THE COMPARABLE ANALYSIS OF TEMPERATURE DISTRIBUTIONS ASSESSMENT IN DISC BRAKE OBTAINED USING ANALYTICAL METHOD AND FE MODEL

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Abstract

In this paper the one-dimensional analytical formulation as well as the two-dimensional FE model to study the temperature distributions in the pad/disc brake system during an emergency braking process was proposed. Both the time-dependent pressure variation and the convective cooling conditions on the free surfaces of a pad have been incorporated in the contact analytical model. The one-dimensional thermal problem of friction of the strip-foundation system during braking was formulated. An exact solution of the problem using the mathematical device of the integral Laplace transform related to Duhamel theorem was obtained. In the FE formulation to discretize the heat conduction equation for the two-dimensional problem, the Galerkin method was employed. The temperature distributions were calculated exclusively for the disc employing the heat partition ratio. Furthermore, due to the symmetry of the problem the computations were restricted to the half of the entire disc. The finite element analysis of the transient heat transfer problem for the pad/disc system was carried out using the MSC Patran/Nastran program package. On order to confront and compare the resulting temperatures distributions, equal operation parameters, the thermophysical properties of materials as well as the dimensions of the brake components were used within the numerical and analytical calculation. The obtained results from the finite element analysis reveal that both the contact temperature evolution and its values in depth of the brake rotor agree well with the analytical solution and experimental data.

Keywords: *frictional heating, FEM, heat conductivity, disc brake*

1. Introduction

Disc brake consists of cast-iron disc which rotates with the wheel, calliper fixed to the steering knuckle and friction material (brake pads) which is shown in Fig 1. When the braking process occurs, the hydraulic pressure forces the piston and therefore pads and disc brake are in sliding contact. Set up force resists the movement and the vehicle slows down or eventually stops. Friction between disc and pads always opposes motion and the heat is generated due to conversion of the kinetic energy.

Under the influence of temperature, the friction elements hence, the conditions of operation of the friction patches become less favourable: their wear intensifies and the friction coefficient decreases, which may lead to emergency situations [1]. Thus, experimental, mathematical and numerical modelling of the temperature is the important problem at a design stage of brake systems [2-4]. Experimental determination of temperature of a surface of contact concerning authentic objects in most cases causes significant technical difficulties and is connected with essential material and time expenses [5]. Therefore, an analytical or numerical definition of a temperature regime of elements of friction couple obtained by the solution of a corresponding thermal problem of friction during braking attracts the great interest.

Mostly, these temperatures are obtained from a solution of a one-dimensional contact problem with transient frictional heat generation [6-10]. The one-dimensional models correspond to those cases when the Peclet number is large and, consequently, the frictional heat flux is normal to the contact surface. The verification of many analytical solutions with the results from the experimental data which refers to the work of braking devices shows that the one-dimensional models are the sufficiently good approximation for the computation of the temperature during braking [11]. Reviews of analytical methods of the solution of thermal problems of friction during braking are given in the articles [12, 13].

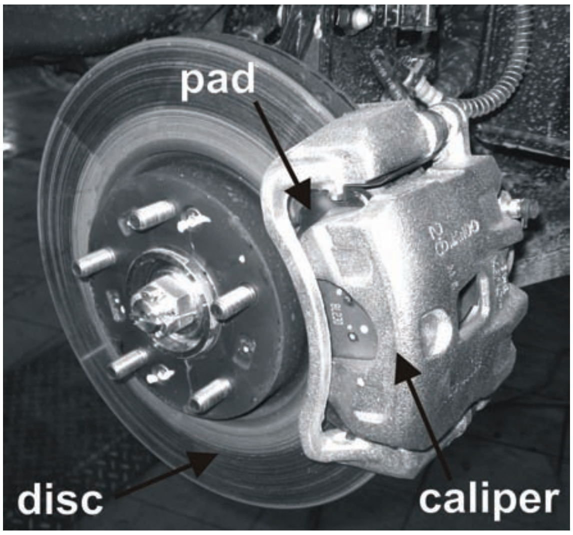

Fig. 1. Front disc brake of the passenger's car

The use of finite element method (FEM) made a significant contribution to the area of heat dissipation problem in disc brakes or dry/wet clutches. Investigated non-axisymmetrical model of brake disc system with moving heat source is presented in article [14]. Appropriate boundary conditions due to analytical model have been imposed. To solve the problem, a transient FE technique has been used. Numerical estimations reveal that the operating parameters of the braking process significantly influence the disc/pad interface temperature distribution and the maximal contact temperature.

According to article [15] it is essential that the analysis is treated as a nonlinear one (thermal conductivity and enthalpy for the disc material vary with respect to temperature). Compares different materials for pad element of automotive disc brakes with its significant weight advantages corresponds to lower maximum operating temperature is shown in work [16]. Three dimensional model of brake system assembly has been imposed with the finite element method simulation. The author examines the effect of the vehicle mass on the peak disc temperatures. Also in paper [17] has been applied to develop influence of all the critical design and material factors. FE modelling of the heat generation process in a mine winder disc brake is proposed in monograph [18].

