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BOUNDARY CONDITIONS FOR FRICTION FORCES IN FUTURE BEARING

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Abstract

The aim of the presented paper is determination of the boundary conditions for friction forces in micro scale during the hydrodynamic flow of viscoelastic non-Newtonian liquid in the thin boundary layer about 0,1 micrometer situated on the deformable surface of the body with anisotropic, hyper-elastic, hypo-elastic properties.

One of the most important features which distinguish the surfaces of cartilage tissues from machinery ones is the cells capability of growing along with growing age of human being, and the joint cartilage capability of regenerating itself after failure. Contemporary machine bearings do not possess such features. Moreover, cartilages are capable of being self-adapted to the environment, whereas machine bearings are not able to adapt themselves to external conditions. The description of kinematics pair of such surfaces leads mainly to friction forces and wear determination what give knowledge about cooperating soft superficial layers in future machinery bearings and enables the realization of super thin material layers in intelligent micro-robotics where bearing material should be adapted to the external conditions.

The research presented in his paper show the construction of modified boundary Beavers conditions for liquid components flowing in super thin lubricant layers situated in bio-bearings and in micro-canals inside human joint cartilage. The methods of friction forces in two directions are determined. Gained experiences are confirmed by the experimental data in the field of performed calculations for bio-bearing and after Authors knowledge could be constitute the first step to application of presented theory for designing of machine slide bearings which can be adapted to the external conditions. Such bearings have the chance to be the bearings of the 21 century.

Keywords: *Boundary conditions, micro-friction forces, future machine slide bearings*

WARUNKI BRZEGOWE SIŁ TARCIA W ŁOŻYSKACH PRZYSZŁOŚCI

Streszczenie

Tematem niniejszej pracy jest wyznaczanie warunków brzegowych dla sił tarcia w mikroskali w trakcie hydrodynamicznego przepływu lepko-sprężystej cieczy o własnościach nie-newtonowskich w cienkiej warstwie granicznej o grubości poniżej 0,1 mikrometra znajdującej się na powierzchni deformowanych ciał o własnościach anizotropowych, hiper-sprężystych oraz hypo-elastycznych. Powierzchnie chrząstki stawowej człowieka mają wspomniane właściwości oraz dodatkowo wykazują dużą zdolność adaptacji do warunków zewnętrznych, w których pracują. Opis współpracy kinematycznej takich powierzchni finalizowany głównie wyznaczaniem sił tarcia i wartości zużycia umożliwi opis cech materiału do wykonania współpracujących warstw wierzchnich w przyszłościowych łożyskach maszynowych a także w łożyskach mikrorobotów, gdzie materiał łożyskowy mógłby adaptować się do warunków zewnętrznych spełniać wymogi dobrego smarowania.

Badania prowadzone w niniejszej pracy zaprezentują konstrukcję zmodyfikowanych warunków brzegowych Beaversa dla prędkości cieczy przepływających w super cienkich warstewkach smarujących zalegających szczeliny biołożysk oraz mikro-kanaliki chrząstki stawowej człowieka. Następnie określone zostaną metody wyznaczania wartości sił tarcia w dwóch kierunkach.

Zdobyte doświadczenia wsparte wynikami doświadczalnymi w zakresie przeprowadzenia obliczeń w biołożyskach i bioreaktorach mogą stanowić zdaniem autora pierwszy krok do wykorzystania przedstawionej teorii przy projektowaniu maszynowych łożysk ślizgowych, które będą przystosowywać się do warunków zewnętrznych. Takie łożyska mają szansę być łożyskami 21 wieku.

Słowa kluczowe: *Warunki brzegowe, siły tarcia, mikro-skala, maszynowe łożyska ślizgowe przyszłości*

1.Introduction

During the lubrication the friction forces between two cooperating cartilage surfaces occur in macro-, micro- and nano-scale. The comparison of two bodies cooperating in macro- micro- and nano-scale in human joints is presented in Fig. 1.a, b, c.

The friction forces in macro- scale are located on the external surfaces of cartilage (see Fig. 1a).The friction forces in micro-scale take place between collagen fibres and between intermediate layers of superficial layer of tissue (see Fig. 1b). The friction forces in nano-scale are tangential to the lateral surfaces of glycoprotein fibres (see Fig. 1c). The glycoprotein fibres are of the diameter ranging from 3 to 100 nm [1], [4], [5], [6], [8].

