant (i.e., one-dimensional) correlation function $C_2(Q_{\rm inv})$. Referring to [3, 4] for motivation and justification and all other details, let us list here only the main points proposed so far:

– Bosonic character of produced secondaries demands that they are produced in bunches (named by us elementary emitting cells – EEC's) in momentum space [2].
– The problem then is how to model production of EEC. It is done by adding to some preselected particle with energy E_1 (according to some distribution $f(E)$, which in our numerical calculations was taken in the Boltzmann form, $f(E) \sim \exp(-E/T)$, with temperature T being a parameter) other particles of the same energy with some probability $P = P_0 \cdot \exp(-E_1/T)$ (where P_0 is another parameter) up to the first failure. Afterwards one starts to build another EEC and continues as long as total energy allows.
– Such procedure results in a number of EEC's (distributed according to Poisson distribution and following boltzmanian energy level distribution)

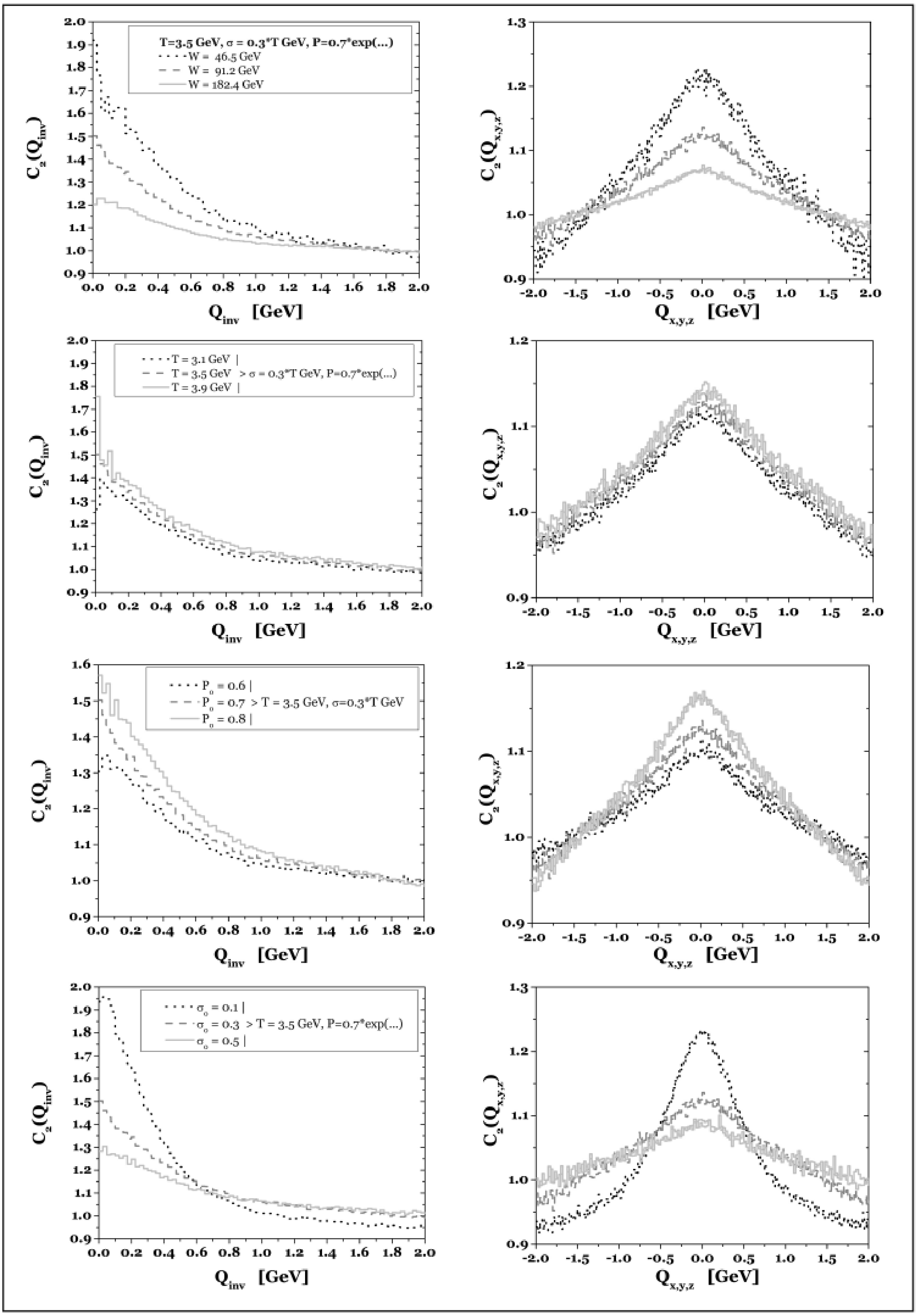

Fig. 1. An example of the results obtained for $C_2(Q_{\mathrm{inv}})$ (left column) and the corresponding $C_2(Q_{x,y,z})$ (right column, for parameters used here all $C_2(Q_{x,y,z})$ are the same). Calculations were performed assuming spherical source $\rho(r)$ of radius $R = 1$ fm (spontaneous decay was assumed, therefore there is no time dependence) and spherically symmetric distribution of $p_{x,y,z}$ components of momenta of secondaries p. Energies were selected from $f(E) \sim \exp(-E/T)$ distribution. The changes investigated are – from the top to the bottom: different energies of hadronizing sources, different temperatures T, different values of parameter P_0 and different spreads $\sigma = \sigma_0 \cdot T$ of the energy in EEC.

