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Colored-noise-induced anomalous transport in periodic structures

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Abstract Colored-noise-induced anomalous transport phenomena of overdamped particles in a tilted periodic sawtooth potential driven by a nonequilibrium three-level noise and an additive thermal noise are considered analytically. All the physical results discussed are computed by means of exact formulas. It is demonstrated that the particles exhibit anomalous transport properties, such as, multiple change in the sign of the particle flow, absolute negative mobility, negative differential mobility, hypersensitive transport, and hypersensitive differential response. The necessary conditions for various anomalous transport properties are found. It is established that in certain parameter regions an increase in noise parameters (noise-flatness, correlation time, temperature) can facilitate the conversion of noise energy into mechanical work – i.e., the dependence of the efficiency of energy transformation on noise parameters exhibits a bell-shaped (resonance) form. Some possible applications to fluctuation-induced separation of particles as well as to the amplification of small signals are also discussed. Our results provide some new perspectives to support elaboration of a model of interaction between plasma beams and construction materials in plasma focus devices.

Key words nonequilibrium fluctuations • noise-induced transport • hypersensitive transport • ratchet system • anomalous mobility

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Introduction

Experiments with plasma focus devices serve a special interest for investigations on the interaction of pulsed plasma streams with materials [21]. Among the various tasks of scientific activity in this field there is one of particular interest, namely, the elaboration of a phenomenological model of interaction between dense plasma beams and construction materials. Two sweeping generalization can be made about the processes between plasma and materials. They are intrinsically nonlinear and operate in noisy environment far from thermo-dynamic equilibrium.

Within the past two decades the behavior of nonequilibrium systems depending on fluctuations (noise) has deserved considerable attention. Some phenomena recently discovered have brought about the idea that noise can induce order in nonlinear nonequilibrium systems. Stochastic resonance [3], noise-induced phase transitions in spatially extended systems [4], stochastic transport in ratchets [22], noise-induced spatial patterns [4], noise-supported traveling structures in excitable media [4], absolute negative mobility [2], hypersensitive response [6], and giant amplification of diffusion [23] are a few new phenomena in this field. Active analytical and numerical investigations of various models in this field were stimulated by their possible applications in chemical physics, molecular biology, nanotechnology and for separation techniques of nano-objects [22].

The most productive abstraction of noise-like influence from the environment is the case of Gaussian white noise. There, the problem is considerably simplified as the corresponding transition probability densities satisfy the Fokker-Planck equation, which in certain specific cases can be solved exactly. However, various physical effects are induced only by colored noise, which has a non-zero correlation time, and in these cases the white noise approximation represents an oversimplification. Theoretical investigations [15] indicate that colored-noise-induced nonequilibrium effects are sensitive to noise flatness, which is defined as the ratio of the fourth moment to the square of the second moment of the noise process. Although its significance is obvious, the role of the flatness of fluctuations has not received due attention. The latter circumstance inspired us to model the nonequilibrium fluctuations as a three-level telegraph process that may be called trichotomous noise [15]. Notably, the flatness of a trichotomous noise can have any value from 1 to ∞.

In this paper we consider, as a toy-model for the elaboration of the above-mentioned phenomenological model, the dynamics of overdamped particles in a periodic, one-dimensional potential landscape subjected to a static (or adiabatic) external force and to both thermal noise and a nonequilibrium three-level colored noise.

The purpose of the present paper is twofold: first, to provide a compact review of a series of our recently published papers [8, 13, 15–17, 19, 25] in which colored-noise-induced nonequilibrium transport phenomena of Brownian particles were considered, and second, to discuss – on the basis of the proposed model-system – some novel phenomena in stochastic systems where the role of noise flatness as a control parameter is crucial. Here we report, for the first time, the following results: (i) the phenomenon of four current reversals vs. noise correlation time, i.e. a change in the sign of the particle flow while the correlation time of noise is varied, found in [16, 19, 25] at the large flatness limit, is also present in the case of moderate flatnesses of noise. The necessary condition for the existence of this effect is that the flatness $\varphi > 2$; (ii) in the region of current reversals, the dependence of efficiency (with which the system converts fluctuations to useful work) on noise flatness as well as on temperature exhibits a bell-shaped resonance form; i.e., the efficiency can be optimized if we vary either the noise flatness or the temperature; (iii) there is a possibility of a new interpretation of the results in terms of cross-correlation between two dichotomous noises.

