HYDRODYNAMIC PRESSURE IN HUMAN HIP JOINT

Krzysztof Wierzcholski

Gdynia Maritime University
Morska 83, 81-225 Gdynia, Poland
tel.: +48 058 6901348, 3476126, fax: +48 058 6901399
e-mail:wierzch@am.gdynia.pl

Abstract

This paper presents modelling and simulations for synovial fluid flow occurring in gap between two co-operating bone surfaces in human hip joint. The present consideration gives an analysis of solutions of systems of non-linear, partial, differential equations for synovial fluid flow in human joint gap. In the hip joint the spherical rotation bone and the pelvis bone create a spherical gap. In this gap between two co-operating bones synovial fluid flows, see Dowson [1], Maquet [4], Mow [5], [6], [7], [8], [9], [10], Wierzcholski [12], [13], Pytko et.al. [11]. The flow of this fluid is caused by the motion of the bone head. The theoretical considerations of the synovial non-Newtonian fluid flow in thin joint gap, taking into account boundary layer simplifications, have practical applications in theory of lubrication in medicine, see Dowson et.al. [1], [2], Gruca [3]. The consideration given in the present section enables to find synovial fluid flow parameters and carrying capacity forces in human hip joint gap between two co-operating cartilage surfaces.

Human hip joint and gap height for deformable cartilage and pressure distribution in normal and pathological spherical hip joint gap for hydrodynamic lubrication caused by rotation, cases of pressure distribution in pathological spherical hip joint gap for hydrodynamic lubrication caused by rotation, lubrication surface, area=20.38 cm' are presented in the paper.

Keywords: *normal, pathological hip, capacity, compressive stresses*

CIŚNIENIE HYDRODYNAMICZNE W BIODROWYM STAWIE CZŁOWIEKA

Streszczenie

Praca przedstawia obliczenia numeryczne rozkładu ciśnienia hydrodynamicznego na sferycznej głowie kostnej stawu biodrowego wykonującego ruch obrotowy. Uwzględniona została deformacja powierzchni chrząstki stawowej oraz brane są pod uwagę własności nie Newtonowskie cieczy synowialnej poprzez zmiany lepkości dynamicznej wywołane zmianami prędkości deformacji. Ponadto uwzględnia się zarówno przypadek zdrowego jak i chorego stawu biodrowego. W tym stawie między dwoma współpracującą kości przepływa płyn synowialny , patrz Dowson [1], Maquet [4], Skoście [5], [6], [7], [8], [9], [10], Wierzcholski [12], [13], Pytko et.al. [11]. Przepływ tego płynu jest spowodowany przez rucch główki kości. Teoretyczne rozważania synowialnego nie-newtonowskiego przepływ płynu w cienkiej w cienkiej szczelinie stawu biorąc pod uwagę uproszczenia warstwy granicznej , mają zastosowania praktyczne teorii smarowania w medycynie , patrz Dowson et.al. [1], [2], Gruca [3]. Rozważanie podane w obecna artykule umożliwią znalezienie parametrów strumienia synowialnego płynui i zdolności do przenoszenia sił w szczelinie stawu biodrowego człowieka między dwoma współpracującymi powierzchniami chrząstki.

Staw biodrowy człowieka oraz wysokość szczeliny dla odkształcalnej chrząstki oraz rozkład ciśnienia w szczelinie zdrowego i chorego stawu biodrowego przy hydrodynamicznym smarowaniu, przypadki rozkładów ciśnienia w chorej, sferycznej szczelinie biodra dla hydrodynamicznego,smarowania wywołanego ruchem obrotowym, przypadki rozkładów ciśnienia w zdrowej, sferycznej szczelinie biodra dla hydrodynamicznego smarowania wywołanego ruchem obrotowym sa przedstawione w artykule.