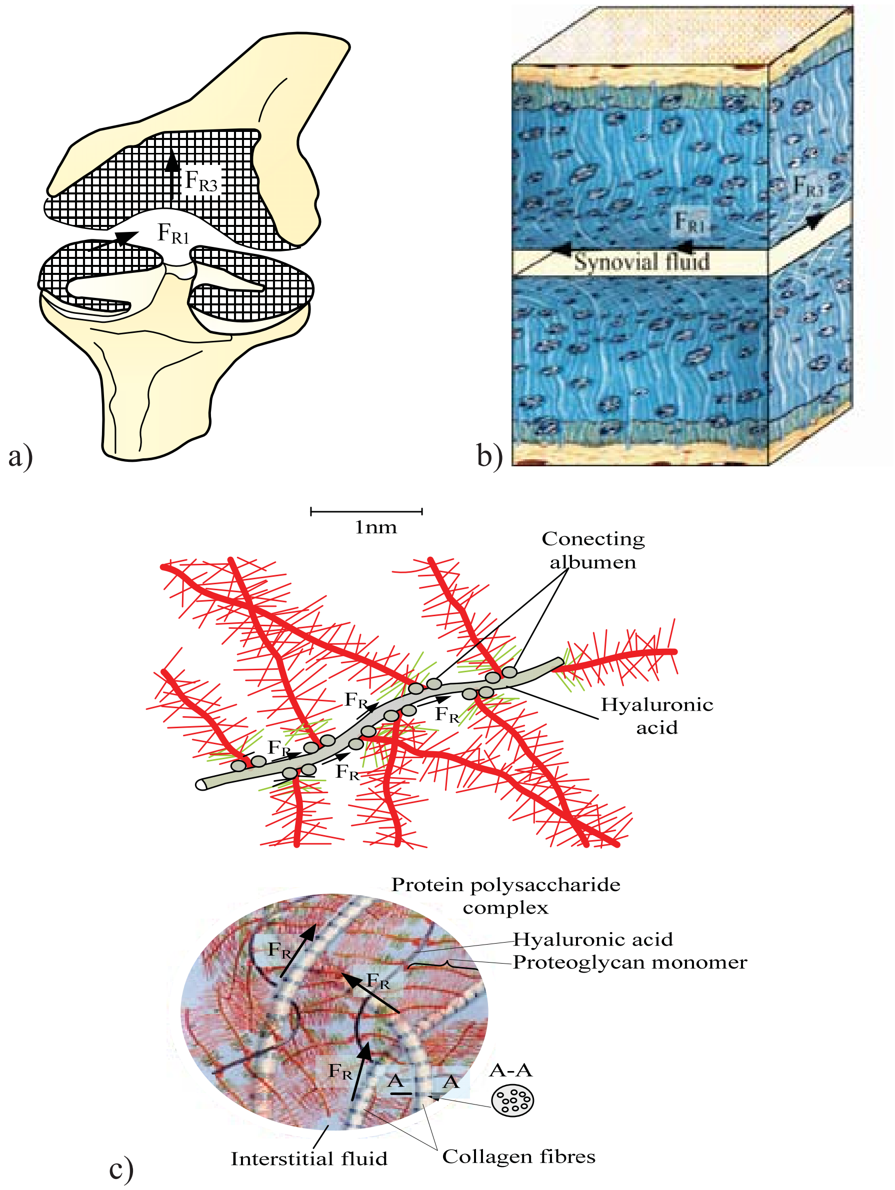
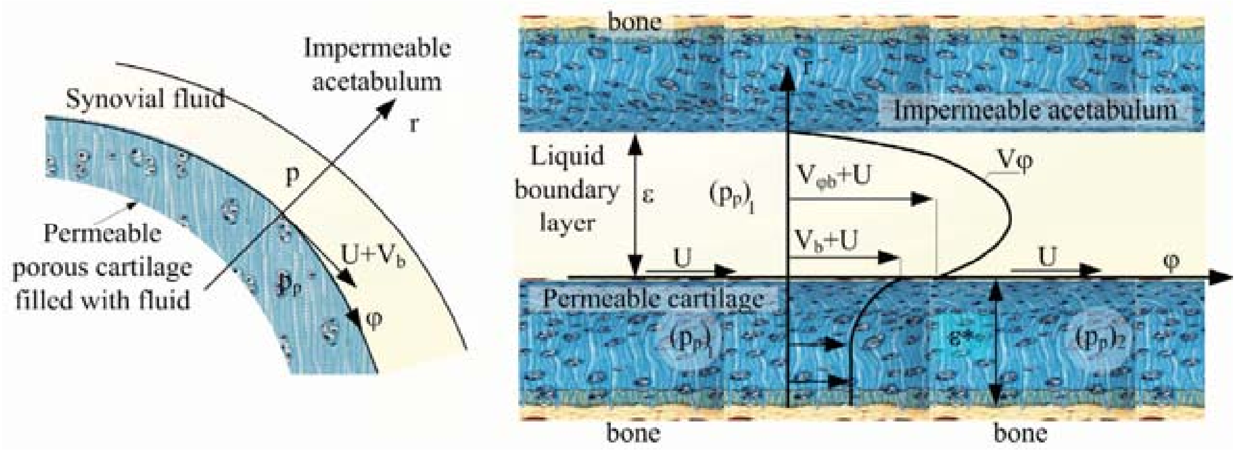