However it has to be noticed that in the analytical or numerical approximation of the problem, simplification of the process may falsify the results, therefore in this paper we compared and analyzed finite element (FE) and analytical modelling technique.

2. Physical problem

Disc brake system consists of two elements: rotating axisymmetric disc and immovable non-axisymmetric pad (Fig. 2). The most important function of disc brake system in automotive application is to reduce velocity of the vehicle by changing the kinetic energy into thermal energy. While braking process occurs total heat is dissipated by conduction from disc/pad interface to adjacent components of brake assembly and hub and by convection to atmosphere in accordance to Newton's law. The radiation may be neglected due to relatively low temperature and short time of the braking process.

For both types' models (FE and analytical) it has been assumed as follows:

1) Material properties are isotropic and independent of the temperature,
2) The real surface of contact between a brake disc and pad in operation is equal to the apparent surface in the sliding contact. Hence pressure is uniformly distributed over all friction surfaces,
3) The frictional heat due to Newton law has been dissipated to atmosphere on the other surfaces. The heat transfer coefficient *h* is constant during braking process;
4) Because of short braking time and hence relatively low temperature the radiation is neglected,
5) The pressure varies with time [1]

$$p(t) = p_0\left(1-e^{-\frac{t}{t_m}}\right),\ 0 \le t \le t_s\,, \tag{1}$$

where:

p_0 - nominal pressure,
t - is the time,
t_m - is the time of the increase of the pressure from zero to the maximal value p_0,
t_s - is the braking time,

6) The velocity corresponds to pressure (2) is equal [8]

$$V(t)=V_0\left[1-\frac{t}{t_s^0}+\frac{t_m}{t_s^0}\left(1-e^{-\frac{t}{t_m}}\right)\right],\ 0 \le t \le t_s\,. \tag{2}$$

where:

t_s^0 - is the time of braking in the case of constant pressure $p(t) = p_0$,
V_0 - nominal velocity.

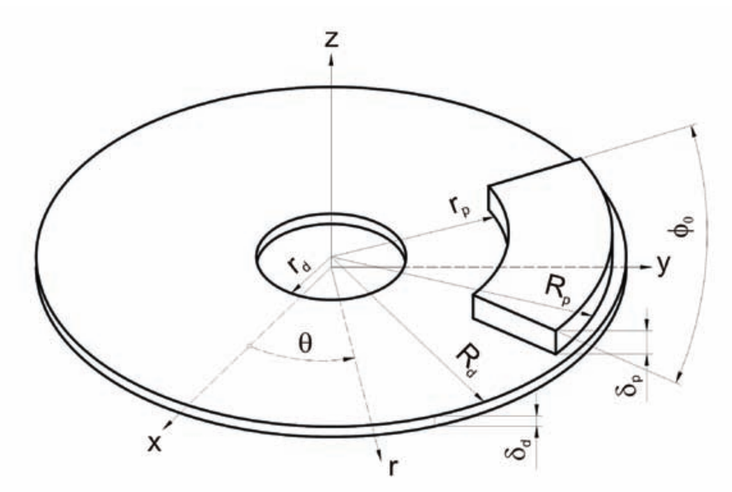

Fig. 2. The schematic assignment of disc brake system

3. Physical problem

In FE model the disc have been analyzed with its simplification to symmetrical problem. Therefore one side of the disc has been insulated. The inner surface of disc was thermally insulated. On the area of sliding contact of disc brake surface intensity of the heat flux has been established. The frictional heat due to Newton law has been dissipated to atmosphere on the other surfaces (Fig. 3).

The average intensity of heat flux into disc on the contact area equals [19]:

$$q_d(r,z,t)\Big|_{z=\delta_d} = \gamma \frac{\phi_0}{2\pi} fp(t) r\omega(t),\ r_p \le r \le R_p,\ 0 \le t \le t_s, \tag{3}$$

and into pad

$$q_p(r,z,t)\Big|_{z=\delta_p} = (1-\gamma)\, fp(t) r\omega(t),\ r_p \le r \le R_p, 0 \le t \le t_s, \tag{4}$$

where:

γ - is the heat partitioning factor,
ϕ_0 - is the cover angle of pad,
f - is the friction coefficient,
ω - is the angular velocity, $r\,\omega(t) = V(t)$,
r - is the radial coordinate,
z - is the axial coordinate,
r_p and R_p - are the internal and external radius of the pad.

The heat partitioning factor representing the fraction of frictional heat flux entering the disc has the form [20]:

$$\gamma = \frac{1}{1+\sqrt{\rho_p c_p K_p / \rho_d c_d K_d}}, \tag{5}$$

where:

ρ - is the density,
c - is the specific heat,
K - is the thermal conductivity.