Słowa kluczowe: *Zdrowe i chore biodra ,nośność, naprężenia ściskające*

1.Introduction

We take into account equations of conservation of momentum and continuity equation for the non-stationary flow of synovial incompressible fluid between two rotational roughness cartilage surfaces in the spherical co-ordinates. Imposing the proper boundary conditions on the basic equations the pressure p we obtain from the following Reynolds equation:

$$\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon^3}{\eta}\frac{\partial p}{\partial\varphi}\right)+R^2\sin\left(\frac{\vartheta}{R}\right)\frac{\partial}{\partial\vartheta}\left[\frac{\varepsilon^3}{\eta}\frac{\partial p}{\partial\vartheta}\sin\left(\frac{\vartheta}{R}\right)\right]=6\omega\,R^2\frac{\partial\varepsilon}{\partial\varphi}\sin^2\left(\frac{\vartheta}{R}\right), \tag{1}$$

in Ω region: $0\le\varphi\le\pi$, $\pi R/8\le\vartheta\le\pi R/2$.We denote: η-dynamic viscosity of synovial fluid, ω-angular velocity of bone head, R-radius of bone head.

2. Deformable gap height

The minimum gap height, see Dowson [1], for spherical hip joint can be obtained from the formula [2]:

$$\frac{\varepsilon_{\min}}{R}\equiv\sqrt[5]{2\pi}\;S_1^{0,4}\left(\frac{\omega R^2\eta_p}{C}\right)^{0,6},$$

$$S_1\equiv\frac{C}{ER^2},\quad \frac{1}{E}=\frac{1}{2}\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right), \tag{2}$$

where:
E_1, E_2, ν_1, ν_2 – the elastic module and Poisson ratios for bone head and cartilage,
C – load, and the quantities η, ω, R are as defined previously.

Relation for share rate $\Theta\approx\omega R/\varepsilon_{\min}$ can be written in the following form:

$$\frac{\omega R^2\eta_p}{C}\equiv\frac{S_2}{S_1}\left(\frac{\eta_\infty}{\eta_o}+\frac{\dfrac{\eta_o-\eta_\infty}{\eta_o}}{1+S_3\dfrac{R}{\varepsilon_{\min}}}\right),\quad S_2\equiv\frac{\omega R\eta_o}{ER},\ S_3\equiv A\omega. \tag{3}$$

By combining equations (2), (3) we obtain the system of two equations for determination of two unknown quantities, namely the dynamic viscosity η_p of synovial fluid and the minimum value $\varepsilon_{\min}$ of gap height, where elastic deformations of cartilage are taken into account. If we assume the following data:
$R=2.6\times10^{-2}$ m, $E=2\times10^5$ Pa, $\omega R=3\times10^{-1}$ m/s, $\eta_\infty=0.10$ Pas, $2\pi R/C=3\times10^{-4}$ m/N, $\eta_o/\eta_\infty\cong1000$, A=1.88 s, C=544.26 N, then from Eqs. (2), (3) we obtain:
$\varepsilon_{\min}=0.0000208$ m=20.88 μm and $\eta_p=$ 0.1036 Pas. If we take in computations the following quantities:
A=1.88 s, $\eta_o=100.00$ Pas, $\eta_\infty=0.10$ Pas, R=0.020 m, C=544 N, $0.50\ s^{-1}\le\omega\le10.00\ s^{-1}$, $2\times10^5\ Pa\le E\le2\times10^7$ Pa, then we obtain the minimum value of gap height in the interval:
$0.29\ \mu m\le\varepsilon_{\min}\le19.90\ \mu m$.