Fig. 1. Friction forces in macro-micro-and nano- level: a) macro-level friction forces in human joint, b) micro-level friction forces between two cartilage surfaces, c) protein Polysaccharide Complex and collagen fibres in nano-scale, appearing in cartilage layer, and friction forces in nano-level scale

2. Beaver's boundary conditions for the flow in bio-bearing gap

The boundary conditions for the unknown synovial fluid velocity components caused by rotational motion in the circumferential direction φ have the known classical form [8]: $v_\varphi=U$ for r=0 and $v_\varphi=0$ for $r=\varepsilon$. In the case of permeable cartilage surface we additionally take into account the Beaver's boundary conditions [2], [3]. The graphical form of the Beaver's boundary conditions is presented in Fig. 2.



a) b)

Fig. 2. Boundary conditions for synovial fluid circumferential velocity component in thin layer resting on the movable and permeable porous cartilage, where r denotes gap height direction: a) acetabulum, joint gap and bone head, b) model of the boundary condition

Porous permeable cartilage lies on the bone head. The peripheral velocity of the bone head is $U=\omega R$ [7].We introduce the following symbols: ω – the angular velocity of the bone in the circumferential direction φ. The value of the height of the human joint gap is $\varepsilon \approx 2\cdot10^{-5}$m. The pressure in pores is p_p. The symbol p denotes hydrodynamic pressure.

The unknown velocity component is obtained from the following boundary conditions [3]:

$$v_\varphi=0 \text{ for } r=\varepsilon, \tag{1}$$

$$v_\varphi=V_{\varphi b}+U \text{ for } r=0, \tag{2}$$

$$\frac{dv_\varphi}{dr}=\frac{c_\alpha}{\sqrt{c_{por}}}\left(V_{\varphi b}-V_b\right), \text{ for } r=0, \tag{3}$$

$$V_b=-\frac{c_{por}}{\eta c_\lambda}\frac{\partial p_p}{\partial\varphi}. \tag{4}$$

The pressure in pores p_p produces the velocity V_b in the horizontal (or circumferential) direction. To determine the unknown value of the velocity $V_{\varphi b}$ we use the boundary Beavers condition (3). The symbol c_{por} (in m^2) stands for the permeability coefficient of the porous cartilage. The dimensionless coefficient c_α depends on the degree of the cartilage surface porosity.

3. Beaver's boundary conditions for the flow in cartilage canals and friction forces

The boundary condition for the liquid flow through the porous canal in the superficial layer of cartilage in human joint is now determined. We take into account that the canal is limited by the impermeable and permeable walls of cartilage. After Darcy and Beavers investigations, the liquid velocity component v_φ can be determined by the following equation [6.2], [6.3]:

$$\frac{\partial^2 v_\varphi}{\partial r^2}=\frac{1}{\eta}\frac{\partial p_p}{c_\lambda \partial\varphi}, \tag{5}$$

where $0\le r\le\varepsilon_{ch}$ and ε_{ch} is the height of the channel in porous tissue.

In this liquid the flow in boundary layer depends only on the pressure in pore, p_p. The horizontal axis φ lies on the lower surface and the vertical axis r indicates the height direction of the canal (see Fig. 3). The symbol c_λ denotes dimensional value of channel length.

If the boundary layer of viscous liquid is limited by the permeable porous superficial layer on the lower surface and by the impermeable tissue laying on the upper surface, then the boundary conditions for the velocity component in φ – direction have the following form:

$$v_\varphi=0 \text{ for } r=\varepsilon_{ch}, \tag{6}$$

$$v_\varphi=V_{\varphi b} \text{ for } r=0, \tag{7}$$

$$\frac{dv_\varphi}{dr}=\frac{c_\alpha}{\sqrt{c_{por}}}\left(V_{\varphi b}+\frac{c_{por}}{\eta c_\lambda}\frac{\partial p_p}{\partial \varphi}\right), \quad \text{for } r=0, \tag{8}$$

where $0 \le r \le \varepsilon_{ch}$.