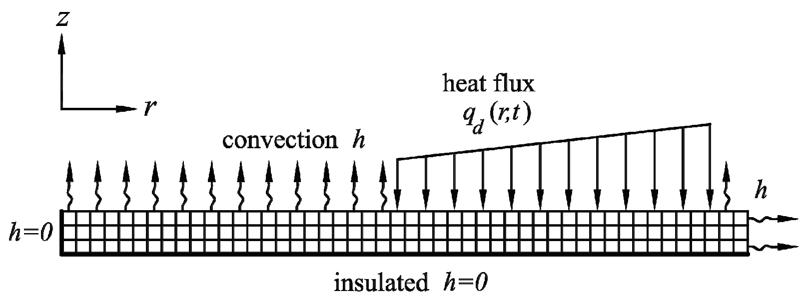

Fig. 3. Finite element model with boundary conditions for the transient analysis

3.1 Mathematical model

To evaluate the contact temperature conditions, both analytical and numerical techniques have been developed. The starting point for the analysis of the temperature field in the disc volume is the parabolic heat conduction equation in the cylindrical coordinate system (r, θ, z) which is centred in the axis of disc and z points to its thickness [21]

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k_d}\frac{\partial T}{\partial t},\, r_d \le r \le R_d,\; 0 \le \theta \le 2\pi,\; 0 < z < \delta_d,\; t > 0, \tag{6}$$

where:

k_d - is the thermal diffusivity of the disc,
r_d - is the internal radius of the disc,
R_d - is the external radius of the disc.

In automotive disc brakes the Peclet numbers almost always are in order 10^5 [1]. Hence the distribution of heat flow will be uniform in circumferential direction, which means that neither temperature nor heat flow will vary in θ direction nor thus the heat conduction equation reduces to

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k_d}\frac{\partial T}{\partial t},\, r_d \le r \le R_d, 0 < z < \delta_d,\; t > 0. \tag{7}$$

The boundary and initial conditions are given as follows:

$$K_d \left.\frac{\partial T}{\partial z}\right|_{z=\delta_d} = \begin{cases} h[T_a - T(r,\delta_d,t)],\, r_d \le r \le r_p \wedge R_p \le r \le R_d, t \ge 0, \\ -q_d(r,\delta_d,t), r_p \le r \le R_p,\, t \ge 0, \end{cases} \tag{8}$$

$$K_d \left.\frac{\partial T}{\partial r}\right|_{r=R_d} = h[T_a - T(R_d,z,t)],\;\; 0 \le z \le \delta_d,\; t \ge 0, \tag{9}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_d} = 0,\;\; 0 \le z \le \delta_d,\; t \ge 0, \tag{10}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0,\; r_d \le r \le R_d,\; t \ge 0 \;, \tag{11}$$

$$T(r,z,0) = T_0,\; r_d \le r \le R_d,\; 0 \le z \le \delta_d. \tag{12}$$

3.2 FE Formulation

The object of this section is to develop approximate time-stepping procedures for axisymmetrical transient governing equations. For this to happen, the following boundary and initial conditions are considered

$$T = T_p \ \text{on} \quad \Gamma_T \,, \tag{13}$$

$$q = -h(T - T_0) \ \text{on} \ \Gamma_h \,, \tag{14}$$

$$q = q_d \ \text{on} \ \Gamma_q \,, \tag{15}$$

$$T = T_0 \ \text{on} \quad \text{at time} \ t = 0 \,. \tag{16}$$

where:

T_p - is the prescribed temperature,
Γ_T, Γ_h , Γ_q, - are arbitrary boundaries on which temperature, convection and heat flux are prescribed.

In order to obtain matrix form of Eq. (7) the application of standard Galerkin's approach was conducted [22]. The temperature was approximated over space as follows

$$T(r, z, t) = \sum_{i=1}^{n} N_i(r, z) T_i(t) \,, \tag{17}$$

where:

N_i - are shape functions,
n - is the number of nodes in an element,
T_i(t) - are time dependent nodal temperatures.

The standard Galerkin's approach of Eq. (7) leads to the following equation

$$\int_\Omega K_d N_i \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} - \rho_d c_d \frac{\partial T}{\partial t} \right] d\Omega = 0. \tag{18}$$

Using integration by parts of the Eq. (14) we obtain

$$\begin{aligned} -\int_\Omega K_d \left[\frac{\partial N_i}{\partial r}\frac{\partial T}{\partial r} + \frac{\partial N_i}{\partial z}\frac{\partial T}{\partial z} - \frac{N_i}{r}\frac{\partial T}{\partial r} + N_i \rho_d c_d \frac{\partial T}{\partial t} \right] d\Omega + \\ + \int_\Gamma K_d N_i \frac{\partial T}{\partial r} l d\Gamma + \int_\Gamma K_d N_i \frac{\partial T}{\partial z} n d\Gamma = 0. \end{aligned} \tag{19}$$