3. The gap height and primary calculations

The gap height has the following form:

$$\varepsilon=\Delta\varepsilon_x\cos\varphi\sin\vartheta/R+\Delta\varepsilon_y\sin\varphi\sin\vartheta/R-\Delta\varepsilon_z\cos\vartheta/R-R+[(\Delta\varepsilon_x\cos\varphi\sin\vartheta/R+\Delta\varepsilon_y\sin\varphi\sin\vartheta/R-\Delta\varepsilon_z\cos\vartheta/R)^2+(R+\varepsilon_{min})(R+2D+\varepsilon_{min})]^{0.5}. \quad (4)$$

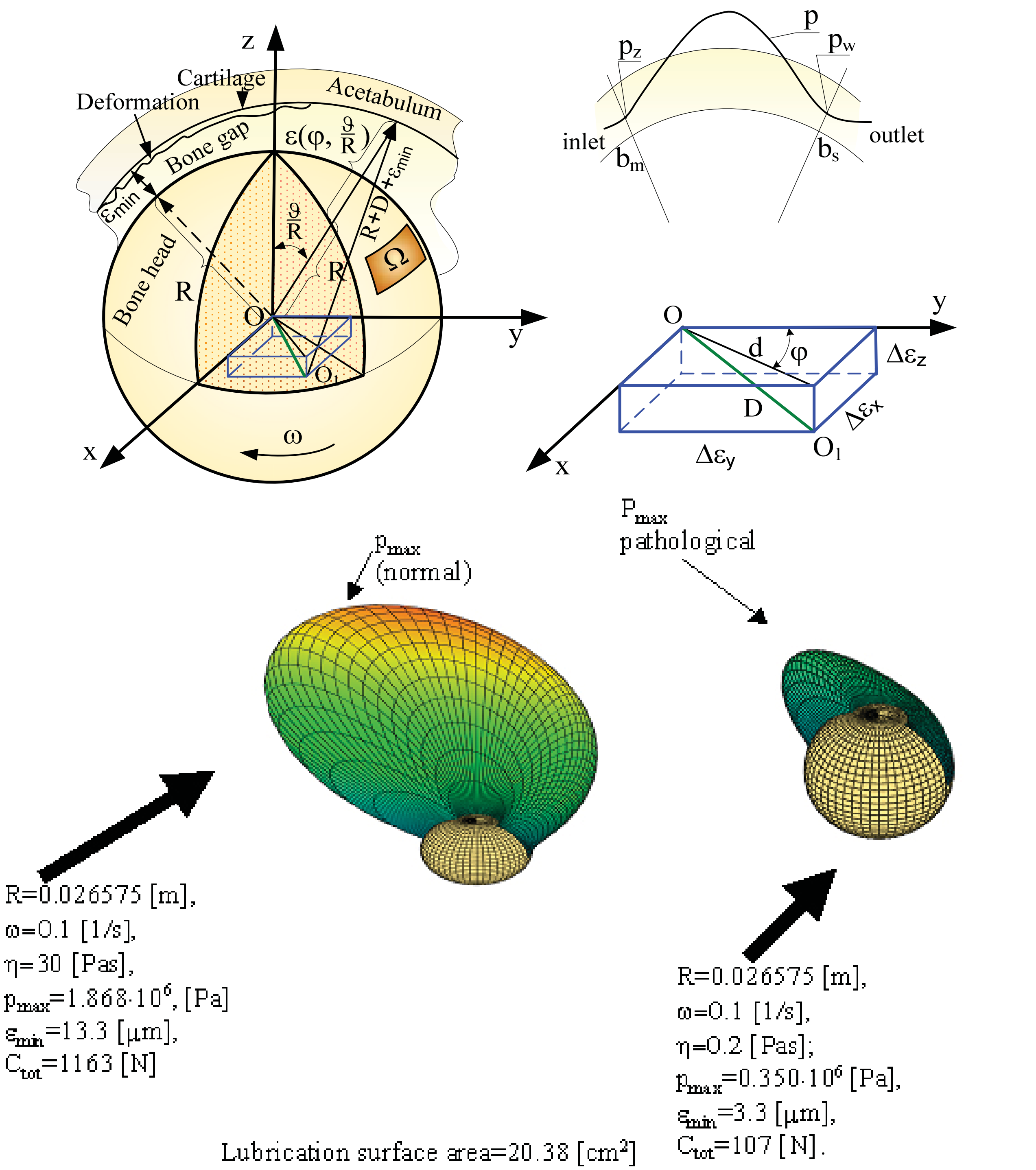

Fig. 1. Human hip joint and gap height for deformable cartilage and pressure distribution in normal and pathological spherical hip joint gap for hydrodynamic lubrication caused by rotation
Rys.1. Staw biodrowy człowieka oraz wysokość szczeliny dla odkształcalnej chrząstki oraz rozkład ciśnienia w szczelinie zdrowego i chorego stawu biodrowego przy hydrodynamicznym smarowaniu

The centre point of bone head (see Fig. 1) can be written in the following form:

$O_1(x-\Delta\varepsilon_x, y-\Delta\varepsilon_y, z+\Delta\varepsilon_z)$, while D is the distance between the centre of bone head and the acetabulum (sleeve) centre.We solve equation (1) for the region $\Omega(\varphi,\vartheta)$ resting on bone head and indicated in Fig. 1. We assume atmospheric pressure on a boundary of the region $\Omega(\alpha_1,\alpha_3)$. The centre point of bone head (see Fig. 1) can be written in the following form: $O_1(x-\Delta\varepsilon_x, y-\Delta\varepsilon_y, z+\Delta\varepsilon_z)$, while D is the distance between the centre of bone head and the acetabulum (sleeve) centre.We solve equation (1) for the region $\Omega(\varphi,\vartheta)$ resting on bone head and indicated in Fig. 1 We assume atmospheric pressure on a boundary of the region $\Omega(\alpha_1,\alpha_3)$For this region we calculate also the