We can use the Beavers condition (8) to determine the unknown value of the tangential velocity $V_{\varphi b}$ on the permeable cartilage surface in the point r=0. This condition determines the angle of inclination of the velocity trajectory on the permeable surface. The flow of the liquid through the canal depends only on the pressure in pores. The dimensionless value c_α and velocity $V_{\varphi b}$ have been already determined above.

The friction forces in φ,ϑ directions have the following forms [8]:

$$F_{R1\varphi}=\iint_\Omega\left(\eta\frac{\partial v_\varphi}{\partial r}\right)_{r=\varepsilon_{ch}} d\Omega, \quad F_{R3\vartheta}=\iint_\Omega\left(\eta\frac{\partial v_\vartheta}{\partial r}\right)_{r=\varepsilon ch} d\Omega, \tag{9}$$

where:

v_φ – synovial fluid velocity component in the circumferential φ direction,
v_ϑ – synovial fluid velocity component in meridional direction ϑ in spherical coordinates,
η – dynamic viscosity of synovial fluid,
ε_{ch} – dimensional height of the channel in porous cartilage, Ω – dimensional value of lubrication area.

The solution of the equation (5) under the boundary conditions (6), (7), (8) gives the following solution of the liquid velocity component in φ direction, inside the canal for $0 \le r \le \varepsilon_{ch}$:

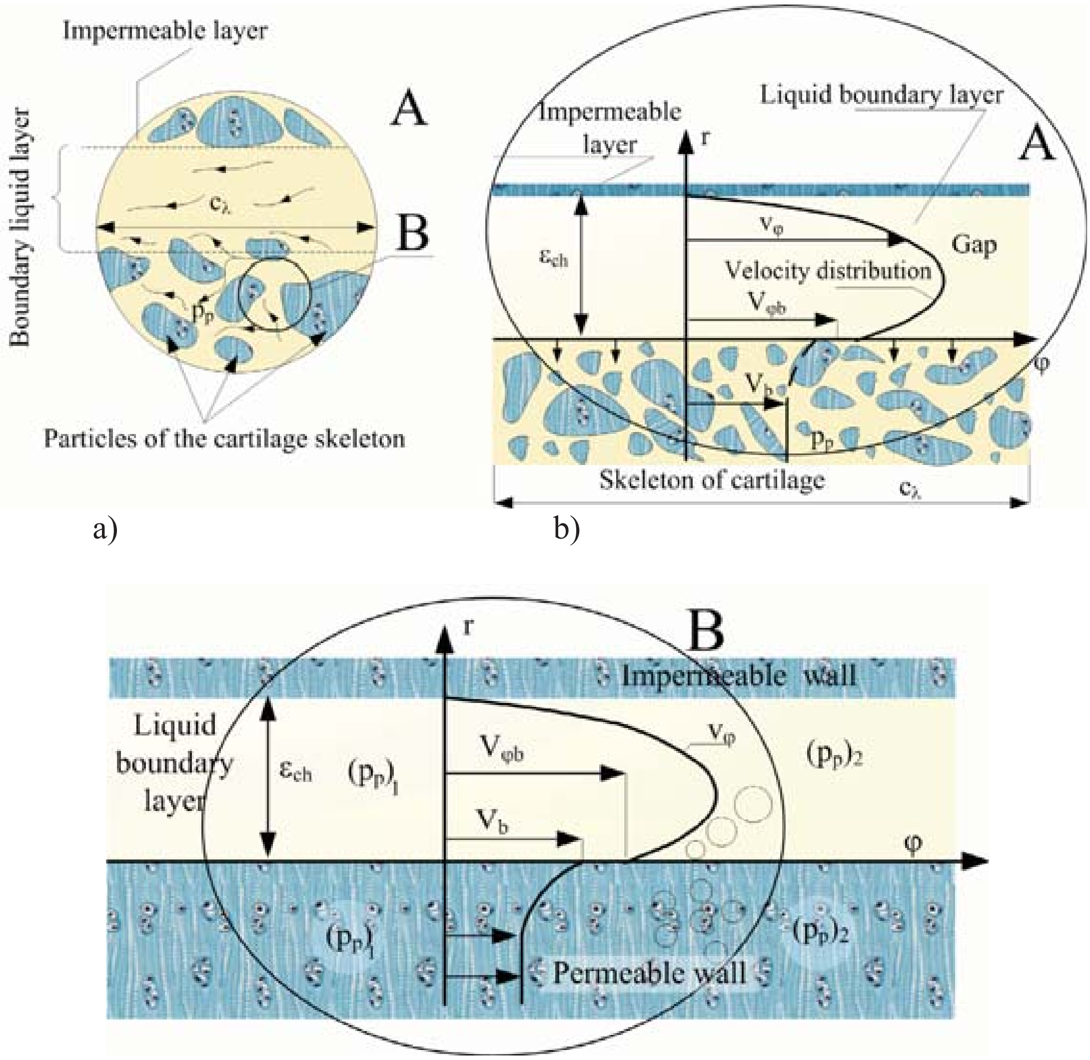
$$v_\varphi=V_{\varphi b}\left(1+\frac{c_\alpha}{\sqrt{c_{por}}}r\right)+\frac{1}{2\eta}\left(r^2+2c_\alpha r\sqrt{c_{por}}\right)\frac{dp_p}{c_\lambda d\varphi}=$$