In particular, we show that the proposed simple toy-model exhibits a rich variety of anomalous transport phenomena, namely, four current reversals, hypersensitive transport, negative differential resistance, hypersensitive differential response, the phenomenon of disjunct "windows" for an external force, and absolute negative mobility. We emphasize that to our knowledge such a rich variety of anomalous transport effects have never been reported before for overdamped Brownian particles in a one-dimensional periodic structure with a simple periodic potential (with one minimum per period).

A model with a three-level noise

We consider overdamped motion of Brownian particles in a one-dimensional spatially periodic potential of the form $V(x,t) = V(x)[\alpha + \beta Z_1(t)]$, where $Z_1(t)$ is a trichotomous process, α and β are constants, and $V(x)$ is a piecewise linear function, which has one maximum per period. There is an additional force that consists of thermal noise $\xi(t)$ of temperature D, an external static force F, and a three-level colored noise $Z_2(t)$. The system is described by the following dimensionless Langevin equation:

$$(1) \qquad \frac{dX}{dt} = \left[\alpha + \beta Z_1(t)\right] h(x) + F + aZ_2(t) + \xi(t)$$

where: $h(x) \equiv -dV(x)/dx$, $V(x) = \tilde{V}(\tilde{x})/\tilde{V}_0$, ($\tilde{V}(\tilde{x})$ is a spatially periodic function of a period $\tilde{L}$, $\tilde{V}_0 = \tilde{V}_{max} - \tilde{V}_{min}$), and α, β, a are constants. The usual dimensionalized physical variables are indicated by tildes and the space and time coordinates read as $X = \tilde{X}/\tilde{L}$ and $t = \tilde{t}\,\tilde{V}_0/\gamma\tilde{L}^2$ with γ being the friction coefficient; $\tilde{F} = F\tilde{V}_0/\tilde{L}$ is a constant external force. The thermal noise $\xi(t)$ with the correlator $\langle\xi(t_1), \xi(t_2)\rangle = 2D\delta(t_1 - t_2)$ satisfies $\langle\xi(t)\rangle = 0$.

As to the random function $Z_i(t)$ $(i = 1, 2)$, we assume it to be a zero-mean trichotomous Markovian stochastic process [15] which consists of jumps among three values $z_1 = 1$, $z_2 = 0$, $z_3 = -1$. The jumps follow in time according to a Poisson process, while the values occur with the stationary probabilities $p_s(z_1) = p_s(z_3) = q$ and $p_s(z_2) = 1 - 2q$. In the stationary state, the fluctuation process satisfies $\langle Z_i(t)\rangle = 0$ and the correlator is $\langle Z_i(t_1), Z_j(t_2)\rangle = 2q\delta_{ij}\exp[-\nu|t_1 - t_2|]$, $i = 1, 2$, where the switching rate ν is the reciprocal of the noise correlation time $\tau_c = 1/\nu$. It is remarkable that for trichotomous noises the flatness parameter $\varphi = \langle Z_i^4(t)\rangle/\langle Z_i^2(t)\rangle^2 = ½q$, contrary to the cases of the Gaussian colored noise ($\varphi = 3$) and the symmetric dichotomous noise ($\varphi = 1$), can be anything from 1 to ∞. This extra degree of freedom can prove useful in modeling actual fluctuations. Moreover, it is interesting that the results of the present paper can be interpreted in terms of the cross-correlation intensity between two noises. Namely, the trichotomous noise $Z(t)$ can be presented as the sum of two cross-correlated zero-mean symmetric dichotomous noises $\tilde{Z}_1(t)$ and $\tilde{Z}_2(t)$, i.e., $Z(t) = \tilde{Z}_1(t) + \tilde{Z}_2(t)$. The dichotomous noises $\tilde{Z}_1(t)$ and $\tilde{Z}_2(t)$ are characterized as follows: $\tilde{Z}_1, \tilde{Z}_2 \in \{½; -½\}$, $\nu_1 = \nu_2 = \nu$ and their correlation function is $\langle\tilde{Z}_i(t'), \tilde{Z}_j(t)\rangle = (\rho_{ij}/4)\exp[-\nu|t - t'|]$, $i, j = 1, 2$, where $\rho_{ij} = 1$, and $\rho_{ij} = \rho \in (-1,1)$ with $i \neq j$ is the cross-correlation intensity of the noises $\tilde{Z}_1$ and $\tilde{Z}_2$. In this case the probability $q = (1 + \rho)/4$, from which it follows that $\rho = (2 - \varphi)/\varphi$. Let us note that such a cross-correlation between dichotomous noises may be a result of the following two reasons: the two noises are either partly of the same origin or influenced by the same factors.