capacity values. In numerical calculations we assume the following values for joint gap: $\Delta\varepsilon_x$=5 μm, $\Delta\varepsilon_y$=5 μm, $\Delta\varepsilon_z$=+5 μm; the radius of bone head R=0.026575 m. Now we calculate pressure distributions in normal and pathological hip joint.

For the angular velocity of bone head $\omega=0.1\ s^{-1}$, and the average value of synovial fluid dynamic viscosity η=30.00 Pas for normal joint and η=0.20 Pas for pathological joint, we obtain the smallest gap height ε_{min}=13.3 μm for normal joint and ε_{min}=3.3 μm for pathological joint.

The hydrodynamic pressure p has its maximum value equal to 18.68×10^5 N/m$^2\approx$ 18.30 at for normal joint and 3.50×10^5 N/m^2 for pathological joint, and the capacity C_{tot}=1163 N for normal joint and the capacity C_{tot}=107 N for pathological joint, (C_{tot}-total capacity,ε-gap height) see Fig. 1.

For the angular velocity of bone head $\omega=1.0\ s^{-1}$ and the average value of synovial fluid dynamic viscosity η_o=2.00 Pas for normal joint and η_o=0.05 Pas for pathological joint, we obtain the smallest gap height ε_{min}=13.3 μm for normal joint and ε_{min}=3.3 μm for pathological joint.

The hydrodynamic pressure p has its maximum value equal to 12.79×10^5 N/m$^2\approx$ 12.79 at for normal joint and 7.26×10^5 Pa for pathological joint, and the capacity C_{tot}=775 N for normal joint and the capacity C_{tot}=268 N for pathological joint. Lubrication surface area has the value $\pi R^2\cos\pi/8\approx20.38$ cm^2.

Numerical calculations are performed by means of Mathcad 12 Professional program with the help of the Method of Finite Differences. This method satisfies the requirement of stability of numerical solutions for the partial differential equation (1).

