

A statistical model analysis of yields and fluctuations in 200 GeV Au-Au collisions

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Abstract We show that the simultaneous measurement of yields and fluctuations is capable of falsifying and constraining the statistical hadronization model. We show how such a measurement can test for chemical non-equilibrium, and distinguish between a high temperature chemically equilibrated freeze-out from a supercooled freeze-out with an over-saturated phase space. We perform a fit, and show that both yields and fluctuations measured at RHIC 200 GeV can be accounted for within the second scenario, with both the light and strange quark phase space saturated significantly above detailed balance. We point to the simultaneous fit of the K/π fluctuation and the K^*/K^- ratio as evidence that the effect of hadronic re-interactions after freeze-out is small.

Key words fluctuations • heavy-ion collisions • statistical models

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The statistical hadronization model (SHM) [12, 18, 22] has been extensively applied to the study of soft particle production in hadronic systems. When it includes the full spectrum of hadronic resonances [14], the SHM can describe quantitatively the abundances of all light and strange hadrons produced in heavy-ion collisions.

The ability of the SHM to describe not just averages, but event-by-event multiplicity fluctuations has, however, not been widely investigated, and its applicability is currently a matter of controversy. Event-by-event particle fluctuations have elicited theoretical [15–17, 23] and experimental interest [1, 4], both as a consistency check for existing models [15, 16], as a test of hadron gas equilibration and the presence of resonances [17], and as a way to search for new physics [16].

This study investigates whether the SHM can explain both yields and fluctuations with the same parameters. We illustrate the use of both yields and fluctuations as a probe capable of differentiating between freeze-out scenarios. We then perform a fit to preliminary 200 GeV RHIC experimental data, and discuss the results in the context of chemical (non)equilibrium within the RHIC system, the freeze-out temperature and the presence (or absence) of hadronic reinteraction after freeze-out.

The statistical hadronization model assumes that particles are produced according to a probability determined by their phase space density. The first and second cumulants of this probability distribution give, respectively, the average value (over all events) of the desired observable, and its event-by-event fluctuation.

Conserved quantities can be treated in several ways, appropriate to different experimental situations: if the totality of the system is observed, then conserved quantities cannot fluctuate. If a small fraction

equilibrated with the rest of the system (the “bath”) is observed, then conserved quantities will fluctuate, event-by-event. Rigorous conservation is known as the microcanonical ensemble, while allowing energy and other conserved quantities to fluctuate between the system and the bath leads, respectively, to the canonical and grand canonical (GC) ensembles. As shown recently [6], all fluctuations are ensemble-specific even in the thermodynamic limit.

In this work, we use the GC ensemble, implemented in a publicly-available software package [27, 30] to calculate fluctuations and yields. We motivate this choice by the fact that RHIC experiments observe the mid-rapidity slice of the system, comprising roughly 1/8 of the total multiplicity, an appropriate fraction for a GC prescription. Boost invariance, a good symmetry around mid-rapidity, links this rapidity slice with a sector in configuration space. If the system observed at RHIC is a nearly ideal fluid, the matter created in this space should be in equilibrium, grand-canonically, with the unobserved regions. If freeze-out temperature throughout observed space is approximately constant, the GC ensemble should be able to describe both yields and fluctuations [9, 28].

The final state yield of particle is computed by adding the direct yield and all resonance decay feed-downs.

$$(1) \quad \langle N_i \rangle_{\text{total}} = \langle N_i \rangle + \sum_{\text{all } j \rightarrow i} B_{j \rightarrow i} \langle N_j \rangle$$

Here $\langle N_i \rangle$ is the ensemble average of the i -th particle (resonance) multiplicity at chemical freeze-out and $B_{j \rightarrow i}$ is the branching ratio of the $j \rightarrow i$ decay. The fluctuation $\langle \Delta X^2 \rangle = \langle X^2 \rangle - \langle X \rangle^2$ also has two parts

$$(2) \quad \langle \Delta N_{j \rightarrow i}^2 \rangle = B_{j \rightarrow i} (1 - B_{j \rightarrow i}) \langle N_j \rangle + B_{j \rightarrow i}^2 \langle \Delta N_j^2 \rangle$$

assuming no primordial correlation between i and j .