For the stationary state, we can solve the master equation corresponding to Eq. (1), assuming that the potential $V(x) = V(x-1)$ in Eq. (1) is piecewise linear (sawtooth-like) and its asymmetry is determined by a parameter $d \in (0,1)$, with $V(x)$ being symmetric when $d = ½$. The effective calculation scheme, appropriate to solving Eq. (1) with sawtooth-like potential, is presented in Refs. [8, 13, 17].

Multiple current reversals

In Refs. [13, 16, 19, 25] we have examined model (1) with $F = \beta = 0$, $\alpha = 1$. Considering the case when the trichotomous noise is very flat, it is shown that the transport direction of Brownian particles can be controlled by thermal noise also in the case it is induced by symmetric trichotomous noise. It is also established that on the basis of a large-flatness limit a number of new cooperation effects occur, namely, for certain system parameters there appear more than two current reversals (CRs) vs. temperature and the switching rate of noise, while at large spatial asymmetries of potential the current $J = \langle dX/dt \rangle$ exhibits characteristic disjunct "windows" of temperature and switching rate, where the direction of the current is opposite to that in the ambient medium. The advantage of such multinoise models involving thermal noise is that temperature as a control parameter can be easily varied both in experiments and in potential technological applications.

Unfortunately, within the framework of the calculation scheme of Refs. [13, 19, 25], the absolute value of the net current is very small (infinitesimal). So, the results of Refs. [19, 25] are mainly of theoretical interest, while applications are possible at not-too-large values of flatness. A more fundamental question is, both from theoretical and practical viewpoints, whether the phenomenon of multiple CRs vs. switching rate of noise also occur in the case of moderate values of the flatness parameter.

Our major result is the establishing of the effects of both 4 CRs and disjunct "windows" at moderate values of noise flatness. Notably, as a rule, for moderate values of flatness ($50 > \varphi > 2$) the phenomenon of four current reversals is related to the effect of disjunct "windows". In Fig. 1, curve 1 demonstrates the phenomenon of 4 CRs at the noise flatness $\varphi = 3.125$. It can be seen that the effect of 4 CRs occurs only in a narrow range of the temperature, $D \in (0.055, 0.133)$, i.e. the effect of disjunct "windows". If the temperature is lower than 0.055 or greater than 0.133, then there occur either two CRs or none. We emphasize that the phenomenon of 4 CRs occurs only at the flatness parameters $\varphi > 2$. Moreover, the numerical calculations show that the growth of the potential asymmetry will increase the region of the phase space (q, a), where 4 CRs with the disjunct "windows" effect appear. It is interesting that in terms of cross-correlation intensity between two dichotomous noises (see previous Section) the necessary condition for the existence of 4 CRs is that the correlation intensity ρ is negative. Thus, in the

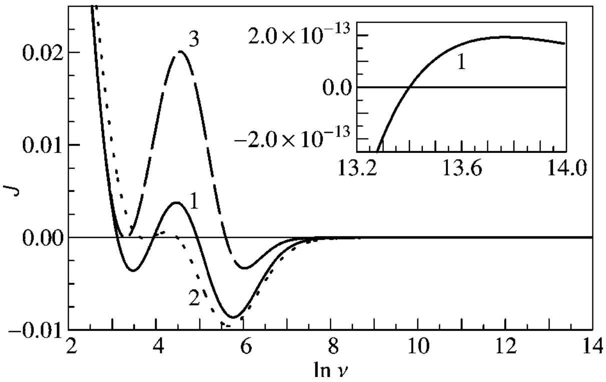

Fig. 1. Four current reversals vs. switching rate ν. For all curves $d = 0.1$, $q = 0.16$, $a = 13.11$, $F = 0$. Solid line (1): $D = 0.090$. Dotted line (2): $D = 0.133$. Dashed line (3): $D = 0.055$. Note the phenomenon of 4 CRs in the case of curve 1. The inset depicts curve 1 in the region of the fourth current reversal.