Integral form of boundary conditions

$$\int_\Gamma K_d N_i \frac{\partial T}{\partial r} l d\Gamma + \int_\Gamma K_d N_i \frac{\partial T}{\partial z} n d\Gamma = -\int_{\Gamma_q} N_i q_d d\Gamma_q - \int_{\Gamma_h} N_i h(T - T_a) d\Gamma_h \,. \tag{20}$$

Substituting Eq. (20) and spatial approximation Eq. (17) to Eq. (19) we obtain

$$-\int_{\Omega} K_d \begin{bmatrix} \frac{\partial N_i}{\partial r}\frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z} \\ -\frac{N_i}{r}\frac{\partial N_j}{\partial r} \end{bmatrix} T_j d\Omega - \int_{\Omega} \rho_d c_d N_i \frac{\partial N_j}{\partial t} T_j d\Omega - \\ - \int_{\Gamma_q} N_i q_d d\Gamma_q - \int_{\Gamma_h} N_i h\left(T - T_a\right) d\Gamma_h = 0, \tag{21}$$

where:

i and j - represent the nodes.

Equation (21) can be written in matrix form

$$[C]\left\{\frac{\partial T}{\partial t}\right\} + [K][T] = \{R\}, \tag{22}$$

where:

$[C_T]$ - is the heat capacity matrix,
$[K_T]$ - is the heat conductivity matrix,
$\{R\}$ - is the thermal force matrix,

or

$$[C_{ij}]\left\{\frac{\partial T_j}{\partial t}\right\} + [K_{ij}][T_j] = \{R_i\}, \tag{23}$$

where:

$$[C_{ij}] = \int_{\Omega} \rho_d c_d N_i N_j d\Omega, \tag{24}$$

$$[K_{ij}] = \int_{\Omega} K_d \left(\frac{\partial N_i}{\partial r}\frac{\partial N_j}{\partial r}\{T_j\} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\{T_j\} - \frac{N_i}{r}\frac{\partial N_j}{\partial r}\{T_j\} \right) d\Omega + \int_{\Gamma} h N_i N_j d\Gamma, \tag{25}$$

$$[R_i] = -\int_{\Gamma_q} q_d N_i d\Gamma_q + \int_{\Gamma_h} N_i h T_a d\Gamma_h \tag{26}$$

or in matrix form

$$[C] = \int_{\Omega} \rho_d c_d [N]^T [N] d\Omega, \tag{27}$$

$$[K] = \int_{\Omega} [B]^T [D][B] d\Omega + \int_{\Gamma} h [N]^T [N] d\Gamma, \tag{28}$$

$$\{R\} = -\int_{\Gamma_q} q_d [N]^T d\Gamma_q + \int_{\Gamma_h} h T_a [N]^T d\Gamma_h. \tag{29}$$

In order to solve the ordinary differential equation (23) the direct integration method was used. Based on the assumption that temperature $\{T\}_t$ and $\{T\}_{t+\Delta t}$ at time t and $t+\Delta t$ respectively, the following relation is specified

$$\{T\}_{t+\Delta t} = \{T\}_t + \left[(1-\beta) \left\{ \frac{\partial T}{\partial t} \right\}_t + \beta \left\{ \frac{\partial T}{\partial t} \right\}_{t+\Delta t} \right] \Delta t \,. \tag{30}$$

Substituting Eq. 30 to Eq. 23 we obtain the following implicit algebraic equation

$$([C] + \beta \Delta t [K]) \{T\}_{t+\Delta t} = ([C] - (1-\beta)[K]\Delta t)\{T\}_t + (1-\beta)\Delta t \{R\}_t + \beta \Delta t \{R\}_{t+\Delta t} \,, \tag{31}$$

where:

β - is the factor which ranges from 0.5 to 1 and is given to determine an integration accuracy and stable scheme.

The heat finite element formulation of disc brake with boundary conditions is shown in Fig. 3. The finite element (FE) type was analyzed using the MD Patran/MD Nastran software package. In the thermal analysis of disc brakes an appropriate finite element division is indispensable. In this paper eight-node quadratic elements were used for finite element analysis. The model consists of 570 elements and 1913 nodes. High order of elements ensures appropriate numerical accuracy.

To avoid inaccurate or unstable results, a proper initial time step associated with spatial mesh size is essential

$$\Delta t = \Delta x^2 \frac{\rho_d c_d}{10 K_d} \,, \tag{32}$$

where:

Δt - is the time step,
Δx- is the mesh size smallest element dimension.

4. Analytical model

The problem of contact interaction of a plane-parallel strip (the pad) and semi-infinite foundation (the disc) is under consideration. The time-dependent normal pressure $p(t) = p_0 p^*(t)$, $0 \le t \le t_s$ in the direction of the z-axis of the Cartesian system of coordinates $Oxyz$ is applied to the upper surface of a strip and to the infinity in semi-space (Fig. 4).