The fluctuation of a ratio N_1/N_2 (a useful experimental observable, since volume fluctuations cancel out event-by-event) can be computed from the fluctuation of the denominator and the numerator [15] ($\sigma_X^2 = \langle \Delta X^2 \rangle / \langle X \rangle$):

$$(3) \quad \sigma_{N_1/N_2}^2 = \frac{1}{\langle N_2 \rangle} (D_{11} + D_{22} - 2D_{12})$$

$$(4) \quad D_{11} = \frac{\langle N_2 \rangle}{\langle N_1 \rangle} F_1, \quad D_{22} = F_2$$

$$(5) \quad D_{12} = \sum_{j \rightarrow 1,2} B_{j \rightarrow 1,2} \frac{\langle N_j \rangle}{\langle N_1 \rangle}$$

$$(6) \quad F_i = \frac{\langle \Delta N_i^2 \rangle_{\text{direct}}}{\langle N_i \rangle} + \sum_{j \rightarrow 1,2} \langle N_{j \rightarrow i}^2 \rangle \frac{\langle N_{j \rightarrow i} \rangle}{\langle N_i \rangle}$$

Note the appearance of a negative correlation term D_{12} , which arises due to the correlation between N_1 and N_2 stemming from a common resonance feed-down ($\Delta \rightarrow \pi\pi$ will be a source of correlation between N_ρ and N_π).

The average yields and fluctuations of hadrons themselves can be calculated by a textbook method given a grand canonical description of the phase space; For a hadron with an energy $E_p = \sqrt{p^2 + m^2}$, the average number, and event-by-event fluctuation will be given by

$$(7) \quad \langle N_i \rangle = gV \int \frac{d^3p}{(2\pi)^3} \frac{1}{\lambda_i^{-1} e^{E_p \beta} \pm 1}$$

$$(8) \quad \langle \Delta N_i^2 \rangle = \lambda_i \frac{\partial}{\partial \lambda_i} \langle N_i \rangle$$

where the upper sign is for Fermions and the lower sign is for Bosons. Here, λ_i is the fugacity of particle i , see Eq. (10). These formulae can be efficiently calculated through a Bessel function expansion [12, 27, 30].

Note that the series for fluctuations starts diverging at $\lambda_i \rightarrow e^{m_i/T}$. For $\lambda_i e^{m_i/T}$, therefore $\langle N_i \rangle$ remains finite but

$$(9) \quad \lim_{\epsilon \rightarrow 0} \langle \Delta N_i^2 \rangle \propto \epsilon^{-1/2}$$

The parameter λ_i corresponds to the particle fugacity, related to the chemical potential by $\lambda_i = e^{m_i/T} = 1 - \epsilon$. Provided the law of mass action holds, it should be given by the product of charge fugacities (flavor, isospin etc.). It is then convenient to parametrize it in terms of equilibrium fugacities λ_i^{eq} and phase space occupancies γ_i . For a hadron with $q/(\bar{q})$ light quarks, $s(\bar{s})$ strange quarks and isospin I_3 the fugacity is then

$$(10) \quad \lambda_i = \lambda_i^{\text{eq}} \gamma_q^{q+\bar{q}} \gamma_s^{s+\bar{s}}, \quad \lambda_i^{\text{eq}} = \lambda_q^{q-\bar{q}} \lambda_s^{s-\bar{s}} \lambda_{I_3}^{I_3}$$

If the system is in chemical equilibrium, then detailed balance requires that $\gamma_q = \gamma_s = 1$. In an expanding system, however, the condition of chemical equilibrium no longer holds. Kinetically, this occurs because collective expansion and cooling will make it impossible for endothermic and exothermic reactions, or for creation and destruction reactions of a rare particle, to be balanced. Provided the system remains in local thermal equilibrium, λ_i can still be used as a Lagrange multiplier for the particle density, and the first and second cumulants can be calculated from the partition function the usual way [20, 31]. However, in this case in general $\gamma_q \neq 1$, $\gamma_s \neq 1$.

