THE SELECTED PROBLEMS OF MULTI-LAYER PAVEMENTS MODELLING IN DESIGN AND DIAGNOSTICS

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Abstract

This paper will present selected problems connected with modelling of multi-layer pavements. Multi-layer pavement models, which utilize solutions such as the one of Boussinesq, Burmister, Kogan, Odemark and others, contain many simplifications, which bas a significant influence on the estimated real value of stress, strain, deflection as well as on the coefficient of pavement layers interaction. In the article based on the experiments carried out, we want to present our own layered-pavement models and solutions to them, which can be used mainly in the diagnostics of bearing capacity. The estimation of bearing capacity is conducted on the basis of full time data obtained with the FWD device. Moreover, the conclusions implicated by the above mentioned analysis are of big importance in design and operation process of road and airfield pavements. They pertain not only to the influence of modulus and thickness of particular layers but also to the way of pavement layers interaction, Poisson's coefficient, thermal characteristics and others.

Keywords: *multi-layer, pavement, modelling, dynamic impact, thermal-impact*

1. Introduction

A road pavement is constructed as a set of layers containing various materials interacting with each other, and during operation it is exposed to dynamic and thermal actions. It is a very complex system, difficult for modelling and mathematical/physical description.

The knowledge of quality and quantity characteristics of the process of dynamic loading of the pavement is crucial for dimensioning, operation use and current estimation of a condition of a road pavement. The above mentioned issues include research on physical phenomena occurring in the pavement under the influence of impacts and vehicles moving along it. Thermal-mechanical waves propagating inside the pavement cause variable stress and strain conditions generating mini fatigue effects of the materials in particular pavement layers. The results of dynamic and thermal actions depend not only on stiffness moduli and expansion coefficients of pavement materials, but also to a significant extent on such parameters as:

- velocity of propagation of waves in layers,
- Poisson's ratio,
- wave impedance ,
- inter-layer connections,
- friction coefficient between a wheel and the pavement,
- thermal match between layers etc.

The influence of the above mentioned factors onto the pavement bas been discussed in relation to its quality or presented graphically in the further part of the paper.

2. Physical model of equivalent pavement layer

The pavement consisting of a few various layers resting on subsoil is substituted by a layer with physical-mechanical parameters equivalent as far as dynamic aspect is concerned (Fig. 1). In order to determine the required parameters of the equivalent layer we will respectively use modified calculation models according to the following equivalence criteria:
1. equivalence of mass density of a layered structure

$$\sum_{i=1}^{n} \rho_i h_i = \rho_z \cdot H_z \;, \quad \rho_{n+1} = \rho_p \;, \tag{1}$$

2. equivalence of wave transition time through layered structure

$$\sum_{i=1}^{n} \frac{h_i}{c_i} = \frac{H_z}{c_z} \;, \quad c_i = \sqrt{\frac{E_i}{\rho_i}} \;, \tag{2}$$

3. equivalence of wave impedance of a layered structure

$$\sum_{i=1}^{n} f(\rho_i c_i) = \rho_z c_z \;, \tag{3}$$

4. equivalence of elastic strain energy for a layered structure

$$E_{n+1} = E_\rho \,, \quad \sum_{i=1}^{n} \sigma_i \varepsilon_i = \sigma_z \varepsilon_z \;, \tag{4}$$

where the parameters for equivalent layer have been designated as:

$\rho_z, c_z, H_z, E_z, \sigma_i, \sigma_z$ - stress tensors,

$\varepsilon_i \varepsilon_z$ - strain tensors.

In the Fig. 1 the adopted principle of equivalence of parameters for multi-layer and substitute structures have been presented.