In addition, the strip slides with the speed $V(t) = V_0 V^*(t)$, $V_0 \equiv V(0)$, $0 \le t \le t_s$ in the direction of the y-axis on the surface of the semi-space. Due to friction, the heat is generated on a surface of contact $z = 0$, and the elements are heated. It is assumed, that:

1) the sum of heat fluxes, directed from a surface of contact inside each bodies, is equal to the specific friction power [10]:

$$q(t) = q_0 q^*(t),\; q_0 = f p_0 V_0,\; q^*(t) = p^*(t) V^*(t), \tag{33}$$

2) on the upper surface of the strip there is a convective exchange of heat to an environment with the heat transfer coefficient h.

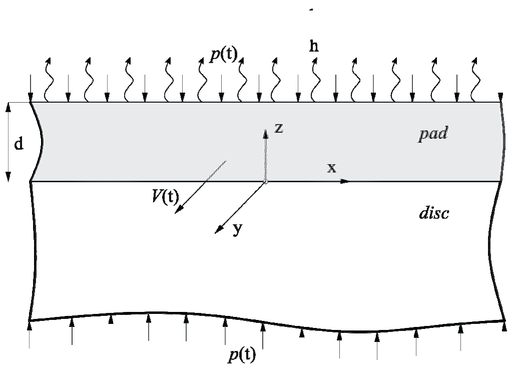

Fig. 4. Scheme of the problem

4.1 Statement of the problem

In accordance with the above-mentioned assumptions the heat conductivity problem at friction takes form:

$$\frac{\partial^2 T_p^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_p^*(\zeta,\tau)}{\partial \tau}, \quad 0<\zeta<1,\ 0\le\tau\le\tau_s, \tag{34}$$

$$\frac{\partial^2 T_d^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k^*}\frac{\partial T_d^*(\zeta,\tau)}{\partial \tau}, -\infty<\zeta<0,\ \ 0\le\tau\le\tau_s, \tag{35}$$

$$K^*\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta=0-} - \left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta=0+} = q^*(\tau), \quad 0\le\tau\le\tau_s, \tag{36}$$

$$T_p^*(0,\tau) = T_d^*(0,\tau), \qquad \tau>0, \tag{37}$$

$$\left.\frac{\partial T_p^*}{\partial \zeta}\right|_{\zeta=1} + Bi\,T_p^*(1,\tau) = 0,\ \ 0\le\tau\le\tau_s, \tag{38}$$

$$T_d^*(\zeta,\tau)\to 0, \quad \zeta\to-\infty,\ \ 0\le\tau\le\tau_s, \tag{39}$$

$$T_{p,d}^*(\zeta,0) = 0, \qquad -\infty<\zeta\le 1, \tag{40}$$

where:

$$\zeta=\frac{z}{d},\ \ \tau=\frac{k_p t}{d^2},\ \ K^*=\frac{K_d}{K_p},\ \ k^*=\frac{k_d}{k_p},\ \ Bi=\frac{hd}{K_p},\ T_0=\frac{qd}{K_p},\ \ T_p^*=\frac{T_p}{T_0},\ \ T_d^*=\frac{T_d}{T_0}. \tag{41}$$

During braking the function $p^*(t) \equiv p^*(\tau)$ in the equation (33) takes the form [14]

$$p^*(\tau) = 1 - e^{-\frac{\tau}{\tau_m}}, \; 0 \le \tau \le \tau_s . \tag{42}$$

where:
τ_m - is the dimensionless time of the increase of the pressure from zero to the maximal value p_0,
τ - dimensionless time.

At the known temporal profile of the pressure (42) the evolution of the sliding speed takes the form [8]:

$$V^*(\tau) = 1 - \frac{\tau}{\tau_s^0} - \frac{\tau_m}{\tau_s^0} p^*(\tau), \; 0 \le \tau \le \tau_s , \tag{43}$$

where:

τ_s^0 - is the dimensionless duration of braking in the case of constant pressure $p(t) = p_0$.

Using the stop condition $V(t_s) = 0$ ($V^*(\tau_s) = 0$), from expression (43) we find the non-linear equation for the dimensionless braking time τ_s:

$$1 - \frac{\tau_s}{\tau_s^0} - \frac{\tau_m}{\tau_s^0} p^*(\tau_s) = 0 . \tag{44}$$

Taking the formulae (33) and (43) into account, we present the dimensionless intensity of the heat flux $q^*(\tau)$ in the boundary condition (36) as

$$q^*(\tau) = \left(1 - \frac{\tau}{\tau_s^0} + \frac{\tau_m}{\tau_s^0} p^*(\tau) \right) p^*(\tau), \; 0 \le \tau \le \tau_s , \tag{45}$$

where the dimensionless temporal profile of pressure $p^*(\tau)$ is given by formula (42).