$$=\frac{dp_p}{c_\lambda d\varphi}\frac{\sqrt{c_{por}}(r^2-\varepsilon_{ch}^2)+c_\alpha\left(\varepsilon_{ch}r+2c_{por}\right)(r-\varepsilon_{ch})}{2\eta\left(\sqrt{c_{por}}+c_\alpha\varepsilon_{ch}\right)}, \tag{10}$$

where:

$$V_{\varphi b}=-\frac{c_{por}}{2\eta}\left(\frac{c_\sigma^2+2c_\alpha c_\sigma}{1+c_\alpha c_\sigma}\right)\frac{dp_p}{c_\lambda d\varphi}, \tag{11}$$

$$c_\sigma \equiv \frac{\varepsilon_{ch}}{\sqrt{c_{por}}}. \tag{12}$$



a) b)

c)

Fig. 3. Boundary conditions for the liquid velocity distribution in the porous canal within cartilage at $(p_p)_1 > (p_p)_2$*; a) canal in porous tissue, b) velocity distribution in the porous canal, c) velocity distribution in the canal between particles*

Fig.3 shows the parabolic distribution of nutrient liquid velocity in the boundary layer inside the canal in porous tissue. Taking into account the component of liquid velocity (10), we can obtain the friction forces produced by liquid flow in canal in the following form:

$$F_{R\varphi} = \iint_{\Omega} \left(\eta \frac{\partial v_\varphi}{\partial r} \right)_{r=\varepsilon_{ch}} d\Omega = \frac{2\varepsilon_{ch}\sqrt{c_{por}} + c_\alpha \varepsilon_{ch}^2 + 2c_\alpha c_{por}}{2c_\lambda\left(\sqrt{c_{por}} + c_\alpha \varepsilon_{ch}\right)} \iint_{\Omega} \frac{dp_p}{d\varphi} d\Omega \, . \tag{13}$$

4. Conclusions

Human joints as slide lubricated frictional units of various geometrical forms are characterized, depending on a need, by an ability to self-regulation of their gap height, eccentricity, lubricating liquid wedge, as well as physical properties of synovial liquid, its lubricity and viscosity in particular, not to be found in machinery bearings. The facts allow calling human joints the intelligent bio-bearings.

Regularity of curves and their curvatures, frequently occurring straight lines, and first of all simplicity of geometrical forms typical for mechanical bearings have been replaced in bio-bearings by irregularity of curvatures of curvilinear surfaces as well as a great variety of different geometrical forms of co-operating surfaces. It seems that dozens thousand years of evolution have made it possible to appropriately optimize geometrical forms of joint surfaces giving them suitable shapes. Hence it may be supposed that the variety of geometrical forms of joint gaps of curvilinear contours is necessary for reaching optimum values of operational parameters of biobearings and minimizing their wear due to friction process.

For biomaterials another factor which differentiates machinery bearings from human joints, is characteristic, namely: synovial liquids and joint cartilage. Joint cartilage ideally satisfies the features of typical bearing material, unreachable in technology and often contradicting to each other. Synovial liquid is capable of changing its dynamic viscosity as well as coefficient of penetration into joint cartilage pores, during its operation. For the reason it can be considered as a time-variable, non-Newtonian rheological liquid having intelligent features. Changes of dynamic viscosity values of synovial liquid in human joints are strictly associated with changes of values of elasticity and hyper-elasticity module of joint's soft cartilage. Also, values of joint cartilage material factors affect values of synovial liquid dynamic viscosity.