4. Numerical calculations

Now we calculate pressure distribution in pathological joint (see Fig. 2.). For the angular velocity of bone head:
$\omega=10.0\ s^{-1}$, $\omega=7.5\ s^{-1}$, $\omega=5.0\ s^{-1}$, $\omega=2.5\ s^{-1}$, $\omega=1.0\ s^{-1}$, $\omega=0.75\ s^{-1}$ and the average value of synovial fluid dynamic viscosity:
η_o=0.0584 Pas, η_o=0.0656 Pas η_o=0.074 Pas, η_o=0.100 Pas, η_o=0.1259 Pas, η_o=0.1359 Pas,
we obtain the following maximum values of hydrodynamic pressure:
p_{max}=1.383 at, p_{max}=1.322 at, p_{max}=1.242 at, p_{max}=1.164 at, p_{max} =1.082 at, p_{max}=1.067 at,
and the carrying capacity:
C_{tot}=25.1607 N, C_{tot}=21.197 N, C_{tot}=15.941 N, C_{tot}=10.771 N, C_{tot}=5.4242 N, C_{tot}=4.3913 N, respectively.

Now we calculate pressure distribution in normal joint (see Fig. 3). For the angular velocity of bone head:
$\omega=10.0\ s^{-1}$; $\omega=7.5\ s^{-1}$, $\omega=5.0\ s^{-1}$, $\omega=2.5\ s^{-1}$, $\omega=1.0\ s^{-1}$, $\omega=0.75\ s^{-1}$
and the average value of synovial fluid dynamic viscosity:
η_o=6.813 Pas, η_o=9.261 Pas, η_o=13.082 Pas, η_o=23.163 Pas, η_o=42.987 Pas, η_o=50.118 Pas,
we obtain the following maximum hydrodynamic pressure values:
$p_{max}=45.63\times10^5$ Pa≈45.63 at, $p_{max}=46.50\times10^5$ Pa ≈46,50 at, $p_{max}=43.75\times10^5$ Pa ≈43.75 at, $p_{max}=39.10\times10^5$ Pa ≈39.10 at, $p_{max}=29.16\times10^5$ Pa ≈29.16 at, $p_{max}=25.62\times10^5$ Pa ≈25.62 at,
and the carrying capacity: C_{tot}=2935.3 N, C_{tot}=2992.5 N, C_{tot}=2818.1 N, C_{tot}=2505.6 N, C_{tot}=1852.0 N, C_{tot}=1619.4 N, respectively.
Lubrication surface area has the value $R^2\cos\pi/8\approx20.38$ cm^2.