case of one symmetric dichotomous noise ($\varphi = 1$) the effect of 4 CRs is not possible. Moreover, if the tilting force is absent and $\varphi = 1$, no current reversals occur. Though, we are not aware of any simple physical explanation for the above-mentioned effects, the distinct behaviors of the current induced by dichotomous and trichotomous noises are not surprising if we remember that there is the so-called flashing barrier effect at $\varphi > 1$, which generates a counter current induced by the dichotomous noise (see [15]).

We now concentrate on another aspect in ratchet systems, namely, the efficiency with which the ratchet converts fluctuations to useful work [9]. To calculate the efficiency, a load F is applied against the direction of the current $J = \langle dX/dt \rangle$. The current flows against the load as long as the load is less than the stopping force F_s, beyond which the current takes the same direction as the load. Thus in the operating range of the load $0 < F < F_s$, the Brownian particles move in the direction opposite to that of the load, thereby storing energy. The input power from nonequilibrium fluctuations $Z(t)$ and the output power done against the load F are given by

$$W_{in} = \left\langle \frac{dX}{dt} Z(t) \right\rangle, \quad W_{out} = -JF$$

and the efficiency of energy transduction is $\eta = -JF/W_{in}$. In context of the present model (1) with $\alpha = 1$, $\beta = 0$, a large efficiency $\eta \approx 1$ can be achieved at low values of the temperature D and the switching rate ν. In this case, it is easy to ascertain that the optimal amplitude of the noise a_m and the optimal load force F_m, maximizing the efficiency, are: $a_m = 1/[2d(1-d)]$, $F_m = -(1-2d)/[2d(1-d)]$. The corresponding η takes the form $\eta_{max} = 1 - 2d$. Hence, it follows that as the potential asymmetry increases, the maximal efficiency of the Brownian motor tends to the ideal limit of unity, i.e., $d \to 0$. The reason is that for a low diffusive Brownian motion the asymmetry of the potential profile ensures suppression of backward motion and hence reduction in the accompanying dissipation.

We now discuss the effect of multiple CRs on the efficiency of energy transformation. Figure 2 shows the efficiency η as a function of the temperature D for four different values of the switching rate ν. The

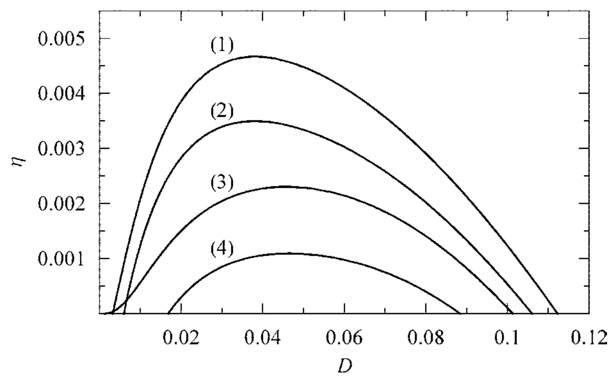

Fig. 2. The efficiency η vs. temperature D in the region of the disjunct "windows" at the system parameters: $q = 0.002$, $F = 0.45$, $d = 0.03$ and $a = 34.4$. The curves (1)–(4) correspond to the following switching rates: (1) $\nu = 164$, (2) $\nu = 78$, (3) $\nu = 430$, (4) $\nu = 33$. The nonmonotonic sequence of the values of ν stems from the bell-shaped dependence $\eta = \eta(\nu)$. Note that the maximum of the efficiency lies at $D_m \approx 0.038$ and $\nu_m \approx 164$.