This picture becomes particularly appropriate if the expanding system undergoes a fast phase transition from a QGP to a hadron gas. In this case, chemical non-equilibrium [24] and supercooling [10] can arise due to entropy conservation: by dropping the hadronization temperature to ~ 140 MeV and oversaturating the hadronic phase space above equilibrium ($\gamma_q \sim 1.5$, $\gamma_s \sim 2$), it is possible to match the entropy of a hadron gas with that of a system of nearly massless partons [24].

Fits to experimental data at both SPS [20] and RHIC energies [25] indeed support these values of $\gamma_{q,s}$ when these parameters are fitted. Moreover, best fit $\gamma_{q,s} > 1$ arises for a critical energy [21] (corresponding to the energy of the K/π “horn” [13]) and system size [26], just as expected from the interpretation of γ_q as a manifestation of a phase transition. However, the fits performed in [21] have not been able to rule out equilibrium models (at SPS and RHIC the difference in statistical

significance between equilibrium and non-equilibrium is $\sim 20\%$).

In the absence of contrary evidence, equilibrium models are usually preferred for their smaller number of free parameters [5, 8]. Equilibrium freeze-out temperature varies between fits, ranging from 155 MeV [21] to 177 MeV [8].

Both scenarios are physically reasonable, can describe the data, and would be instrumental in our understanding of hadronic matter if proven correct. In particular, the HBT puzzle suggests we lack understanding of the last stages of the fireball evolution. Non-equilibrium is useful in this respect, since it affects both system volume [21, 29] and emission time [10].

The reason both equilibrium and non-equilibrium are compatible with data is that in a fit to yields the non-equilibrium phase space occupancies γ_s and γ_q correlate with freeze-out temperature [21], making a distinction between a $T = 170$ MeV equilibrated freeze-out ($\gamma_q = 1, \gamma_s \leq 1$) scenario and a supercooled scenario where $\gamma_{q,s} > 1$ problematic.

A related ambiguity is the difference between chemical freeze-out (where particle abundances are fixed) and thermal freeze-out (where particles cease to interact). Equilibrium models generally assume a long phase between these two points, which would alter considerably the multiplicity of directly detectable resonances. In a non-equilibrium supercooled freeze-out, on the other hand, it is natural to assume that particle interaction after emission is negligible. Once again, a reliable way to probe the extent of the reinteraction would be instrumental for our understanding of how the fireball produced in heavy-ion collisions breaks up.

We have recently shown [28] that event-by-event fluctuations can be used to solve both of the dilemmas discussed above. From the equations in the previous section, it is clear that the dependence of fluctuations of T and γ_q is different, allowing us to decouple these two variables. A higher temperature tends to decrease fluctuations w.r.t. the Poisson value expected from Boltzmann statistics ($\sigma = 1$), since it introduces greater correlations due to an increased contribution of resonances (D_{12} in Eq. (3)). Vice-versa, increasing γ_q will rapidly increase fluctuations of quantities related to pions, due to the fact that at $\gamma_q > 1\lambda_\pi$ rapidly approaches $e^{m_\pi/T}$, giving fluctuations an extra boost w.r.t. yields as per Eq. (9).

In addition, comparing fluctuations to directly detected resonances probes the interval between chemical and thermal freeze-out. Consider, for example, the K^+/π^- fluctuation. The top and the bottom terms in this ratio are linked by a large correlation term due to the $K^{*0}(892)$ decay. This correlation probes the K^{*0} abundance at chemical freeze-out, since subsequent rescattering/regeneration does not alter the fact that the K^{*0} decay produced a π^+ and a K^- . On the other hand, a direct measurement of the K^*/K^- ratio through invariant mass reconstruction measures the K^* abundance at thermal freeze-out, after all rescattering. Hence, comparing the K/π fluctuation to the K^*/K^- ratio provides a gauge for effect of the hadronic reinteraction period on particle abundances. In particular, a sudden freeze-

out model should describe both with the same parameters.