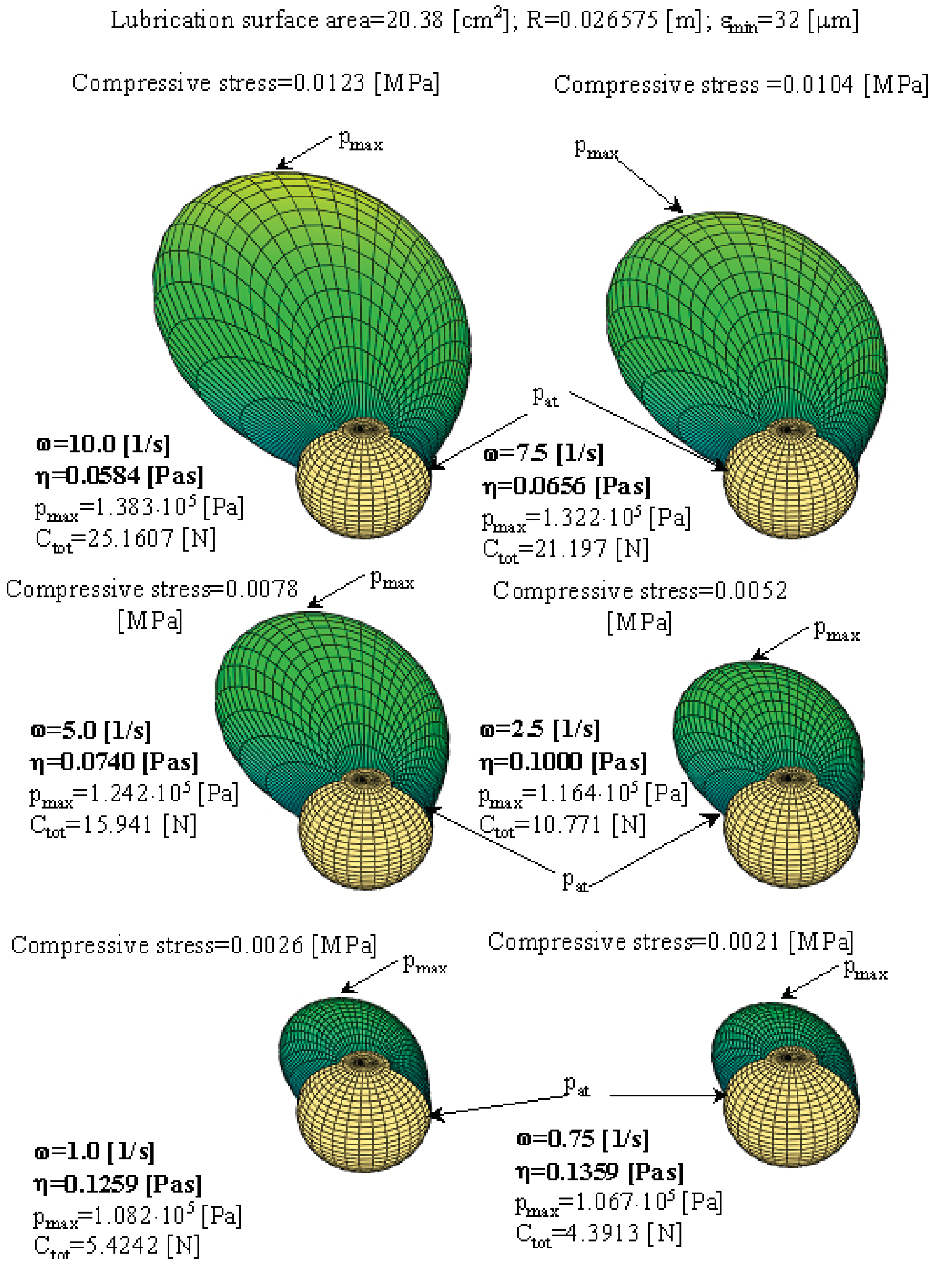

Fig. 2. Cases of pressure distribution in pathological spherical hip joint gap for hydrodynamic lubrication caused by rotation, lubrication surface, area=20.38 cm^2

Rys. 2. Przypadki rozkładów ciśnienia w chorej, sferycznej szczelinie biodra dla hydrodynamicznego smarowania wywołanego ruchem obrotowym