4.2 Solution to the problem of constant pressure at uniform sliding

The solution to a boundary-value problem of heat conductivity (34)-(40) in the case when the bodies are compressed by constant pressure p_0 and the strip is sliding with a constant speed V_0 on a surface of foundation ($q^*(\tau) = 1, \; \tau \ge 0$), has been obtained in article [23].

In case convective cooling on the upper surface of the strip $(Bi \ge 0)$ and perfect thermal contact of the strip and the foundation, the solution $\hat{T}^*(\zeta, \tau)$, has the form [23]:

$$\hat{T}^*(\zeta, \tau) = T_0^*(\zeta) - \frac{2}{\pi} \int_0^\infty F(x) G(\zeta, x) e^{-x^2 \tau} \, dx, \quad -\infty < \zeta \le 1, \; \tau \ge 0 , \tag{46}$$

where

$$T_0^*(\zeta,\tau)=\begin{cases}\dfrac{1+(1-\zeta)Bi}{Bi}, \ 0\le\zeta\le 1,\\ \dfrac{1+Bi}{Bi}, \ -\infty<\zeta\le 0,\end{cases} \tag{47}$$

$$F(x)=\frac{\cos x+Bi\,x^{-1}\sin x}{(Bi\cos x-x\sin x)^2+\varepsilon^2(Bi\sin x+x\cos x)^2}\,, \tag{48}$$

$$G(\zeta,x)=\begin{cases}\varepsilon[\cos(1-\zeta)x+Bi\,x^{-1}\sin(1-\zeta)x], \ 0\le\zeta\le 1,\\ \varepsilon(Bi\,x^{-1}\sin x+\cos x)\cos\left(\dfrac{\zeta x}{\sqrt{k^*}}\right)-(Bi\cos x-x\cos x)x^{-1}\sin\left(\dfrac{\zeta x}{\sqrt{k^*}}\right), \\ \qquad -\infty<\zeta\le 0,\end{cases} \tag{49}$$

$$\varepsilon\equiv\frac{K^*}{\sqrt{k^*}}=\frac{K_d}{K_p}\sqrt{\frac{k_p}{k_d}}\,. \tag{50}$$

4.3 Solution to the problem at braking

The solution to a boundary-value problem of heat conductivity (34)-(40) in case, when the function $q^*(\tau)$ on the right side of a boundary condition (36) has the form (45), we find using the Duhamel's theorem [24]

$$T^*(\zeta,\tau)=\int_0^{\tau}q^*(s)\frac{\partial}{\partial s}\hat{T}^*(\zeta,\tau-s)ds\,,\ -\infty<\zeta\le 1,\ 0\le\tau\le\tau_s\,, \tag{51}$$

where the dimensionless braking time τ_s is the root of the equation (44).

Substituting the temperature $\hat{T}^*(\zeta,\tau)$ (46) to the right parts of equation (51) and changing the order of the integration, we obtain

$$T^*(\zeta,\tau)=\frac{2}{\pi}\int_0^{\infty}F(x)G(\zeta,x)Q_m(\tau,x)\,dx,\quad -\infty<\zeta\le 1,\ 0\le\tau\le\tau_s\,, \tag{52}$$

where:

$$Q_m(\tau,x)=x^2\int_0^{\tau}q^*(s)e^{-x^2(\tau-s)}ds\,, \tag{53}$$

functions $F(x)$ and $G(\zeta,x)$ take the form (48), (49), accordingly. Taking the form of the dimensionless intensity of a heat flux $q^*(\tau)$ (45) into account, the function $Q_m(\tau,x)$ (53) can be written as

$$Q_m(\tau,x)=x^2e^{-x^2\tau}\left[I_1(\tau,x)+\frac{\tau_m}{\tau_s^0}I_2(\tau,x)\right],0\le x<\infty\,,0\le\tau\le\tau_s\,, \tag{54}$$

where:

$$I_1(\tau,x)=\int_0^\tau\left(1-\frac{s}{\tau_s^0}\right)p^*(s)e^{x^2s}ds\,,\ I_2(\tau,x)=\int_0^\tau[p^*(s)]^2e^{x^2s}ds\,. \tag{55}$$

Inserting the dimensionless pressure $p^*(\tau)$ (42) into the right-hand side of the formulae (55) after integrating we find

$$I_1(\tau,x)=x^{-2}(e^{x^2\tau}-1)-\frac{1}{\tau_s^0}[x^{-2}\tau e^{x^2\tau}-x^{-4}(e^{x^2\tau}-1)]-\frac{1}{(x^2-\tau_m^{-1})}[e^{(x^2-\tau_m^{-1})\tau}-1]+$$
$$+\frac{1}{\tau_s^0}\left\{\frac{\tau}{(x^2-\tau_m^{-1})}e^{(x^2-\tau_m^{-1})\tau}-\frac{1}{(x^2-\tau_m^{-1})^2}[e^{(x^2-\tau_m^{-1})\tau}-1]\right\}, \tag{56}$$

$$I_2(\tau,x)=x^{-2}(e^{x^2\tau}-1)-\frac{2}{(x^2-\tau_m^{-1})}[e^{(x^2-\tau_m^{-1})\tau}-1]+\frac{1}{(x^2-2\tau_m^{-1})}[e^{(x^2-2\tau_m^{-1})\tau}-1]\,. \tag{57}$$