The main experimental problem with fluctuation measurements is the vulnerability to effects resulting from limited detector acceptance. This difficulty can be lessened, to some extent, by considering ‘‘dynamical’’ fluctuations, obtained by subtracting a ‘‘static’’ contribution which should be purely Poisson in an ideal detector. The exact nature of the subtraction varies from collaboration to collaboration, for example [11] using

$$(11) \quad \sigma^{\text{dyn}} = \sqrt{\sigma^2 - \sigma_{\text{stat}}^2}$$

σ_{stat} , usually obtained through a Mixed event approach [23], includes a baseline Poisson component, which for a ratio N_1/N_2 can be modeled as

$$(12) \quad \sigma_{\text{stat}}^2 = \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle}$$

as well as a contribution from detector efficiency and kinematic cuts. Provided certain assumptions for the detector response function hold (see appendix A of [23]), subtracting σ_{stat} from σ should yield a ‘‘robust’’ detector-independent observable.

When using measured σ^{dyn} in fits to experimental data, the data-sample should include σ^{dyn} , particle yields (which are needed to determine the Poisson contribution to σ_{stat} as per Eq. (12)) and particle ratios.

We have performed a fit incorporating all ratios from [5], with the exception of the Δ^{++}/p , which the STAR collaboration has taken back, and the $\bar{\Omega}/\Omega$, measured by STAR to be above unity. (The latter measurement is a priori incompatible with statistical production [7].) In addition, we have included the preliminary value for the K/π event-by-event fluctuation measured by STAR [11], as well as the published yield for ϕ [3] and π^- [2].

Our fit parameters include the normalization (hopefully related to the system ‘‘volume’’ at chemical freeze-out), temperature, λ_{q,s,I_3} and $\gamma_{q,s}$. We also require, by implementing them as data-points, strangeness, charge and baryon number conservation ($\langle s - \bar{s} \rangle = 0, \langle Q \rangle / \langle B \rangle = (\langle Q \rangle / \langle B \rangle)_{\text{Au}} = 0.4$).

The result is shown in Fig. 1. The fit gives an adequate description of all particle yields, including the resonance K^*/K^- and $\Lambda(1520)/\Lambda$. It can also, adequately describe

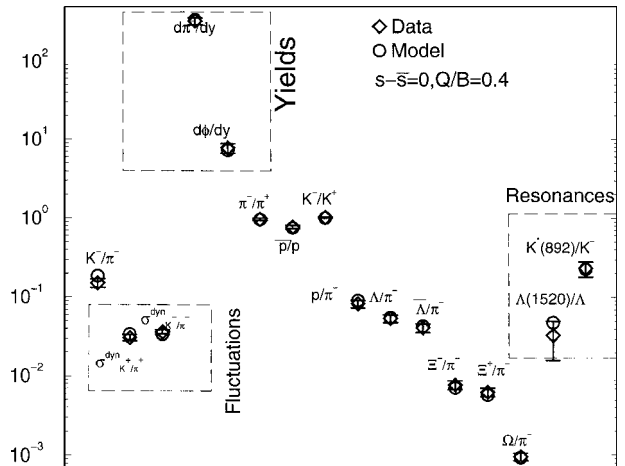


Fig. 1. Fit of preliminary 200 GeV data, including the K^\pm/π^\pm fluctuations and the $K^*(892)$ and $\Lambda^*(1520)$ resonance.

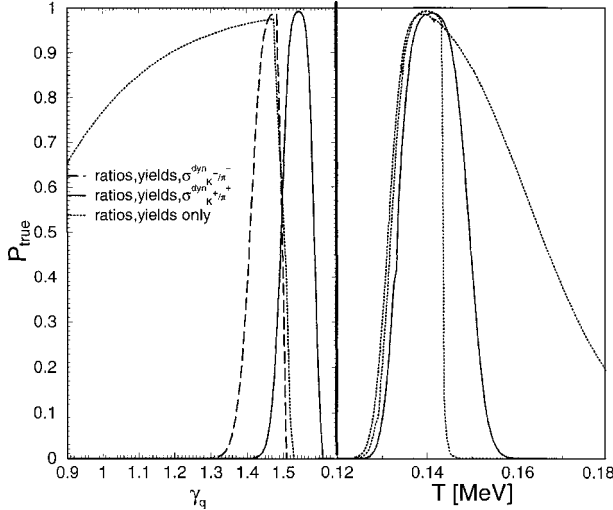


Fig. 2. Temperature and γ_q statistical significance (P_{true}) profiles for the fit in Fig. 1. At each point in the abscissa a fit is performed varying all fit parameters except the one on the abscissa. P_{true} is calculated from the best fit parameters.