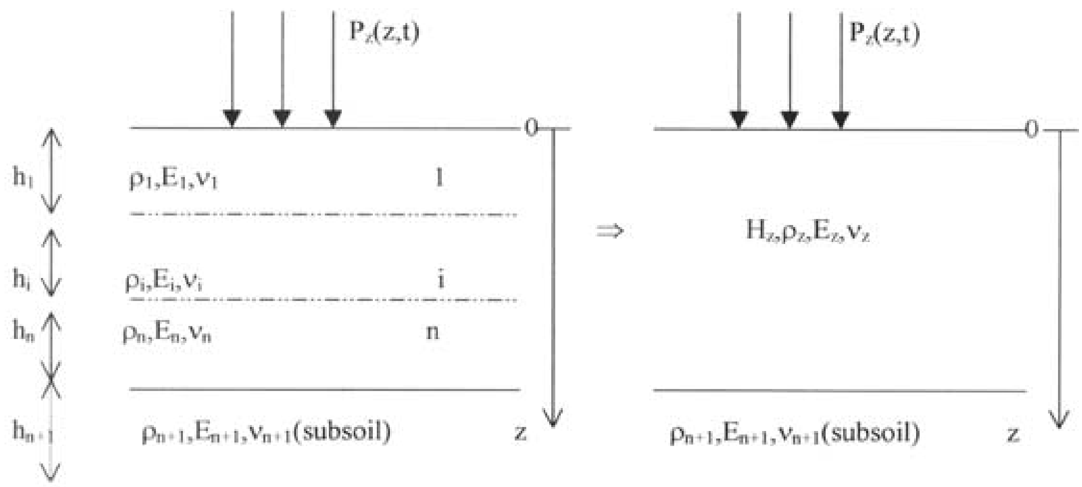

Fig. 1. Equivalence of multi-layer and substitute systems

3. Analysis of the influence of dynamic impacts

3.1 Wave impacts

Dynamic load results in the propagation of stress waves inside the pavement, which undergo the phenomenon of refraction at the limits of the layers. Geometric set of waves in the pavement depends on configuration of subsequent layers and their physical-mechanical properties.

The volume waves, head waves and cone seismic waves propagate in the pavement. Also, on the free surface there is the Rayleigh's wave, and at the limits of the layers the Love's waves.

Issues of propagation and interaction of waves generated in media with surface loads constitute an important research domain of wave phenomena, as far as practical and empirical aspects are concerned.

Influence of this kind of a load onto a given medium, can be generally described as composed of :

– static impact, proportional mainly to a force imposed on a surface (slow-changing load),
– pulsating impact: load or structural model being a load (e.g. a beam or a plate on elastic foundation), in which the crucial thing is a ratio of the free vibrations frequency to the frequency of forcing,
– impact of wave type (they accompany previously mentioned impacts), whose complexity and poses creates a main obstacle of a model and mathematical nature in their thorough recognition.

A complex quantitative presentation requires research into using a model of the phenomenon taking into account number characteristics of a structure itself (value of pressure imposed onto the pavement, speed of structure) and mechanical properties of pavement material and soil underneath the pavement (wave speeds, mass density, pavement thickness etc.).

In order to present the problem from the quality point of view, geometric structure and kinds of generated waves have been discussed, using the example of load acting onto the surface of an elastic medium.

The developing set of waves being shaped has a direct influence on the results of activity of a load onto the medium, i.e. on the magnitude of strains and stresses produced in the medium. The major factor influencing the values of the produced results is the value of speed of loading in relation to the wave speed in the medium, i. e. longitudinal, transversal and especially to the Rayleigh's wave. They are the underlying cause of damage to the surface structures.

This phenomenon is presented in the Fig. 2 and 3.