Substituting functions $I_n(\tau,x),\ n=1,2$ (56) and (57) into equation (54) we obtain

$$Q_m(\tau,x)=\left(1+\frac{\tau_m}{\tau_s^0}+\frac{1}{\tau_s^0x^2}\right)(1-e^{-x^2\tau})-\left[1+\frac{2\tau_m}{\tau_s^0}+\frac{1}{\tau_s^0(x^2-\tau_m^{-1})}\right]\frac{x^2}{(x^2-\tau_m^{-1})}\left(e^{-\frac{\tau}{\tau_m}}-e^{-x^2\tau}\right)+$$
$$+\frac{\tau_mx^2}{\tau_s^0(x^2-2\tau_m^{-1})}\left(e^{-2\frac{\tau}{\tau_m}}-e^{-x^2\tau}\right)+\frac{\tau x^2}{\tau_s^0(x^2-\tau_m^{-1})}e^{-\frac{\tau}{\tau_m}}-\frac{\tau}{\tau_s^0},0\le x<\infty,0\le\tau\le\tau_s. \tag{58}$$

In the limiting case of braking with constant deceleration at $\tau_m\to0$ from formula (58) we find the results of the paper [25]

$$Q_0(\tau,x)=\left(1+\frac{1}{\tau_s^0x^2}\right)(1-e^{-x^2\tau})-\frac{\tau}{\tau_s^0},\ 0\le x<\infty,0\le\tau\le\tau_s\,. \tag{59}$$

5. Numerical analysis

To compare the analytical model and MES, the non-stationary temperature fields were calculated, with the same initial conditions and the materials as in article [14].

Evaluation of the pressure p and angular velocity ω of the disc is shown in Fig. 5.

Material properties and operation conditions adopted in the FEM and analytical analysis are given in Tab. 1 and Tab. 2.

Tab. 1. Material properties

material properties	disc	pad
thermal conductivity, [W/mK]	48.46	1.212
specific heat, [J/kgK]	419	1465
density, [kg/m^3]	7228	2595

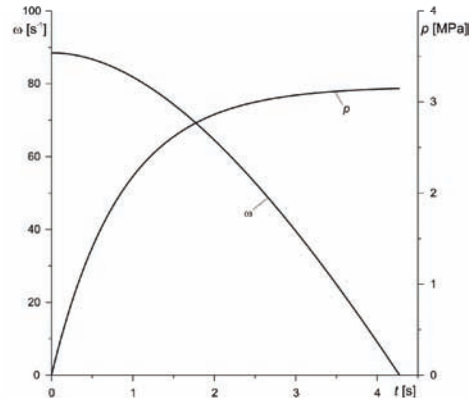

Fig. 5. Evolution of the pressure and angular velocity during braking

Tab. 2. Operation conditions

operation conditions	disc	pad
inner radius, [mm]	32.5	77
outer radius, [mm]	128	125
disc thickness [mm]	6	
initial angular velocity [s^{-1}] / sliding velocity [m/s]	87.23 / 27.78	
braking time, [s]	4.274	
maximum pressure [MPa]	3.17	
coefficient of friction	0.5	
heat transfer coefficient *h* [W/m^2K]	100	
initial temperature [^0C]	20	

The evolutions of the temperature on a contact surface during braking for analytical and FE model are shown in Fig. 6. For botch cases the temperature increases with the beginning of the process of braking. The maximal temperature $T_{\max}$ is reached at the time moment $t_{\max}$ and for analytical model is equal $T_{\max} = 296^{\circ}\mathrm{C}$ at $t_{\max} = 3.27s$ and for FE model: $T_{\max} = 281^{\circ}\mathrm{C}$ at $r = 113mm$ and $t_{\max} = 3.49s$. It agrees well with data in paper [14] where the maximal temperature for the same parameters is equal $T_{\max} = 279.8^{\circ}\mathrm{C}$ at $t_{\max} = 3.287s$.