the event-by-event fluctuations of of K^+/π^+ or K^-/π^- , but not the difference between them. Describing the difference within the SHM is impossible without introducing a sizable isospin fugacity $\lambda_{I_3} \sim 0.96$ which is in turn excluded by ratios such as π^+/π^- and Q/B . We wait for published results, and systematic error computation before studying this discrepancy further.

Figure 2 shows that the correlation between T and γ_q disappears when fluctuations are taken into account, and that the fit tightly constrains γ_q well above the equilibrium value, forcing temperature to be about 140 MeV, in good agreement of the prediction of the supercooled hadronization scenario. In accordance with this scenario, our model can describe both the K/π fluctuation and the directly observed K^*/K^- value with the same statistical parameters, as is expected for a system where hadronic reinteraction (the phase between chemical freeze-out and thermal freeze-out) is negligible.

Figure 3 confirms that fluctuations are indeed responsible for the tight constraining of T and γ_q . The best fit at light quark equilibrium misses $\sigma_{K/\pi}^{\text{dyn}}$ by several standard deviations. Introducing exact conservation for strangeness within the observed window would increase the discrepancy by reducing σ_K [6]. It is only through $\gamma_q > 1$ that $\sigma_{K/\pi}^{\text{dyn}}$ increases until it becomes compatible with the experimental value.

Conclusions

In conclusion, we have used preliminary experimental data to show that, at 200 GeV, the equilibrium model is unable to describe both yields and fluctuations within the same statistical parameters. The non-equilibrium model, in contrast, succeeds in describing almost all of the yields and fluctuations measured so far at RHIC, with the parameters expected from a scenario where non-equilibrium arises through a phase transition from a high entropy state, with supercooling and oversaturation of phase space. We await more published data to

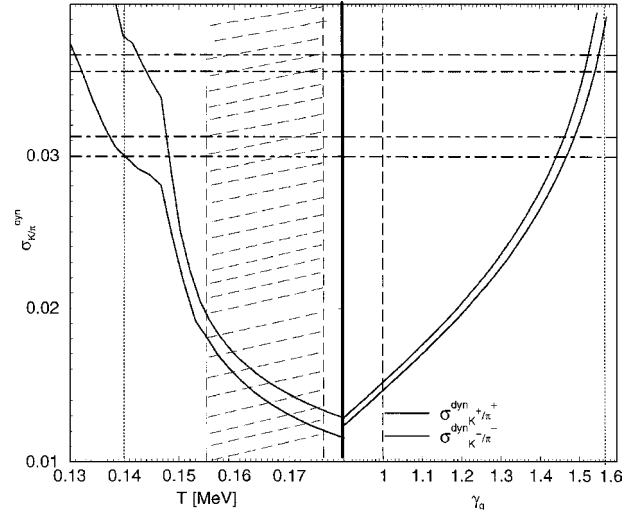


Fig. 3. Sensitivity of the K/π fluctuation ($\sigma_{K/\pi}^{\text{dyn}}$) to γ_q and temperature for the fit in Fig. 1. At each point in the abscissa, a fit is performed (only yield data-points used) varying all fit parameters except the one on the abscissa. $\sigma_{K/\pi}^{\text{dyn}}$ is calculated using the best fit parameters. Dot-dashed lines refer to the experimental limits [11] for the σ_{K^+/π^+} and σ_{K^0/π^0} . Dotted vertical lines give published non-equilibrium best fit parameters [21], while the long-dashed line and shaded area gives published equilibrium best fit [5, 8, 21].

determine whether the non-equilibrium model is really capable of accounting for both yields and fluctuations in all light and strange hadrons produced in heavy-ion collisions.

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