nonmonotonic sequence of the values of ν stems from the bell-shaped dependence $\eta = \eta(\nu)$. The efficiency η exhibits a bell-shaped (resonance) form as D or ν is varied. The optimal temperature D_m and the optimal switching rate ν_m maximizing the efficiency are: $D_m = 0.038$, $\nu_m = 164$. The fact that the efficiency can be maximized as a function of the temperature shows that thermal fluctuations can facilitate the efficiency of energy transformation. Unfortunately, the efficiency in the region of CRs is very small. In the corresponding parameter regime, particles can move in both directions and hence the variance of particle velocity is large. The latter circumstance causes an increase of dissipation and thus a decrease of efficiency. Moreover, it is remarkable that the dependence of the efficiency on the flatness φ, as well as on the amplitude a, also reveals a broad resonance-like peak. This result indicates that an increase in the flatness of nonequilibrium noise can sometimes facilitate energy conversion. Finally, it is suggested that the ratchet mechanism with CRs can be used for obtaining efficient separation methods of nanoscale objects, e.g., DNA molecules, viruses, etc. In this context, the CR phenomenon is one of the most interesting aspects of the theory of Brownian ratchets. To date, the feasibility of particle transport by man-made devices has been experimentally demonstrated for several ratchet types [11].

Noise-flatness-induced hypersensitive transport

In one of our previous papers [17], model (1) with $a = \alpha = 0$ and $\beta = 1$ was considered. It was established that at low temperatures the phenomenon of hypersensitive transport exists, i.e. a macroscopic flow (current) of matter appears under the influence of an ultrasmall dc driving, and the corresponding mobility, $m = J/F$ at $F = 0$, depends nonmonotonically on the flatness of the multiplicative noise. In the case of dichotomous noise ($\varphi = 1$), which is the case addressed in Ref. [6], the effect, for a piecewisely linear sawtooth potential, disappears.

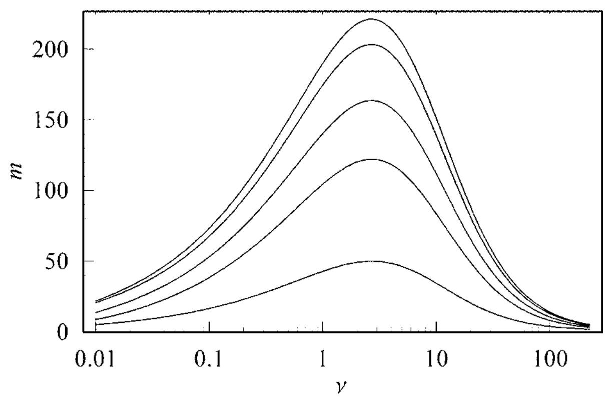

Fig. 3. The mobility m vs. the switching rate ν at $d = ½$, $D = 4 \times 10^{-8}$, and $F = 10^{-5}$. The curves correspond, from top to bottom, to the following values of the flatness parameter: $\varphi = 3$, $\varphi = 2$, $\varphi = 10$, $\varphi = 20$, $\varphi = 1.1$.

The dependence of the mobility m on the parameters ν and φ for the fixed force value $F = 10^{-5}$ and for the fixed temperature $D = 4 \times 10^{-8}$ is shown in Fig. 3. It can be seen that the functional dependence of the mobility on the correlation time $\tau_c = 1/\nu$ and on the flatness $\varphi = ½q$ is of bell-shaped form. A major novel virtue of the model is that the noise flatness can induce the phenomenon of hypersensitive transport. Notably, for fixed low values of temperature the optimal system parameters at which the mobility is maximized, are determined as follows: the flatness parameter $\varphi \approx 3$, the correlation time $\tau_c \approx 3/8$, and asymmetry parameter $d = 1/2$, i.e., the potential $V(x)$ is symmetric. For sufficiently low values of temperature, $D << \min\{2q\nu, 8q/\nu\}$ the mobility m is given by

$$m \approx \frac{8(1-q)}{(\nu+8)^2}\sqrt{\frac{2q\nu}{D}}, \quad F < \sqrt{2q\nu D}. \tag{2}$$

The condition $F < \sqrt{2q\nu D}$ has a distinct physical meaning: the characteristic distance of thermal diffusion $\sqrt{D/\nu}$ is larger than the typical distance F/ν for the particle driven by the deterministic force F in the state $z = 0$. Let us note that the formula (2) for hypersensitive response is qualitatively valid, i.e. $m \sim 1/\sqrt{D}$, also in the case of systems where trichotomous noise is replaced by a multiplicative deterministic periodic stimulus. The physical mechanism which generates the result (2) has been considered in Ref. [17]. The phenomenon of hypersensitive transport is robust enough to survive a modification of the multiplicative noise (or deterministic periodic stimulus) as well as of the shape of the potential (i.e. asymmetric potentials and potentials with several extrema per period). In a general case, if the potential is smooth and the flatness of multiplicative noise is greater than 1, another mechanism described in Ref. [6] plays an important role and should be taken into account when calculating the mobility. However, in the adiabatic case, $\nu << 1$, the reported mechanism of generating hypersensitive transport by noise flatness induces hypersensitive transport more effectively than the one proposed by Ginzburg and