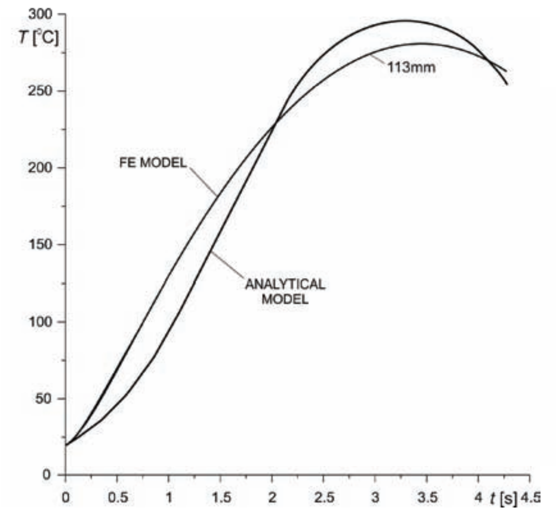

Fig. 6. Evolution of the disc temperature on the friction surface

Fig. 7 shows disc temperature surface variations along radial direction obtained with FEM model. Also angular velocity has been assumed as a nonlinear. As it can be seen temperature at inner disc surface (r=52 mm) has a constant value $200^{\circ}C$. It corresponds to boundary conditions, where surface was insulated. Maximum temperature rise up to $281^{\circ}C$ at 113 mm of radial position and 3.49s of time.

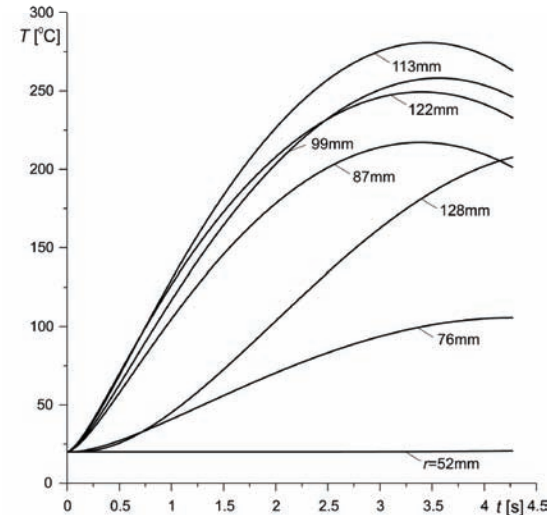

Fig. 7. Evolution of the disc temperature on the friction surface for different values of the radial position (FE model)

The change of temperature in the disc in depth from a surface of friction for FE model at $r = 113mm$ is shown in Fig. 8a and for analytical model – Fig. 8b.

As the distance from the surface of friction grows, the temperature decreases, and the duration of achieving the maximal value in depth – increases. It may be noticed, that the results of calculations corresponds closely with each other.

a) b)

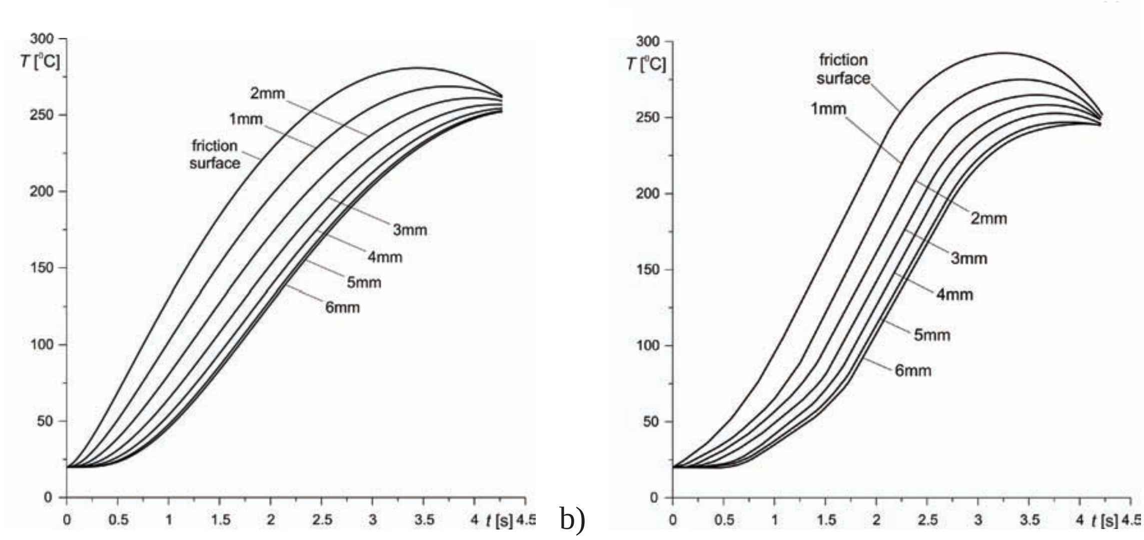

Fig. 8. Evolution of the disc temperature at different distances: a) FE model at radial position r=113mm; b) analytical model

5. Conclusion

In this paper transient thermal analysis of disc brakes in single brake application was performed.

To compare two (analytical and FE) models of calculating the non-stationary temperature fields in disc, the material- and operating parameters as in article [14] were used. The maximal temperature, found with the analytical method, is equal $T_{\max} = 296^{\circ}\mathrm{C}$, and it is slightly higher than the one found with FE model, i.e. $T_{\max} = 281^{\circ}\mathrm{C}$, and than the data of article [14] - $T_{\max} = 279.8^{\circ}\mathrm{C}$.

The temperature values obtained using FEM agree well with the analytical and experimental results.

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