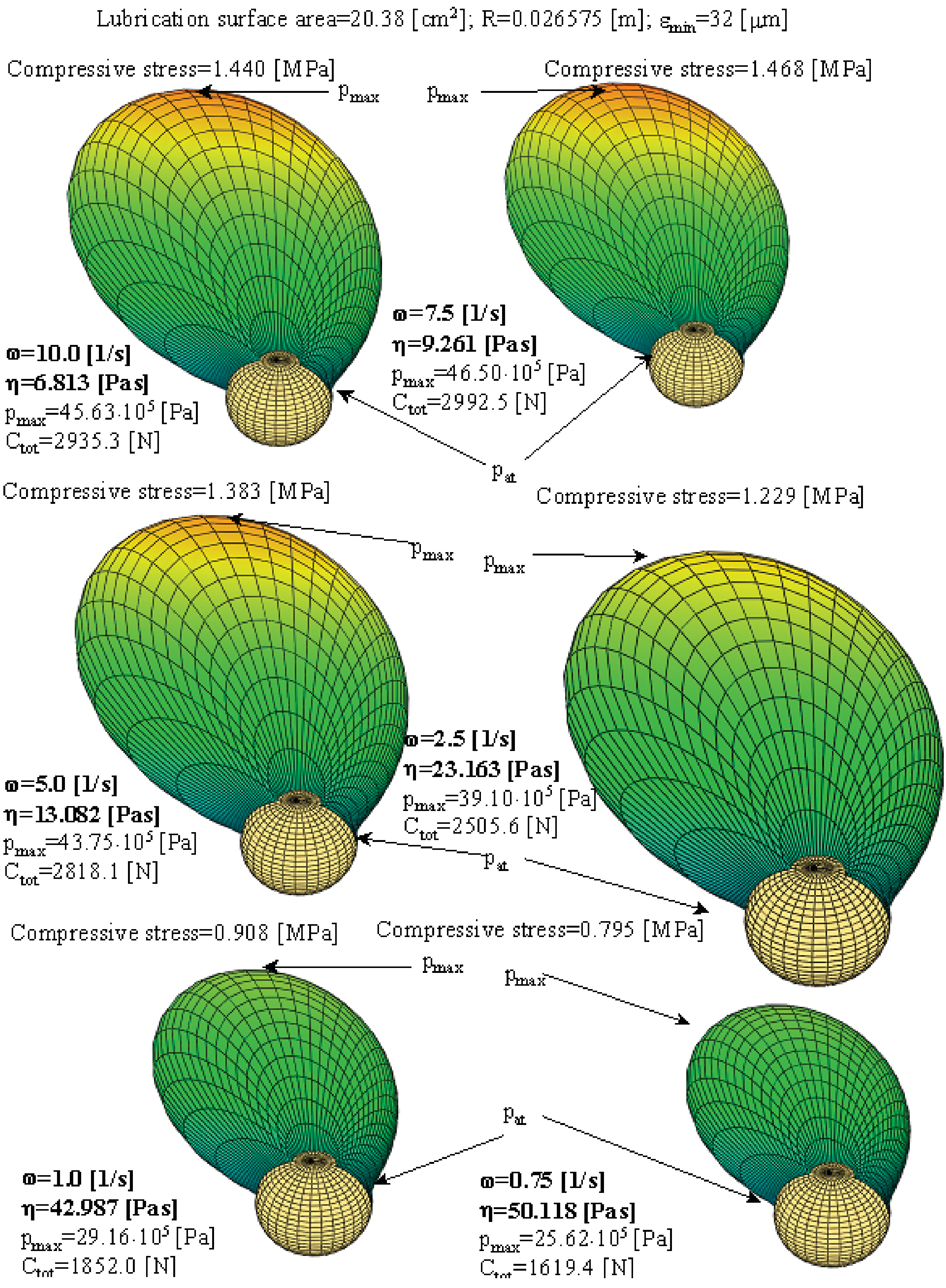

Fig. 3. Cases of pressure distribution in normal spherical hip joint gap for hydrodynamic lubrication caused by rotation, lubrication surface area =20.38 cm²

Rys. 3. Przypadki rozkładów ciśnienia w zdrowej, sferycznej szczelinie biodra dla hydrodynamicznego smarowania wywołanego ruchem obrotowym

5. Final comments

For the above given capacities in normal joint we obtain the following compressive stresses: $\tau_s = 2935.3$ N / 20.38 cm^2 = 1.440N/mm^2 = 1.440 MN/m^2, τ_s=1.468 MN/m^2, τ_s=1.383 MN/m^2, τ_s=1.229 MN/m^2, τ_s = 0.908 MN/m^2, τ_s = 0.795 MN/m^2.
In pathological joint, the compressive stresses are as follows: τ_s=25.16N/20.38cm^2=0.0123 N/mm^2=0.0123 MN/m^2, τ_s=0.0104 MN/m^2, … . These stresses are smaller than the compressive strength 21 MN/m^2 of normal human cartilage. Compressive strength of pathological human cartilage has the value of about 1MPa [2], [3].

The present paper shows the method of determination of approximate solutions of the partial non-linear differential equations of non-Newtonian, asymmetrical synovial fluid flow in the thin gap occurring in human joint in curvilinear, orthogonal co-ordinates.