Pustovoit in Ref. [6]. Consequently, for sufficiently small switching rates the leading-order term of the mobility is generated by the mechanism described in the present paper (cf. [17]). It is remarkable that the phenomenon of noise-flatness-induced hypersensitive transport seems to be applicable for amplifying adiabatic time-dependent signals $F(t)$, i.e., signals of much longer periods than the characteristic time of reaching a stationary distribution, even in the case of a small input signal-to-noise ratio $|F(t)|/\sqrt{D} << 1$ (cf. [17]).

Absolute negative mobility and hypersensitive differential response

A characteristic feature of models with absolute negative mobility (ANM) is that upon the application of an external static force F, these models respond with a current that always runs in the direction opposite to that of the force (if the force is small enough) [2]. Notably, for $F = 0$ no current appears due to the spatial symmetry of the system. The effect of ANM is distinct from the phenomenon of negative differential mobility (or resistance) which is, for a sufficiently large F, characterized by a decrease of the current as the driving force F increases, but the system does not exhibit ANM [2]. Devices that display both ANM and negative differential resistance exist and they have important biophysical and technological applications, e.g. semiconductor devices [10], biological ion channels [7], tunnel junction in superconductor devices [5], etc.

We consider overdamped motion of Brownian particles described by Eq. (1) with $\alpha = 1$, $\beta = 0$. The term $Z_2(t)$ in Eq. (1) represents spatially nonhomogeneous fluctuations assumed to be a three-level Markovian stochastic process taking the values $z_1 = -1$, $z_2 = 0$, $z_3 = 1$. That is, the three-level noise $Z_2(t)$ is assumed to be spatially nonhomogenous, so that transitions between the states $z_3 = 1, z_2 = 0$ and between the states $z_2 = 0, z_1 = -1$ can take place only in the left half-period and in the right half-period of the potential, respectively (for more details see [8]). For brevity's sake, from now on we will confine ourselves to the case of a symmetric potential $V(x)$, i.e., $d = ½$. Regarding the symmetry of the dynamical system (1), we notice that $J(-F) = -J(F)$. Obviously, for $F = 0$ the system is effectively isotropic and no current can occur. For the stationary case, the dependence of the current J on the tilting force F enables us to establish a number of effects characterizing anomalous behavior of the resistance (or mobility). Figure 4 illustrates the behavior of the current J as a function of the tilting force F in the region of anomalous resistance. It appears that the curve is highly nonlinear and the phenomenon of ANM occurs: the particles move in the direction opposite to a small external force F. Moreover, the curve exhibits intervals of F where particle speed decreases as the applied drive is increased – an effect that is termed negative differential resistance. Let us note that two additional effects occur. First, one can see a hypersensitive differential response (HDR) in jumps of the current at $F = 1$ and $F \approx 2$. Second, at a low temperature and a large switching rate ν the current exhibits characteristic "disjunct windows" of the tilting force $2 < F < 3.5$, where the value of the current is very small.

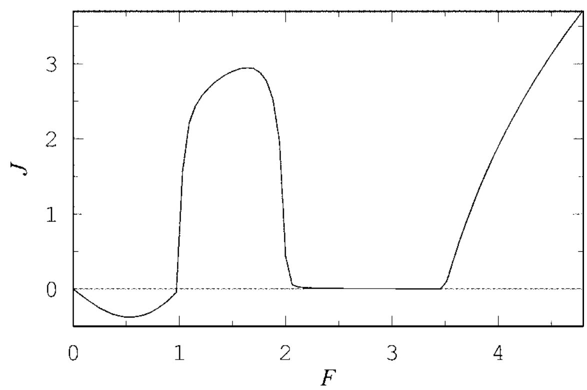

Fig. 4. The current J vs. applied force F in the region of anomalous mobility. $D = 10^{-7}$, $\nu = 10^3$, $a = 3$. The phenomenon of absolute negative mobility occurs.