Presented scientific considerations have applications in MEMS (Micro Electro-Mechanical Systems and NEMS (Nano-Electro-Mechanical Systems) and particularly in micro-motors, micro turbines, micro mechanisms with dimension about 50 nm [9],[10] see Fig.4.

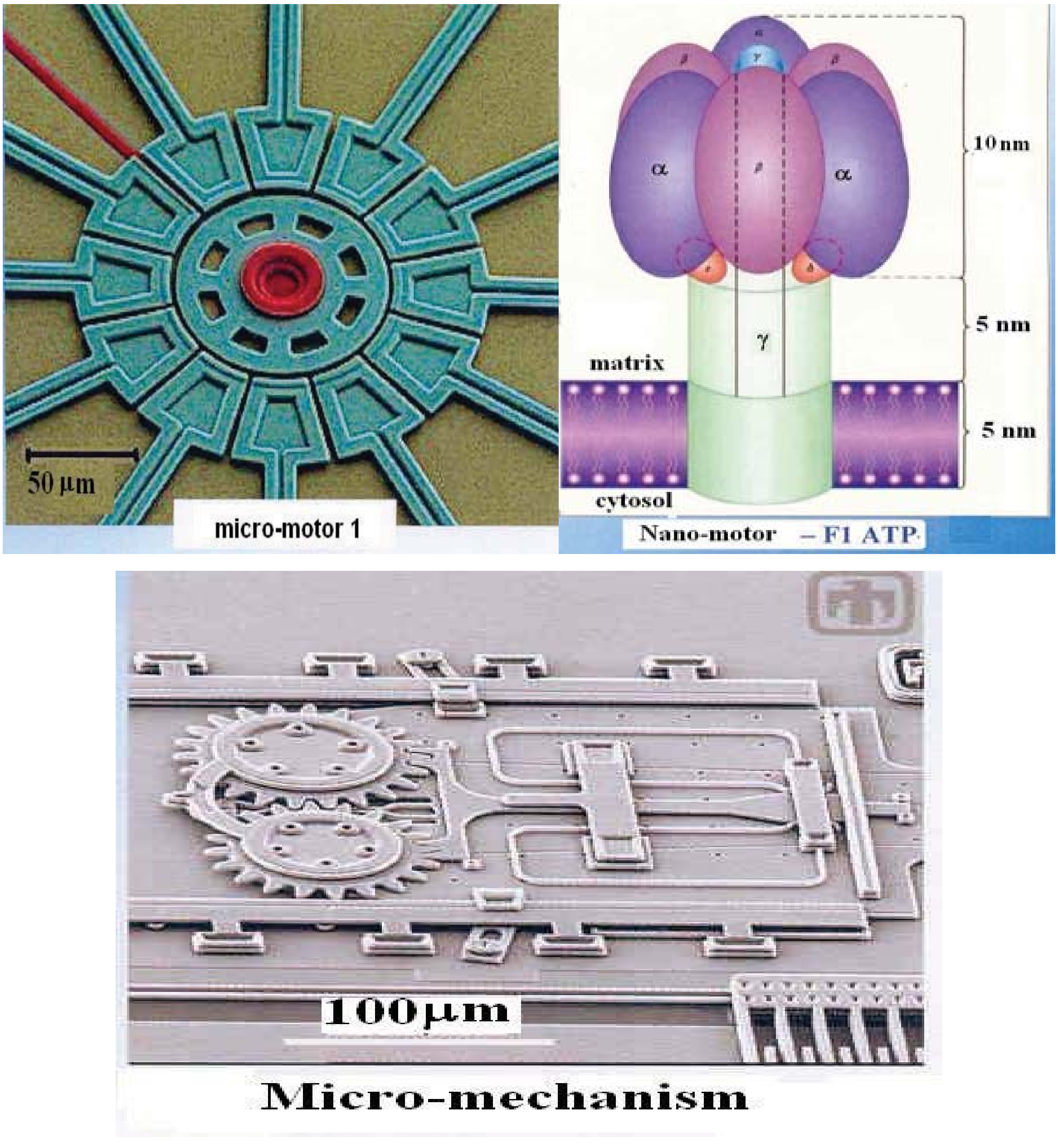


Fig. 4. Micromoto, Nanomotor and Micro-mechanism [9],[10].

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