The presented method enables to obtain solutions in the form of Taylor series with increasing powers of the small parameter A obtained in experimental way for synovial fluid.

In the particular case of the symmetrical flow we can, by virtue of the presented theory, find analytical solutions in a simple form. The percentage corrections of pressure caused by the non-Newtonian properties of synovial fluid are examined numerically. For large shear rates within the range: 100 $s^{-1} \le \Theta \le 1000$ s^{-1}, the viscosity of synovial fluid is small and has values: 10^{-1} Pas$\le \eta \le 1$ Pas. In this case we obtain small pressure changes from 2% to 4%. For small shear rates within the range: 10^{-1} $s^{-1} \le \Theta \le 10$ s^{-1}, when viscosity of synovial liquid is large, i.e. within the range: 10 Pas$\le \eta \le 100$ Pas, from Eq. (5) we obtain pressure changes from 7% to 15%.
- Note 1: If Young Modulus of cartilage and bone decreases then the least value of joint gap height increases.
- Note 2: If Young Modulus of cartilage decreases then viscosity of synovial fluid increases.

Acknowledgement

The present research is financially supported within the frame of the TOK–FP6, MTKD-CT-2004-517226

5. References

[1] Dowson, D., Van Mow, C., Ratcliffe, A., Woo, S. L. Y., *Biomechanics of Diarthrodial Joint.* Springer-Verlag, Chap.29, New York-Berlin, 1990.

[2] Dowson, D., *Advances in Medical Tribology*, Mech. Eng. Publ. Limited Bookcraft UK Limited Bath., 1998.

[3] Gruca, A., *Chirurgia Ortopedyczna*, (in Polish), PZWL, Warszawa, 1993.

[4] Maquet, P. G. J., *Biomechanics of the knee*, Springer-Verlag, Berlin-Heidelberg- New York, 1984.

[5] Mow, V. C., Ratcliffe A., Woo S., *Biomechanics of Diarthrodial Joints*, Springer-Verlag, Berlin-Heidelberg-New York,1990.

[6] Mow, V. C., Guilak, F., *Cell Mechanics and Cellular Engineering,* Springer-Verlag, Berlin-Heidelberg-New York, 1994.

[7] Mow, V. C., Foster R. J., *Tribology.* (Chapter 15) In: *The Adult Hip*, edited by JJ Callaghan, Rosenberg AG, Rubash HE; Lippincott-Raven Publishers, Philadelphia, 1998.

[8] Mow, V. C., Soslowsky L. J., Friction, lubrication and wear of diarthrodial joints. In: Mow V. C., Hayes W. C., Eds. *Basic orthopedic biomechanics*, Raven Press; pp.254-291. New York, 1991.

[9] Mow, V. C., Holmes, M. H., Lai, W. M., *Fluid transport and mechanical properties of articular cartilage*, Journal of Biomechanics, Vol.17, pp.337-394, 1984.

[10] Mow, V. C., Wilson, C. Hayes, W. C., *Biomechanics,* Lippincott-Wiliams, Wilkins Publishers, Philadelphia-Baltimore-New York-London, 1998.
[11] Pytko, S., Wierzcholski, K., *Analytical Biobearing Calculation for Experimental Dependencies Between Shear Rate and Synovial Fluid Viscosity*, Proceedings of International Tribology Conference in Yokohama, 3, pp.1975-1980, Yokohama 1995.
[12] Wierzcholski, K., *The method of solutions for hydrodynamic lubrication by synovial fluid flow in human joint gap*, Control and Cybernetics, Vol. 31 No.1, pp.91-116,Warszawa 2002.
[13] Wierzcholski, K., *Synovial fluid squeeze film flow in curvilinear biobearing human gap*, Polish Society of Medical Informatics, Computers in Medicine 5, Volume II pp. 151-156. Łódź 1999.