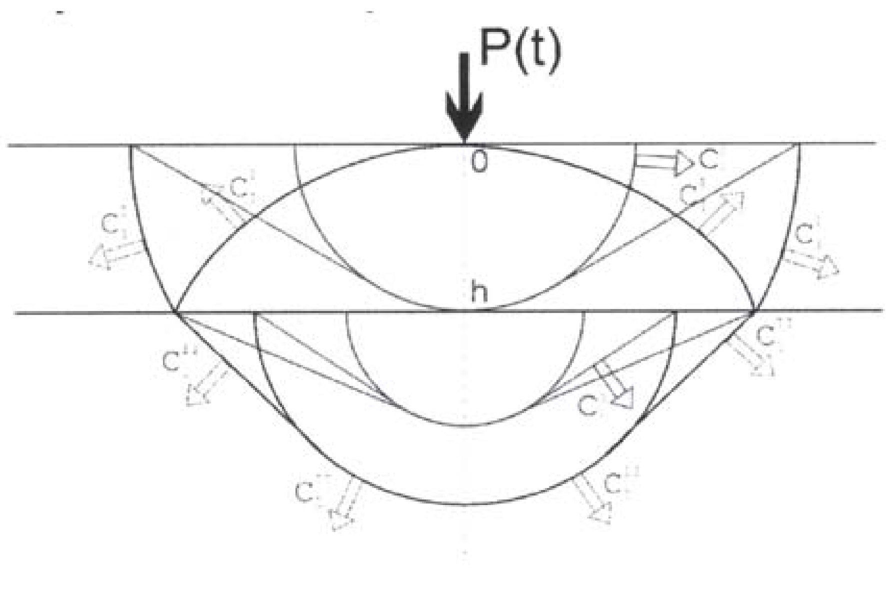

Fig. 2. The scheme of longitudinal and transversal waves propagation in a layer on flexible subsoil (case $C_1^I > C_1^{II}$, in practice $E_{layer} > E_{subsoil}$)

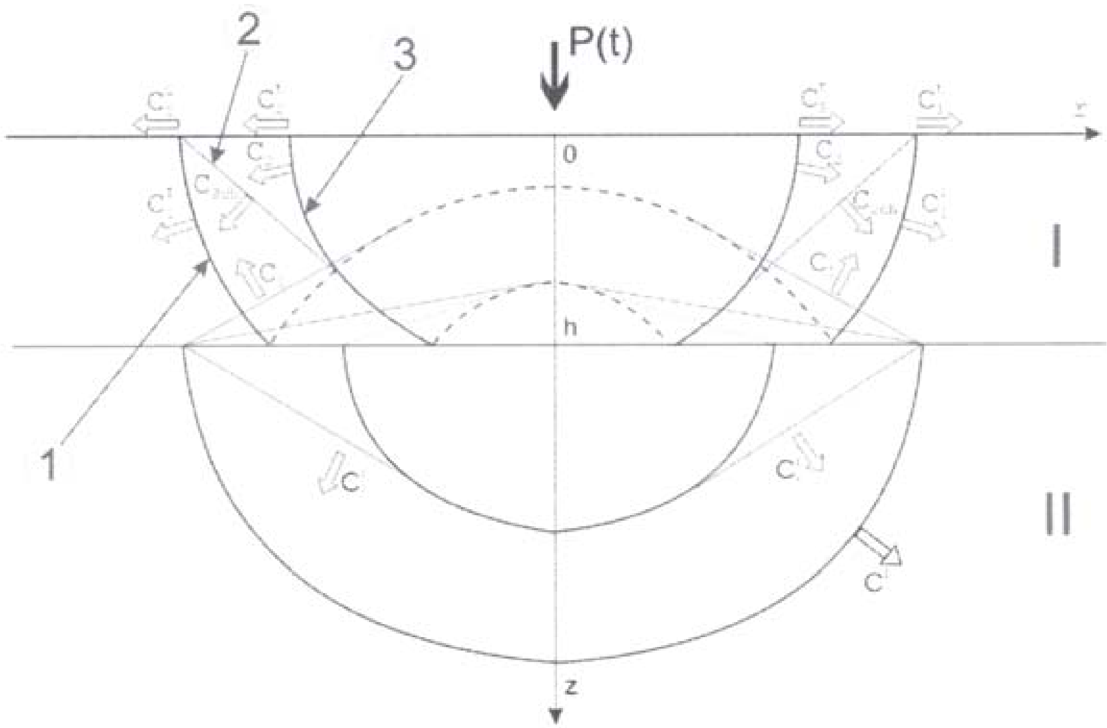

Fig. 3. The scheme of longitudinal and transversal waves propagation in a layer on rigid subsoil, (case $C_1^I<C_1^{II}$, in practice $E_{layer}<E_{subsoil}$), where the presented kinds of waves have been designated as follows: 1 - longitudinal waves, 2 - head waves of Schmit, 3 - transversal waves

Refraction of waves is described as k_i parameter expressing impedance (wave resistance) of layers:

$$k_i = \sqrt{\frac{E_i \rho_i}{E_{i+1}, \rho_{i+1}}}\ , \tag{5}$$

where:

E_i - stiffness modulus of a layer no i,

ρ_i - mass density of a layer no i.

The influence of the values of the match parameter k_i bas been presented in the Tab. l .