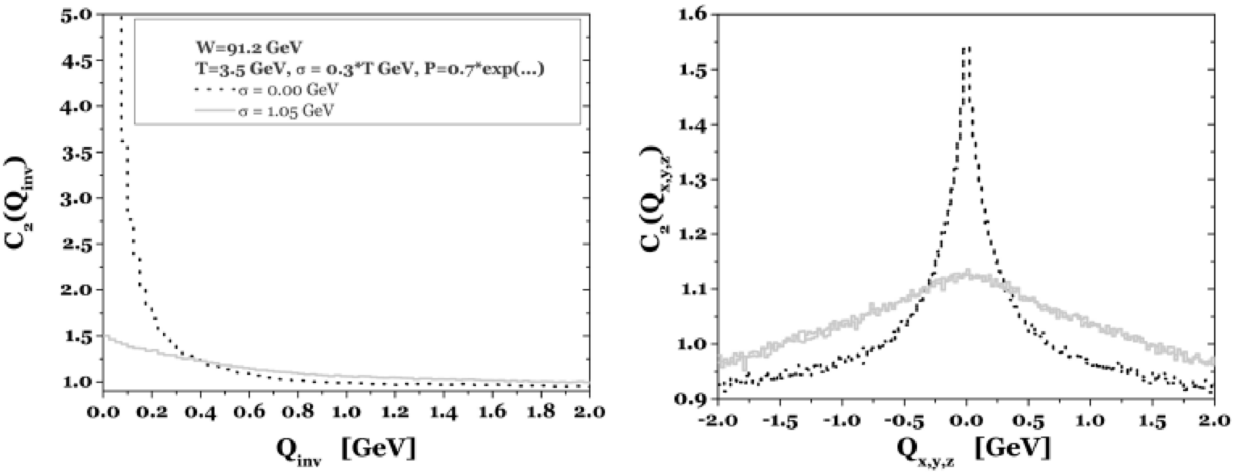

Fig. 2. Demonstration of a great role played by parameter σ defining the energy spread of particles in the EEC.

with a number of identical particles in each of them distributed according to geometrical (Bose-Einstein) distribution with energy level distribution following Bose-Einstein statistics.

– In this way, one gets $C_2(Q_{\rm inv}) = 2$, but only in the first bin, i.e., for $Q_{\rm inv} = 0$. In order to get the characteristic shape of $C_2(Q_{\rm inv})$ function, one has to allow for particles in each EEC to have slightly different energies, for example, distributed around E_1 according to the gaussian distribution with the width σ, which is our next and last parameter at this stage. It should be stressed at this point that in the field theory approach to BEC, as, for example, that presented in [1], $\sigma = 0$ corresponds to infinite source and commutation relations with Dirac-delta functions, whereas non-zero values of sigma arise for finite space-time extensions of the hadronizing sources.

To obtain correlation functions $C_2(Q_{x,y,z})$ as well (i.e., three-dimensional, calculated for different components of the differences of particle momenta Q), one has to proceed further. What we propose here is the following:

– Change of $Q_{\rm inv}$ to $\vec{Q}$ means that we shall now be sensitive not only to the overall spatial difference between particle production points r, but to the whole vector $\vec{r}$ as well. Then we have to assume that particles are produced from some spatial region and that density of production points is given by some function (our additional input) $\rho(x,y,z)$.[1)]

– Momenta of each particle in a given EEC, p_i, obtained in the first part of algorithm must now be decomposed into their components, $(p_x^{(i)},p_y^{(i)},p_z^{(i)})$. To do this, let us observe that BE statistics demands that multiparticle wave functions must be symmetrized accordingly and this results in correlations between production points represented by $\rho(x,y,z)$ and momenta $(p_x^{(i)},p_y^{(i)},p_z^{(i)})$. Denoting by $\delta_{i=x,y,z}$, the corresponding differences in position this correlation is given (in the plane wave approximation) by the known $1 + \cos(\delta_i{\cdot}Q_i)$ term.

– We proceed then in the following way. In each EEC we select the $(p_x^{(i)},p_y^{(i)},p_z^{(i)})$ for the first, $i = 1$ particle in some prescribed way (here isotropically but one can introduce at this moment p_T cutting or something else as well) and then establish $(p_x^{(i)},p_y^{(i)},p_z^{(i)})$ for every one of additional particles, $i \geq 2$, in such a way that $\cos(\delta_i{\cdot}Q_i) \leq 2{\cdot}Rand\text{-}1$ where $Rand$ is a random number uniformly chosen from interval (0,1).

This leads us to results presented in Figs. 1 and 2. We regard them as very promising, but we are aware of the fact that our proposition is still far from being complete. To start with, one should allow for time-dependent emission by including $\delta E{\cdot}\delta t$ term in the cos(…) above. The other is the problem of Coulomb and other final state interactions. Their inclusion is possible by using some distorted wave function instead of the plane waves used here. Finally, so far only two particle symmetrization effects have been accounted for: in a given EEC all particles are symmetrized with the particle number i being its seed, they are not symmetrized between themselves. To account for this one would have to add other terms in addition to the cos(…) used above – this, however, would result in a dramatic increase of the calculational time.

To summarize – we propose a new method of numerical modelling of hadronization events in such way as to respect the bosonic character of produced secondaries and, therefore, leading to BEC. It seems to converge in some sense to the proposition presented long time ago in [5], which was, however, in practice impossible to be implemented. The only hope that it could work now is that in our case symmetrization is within a given EEC and not for the whole bunch of particles produced. Therefore, the number of terms involved is rather limited, whereas in [5] the whole source had to be symmetrized at once. But the effect of including at least terms when symmetrization between, say, particles 2 and 3 are added to the already present symmetrization between 1 and 2 and 1 and 3, must be carefully investigated and estimated before any conclusion is to be reached.

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[1)] For the time being, we are assuming for simplicity that hadronization is instantaneous, so ρ is time-independent.

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