For both slow and fast fluctuating forces, and for low temperatures, we have presented analytical approximations that agree with the exact numerical results (Ref. [8]). One of our major results is a resonance-like enhancement of absolute negative mobility at intermediate values of the switching rate of a nonequilibrium noise (see [8]). Two circumstances should be pointed out. (i) A resonance-like behavior of ANM can occur in a system parameters domain where the characteristic distance of thermal diffusion $\sqrt{D/\nu}$ is comparable with typical deterministic distances for the driven particles during the noise correlation time. (ii) There is an upper limit temperature D_c beyond which the phenomenon of ANM disappears. Notably, at increasing the noise amplitude a the critical temperature D_c grows as $D_c \sim \sqrt{a}$. It is obvious that the presence and intensity of ANM can be controlled by thermal noise or by the nonequilibrium noise amplitude a. The advantage of this model is that the control parameter is temperature, which can easily be varied in experiments. Moreover, as in Eq. (1) the friction coefficient γ is absorbed into the time scale, so, in the original (unscaled) set-up, the particles of different friction coefficients are controlled by different switching rates. This can lead to an efficient mechanism for the separation of different types of particles.

Our other major result is the establishing of the effects of both HDR and a disjunct "window" at large values of the switching rate ν and low values of the temperature D. We emphasize that here the mechanism of HDR is of a qualitatively different nature from the mechanism of hypersensitive response considered in the previous Section. It should be pointed out that in the present model the effect of HDR is pronounced in the case of a fast switching of the nonequilibrium noise, while in the models of the previous Section hypersensitive transport is generated by low or moderate values of the switching rate. Note, that the results of HDR also seem to be applicable for amplifying adiabatic time-dependent signals (cf. previous Section). Surprisingly enough, at a low temperature and a large switching rate, $D\nu << 1$, the current is very small in the finite interval of the tilting force $2 + (a/2) > F >$

max{2, $a-2$}. It seems that the last mentioned behavior is a new anomalous transport phenomenon for Brownian particles.

Concluding remarks

We have presented some results for the dynamics of overdamped particles in a periodic, one-dimensional potential landscape subjected to a static tilting force and to both thermal noise and a nonequilibrium three-level colored noise. A major virtue of the proposed model is that an interplay of three-level colored and thermal noises in tilted ratchets with simple periodic potentials can generate a rich variety of nonequilibrium cooperation effects.

An amazing noise-induced nonequilibrium phenomenon is a giant enhancement of free diffusion by the use of tilted periodic potentials and also in the case of randomly switching potentials [1, 23]. The corresponding calculations for model (1) are in progress. Futhermore, our earlier results [14, 18] about the influence of colored noise in *N*-species stochastic Lotka-Volterra systems show that mutual interplay of interaction intensities of species (particles of plasma, atoms, photons, molecules, predators and preys, infected individuals, etc.) and colored noise can induce bistability and cause discontinuous nonequilibrium phase-transitions, even if the system is monostable in the absence of noise.

As the nonequilibrium phenomena considered in this paper are robust enough to survive a modification of the colored noise (or deterministic periodic stimulus) as well as the potential landscape (i.e. potentials with several extrema per period), the results of the investigations of the basic model (1), belonging to a highly topical interdisciplinary realm of studies, can be applied for a variety of purposes, starting from a description of Josephson connections [12, 20] and ending with a possible explanation of the catastrophic shifts sometimes occurring in ecosystems [14, 24]. Possible applications range from superconductors to intracellular protein transport in biology, or to methods of particle separation in nanotechnology [11, 22].

We believe that the described model may shed some light on the stochastic interaction processes of plasma with wall materials in plasma focus devices and it could be used for the elaboration of more realistic physical models for nonequilibrium transport and diffusion of particles as well as the nonequilibrium phase transitions, which take place at interactions between hot dense plasma beams and wall materials. Such a realistic model would be of utmost importance for experiments planned with small-scale dense plasma focus devices, working in a short periodic pulse regime. Note that in such devices the role of nonequilibrium noise is played by periodic pulses of plasma.

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