Tab. l. The change of values of displacements (u) and stresses (σ) in upper layer and in base course depending on changes in values of wave matching of layers

Value of parameter k	Displacements		Stresses	
	Upper layer	Base course	Upper layer	Base course
$k < 1$	u_1↗	u_2↗	σ_1↗	σ_2↘
$k > 1$	u_1↘	u_2↘	σ_1↘	σ_2↗

3.2 The influence of Poisson's ratio

Conclusions regarding the influence of Poisson's ratio $-v$ on deflections on the surface of a layer are based directly on the solution of a problem, formulated for half-space. For the half-space deflections are proportional to the known formula (2), which results in its little influence on deflection values.

$$u = (1 - v^2)u_0, \tag{6}$$

where:

u - def1ections at surface, v - Poisson's ratio, u_0 - deflection function

In the case of a layer interacting with subsoil, the relation of deflection at the surface with Poisson's ratio of the layer becomes more complex, as it also depends on the kind of interaction between the layer and subsoil, layer thickness and its location in the structure.
The parameter characterizing influence of Poisson's ratio on the layered structure is expressed by the approximate relation (7), which results, among others, from the formulas of D.M. Burmister (1).

$$\overline{K} = \frac{a-1}{a-x}, \tag{7}$$

where: $a \cong \frac{E_1}{E_2}$ and $x \cong 3 - 4v$.

As it can be seen, the influence of Poisson's ratio for the layered structure is considerably stronger than half-space. The results of a comparative analysis of the influence of Poisson's ratio on deflections at surface for both cases have been illustrated in the Fig. 4 and 5.

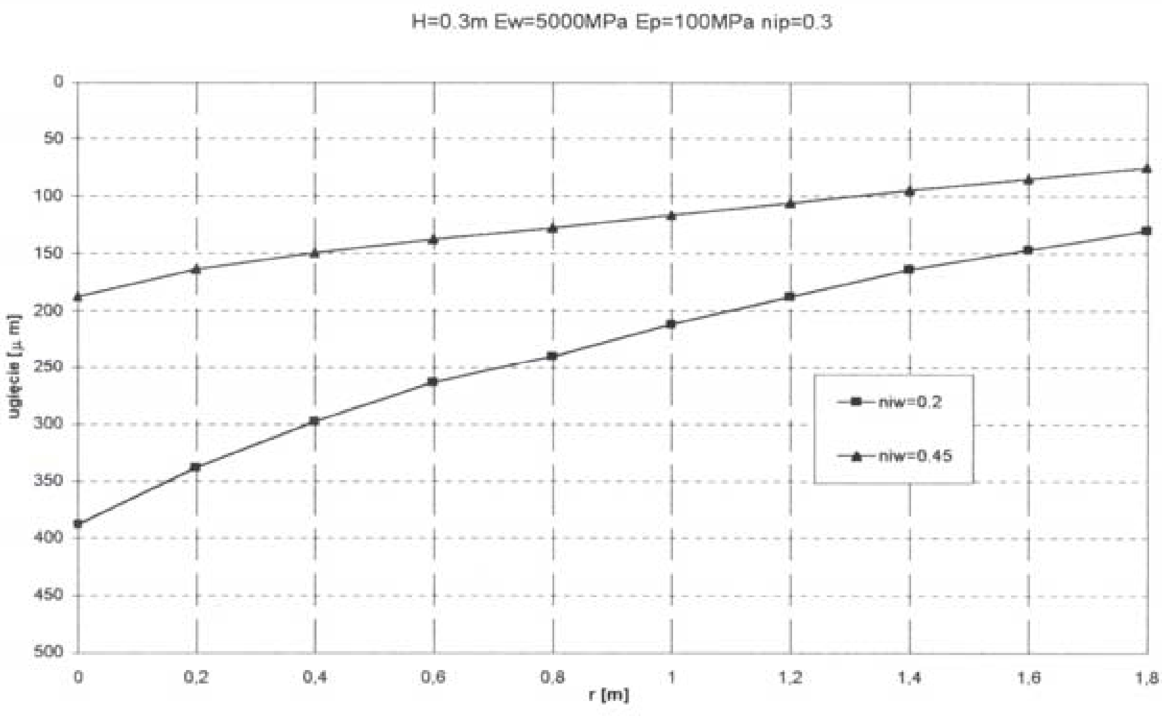

Fig. 4. The influence of Poisson's ratio values on deflections at surface in layered model

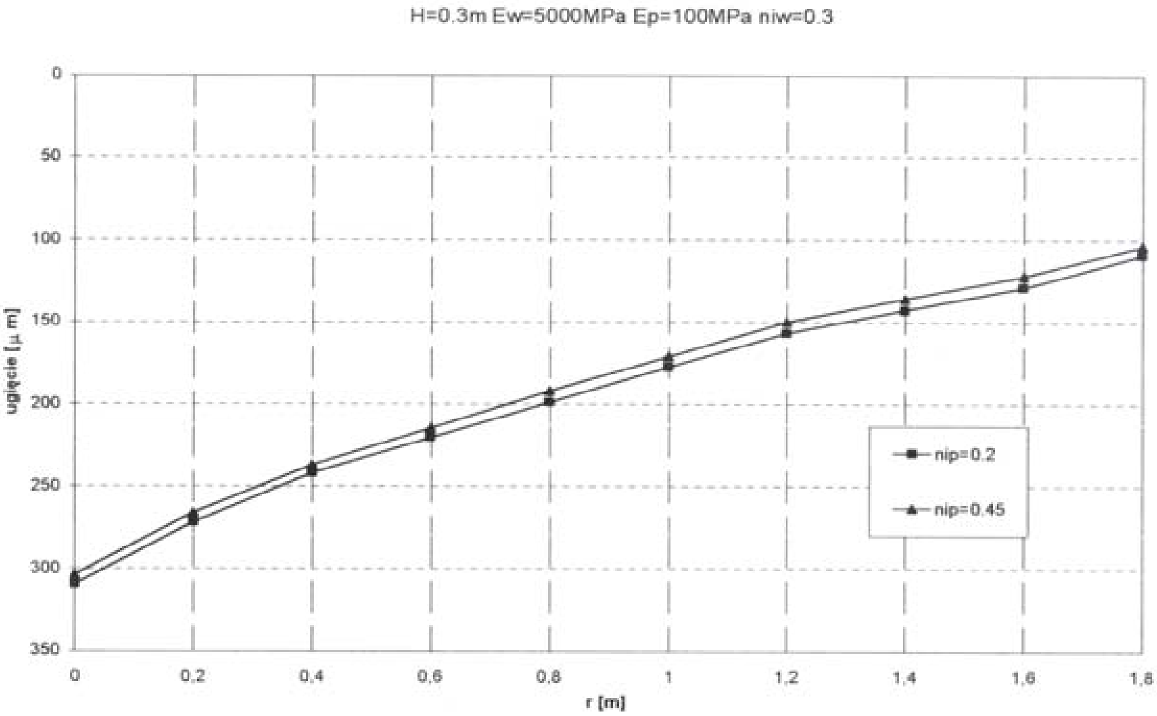

Fig. 5. The influence of Poisson's ratio values on deflections at surface in half-space model

3.3. The influence of friction at surface

Taking into account the loading on the pavement with tangential stresses caused by friction of a vehicle tire with the pavement, results in increased displacements in the pavement, which can be determined by the following formulas (8) and (9):

- for vertical displacements:
$$\frac{u\big|_{\eta_0 \neq 0}}{u\big|_{\eta_0 = 0}} = 1 + \eta_o (1 - 2\beta^2), \qquad (8)$$

- for radial displacements:
$$\frac{w\big|_{\eta_0 \neq 0}}{w\big|_{\eta_0 = 0}} = 1 + \frac{\eta_o}{\beta^2}, \qquad (9)$$

where: η_0 - coefficient of friction of tire with pavement,

$$\beta^2 = \frac{1 - 2v}{2(1 - v)},$$ v - Poisson's ratio.

To illustrate the influence of friction forces on the magnitude of deflections, the calculation example bas been presented below.
It bas been assumed: $v = 0.3$; $\eta_0 = 0.4$
Hence, it bas been obtained:

$$\frac{u|_{\eta_0 \neq 0}}{u|_{\eta_0 = 0}} \approx 1{,}2, \qquad \frac{w|_{\eta_0 \neq 0}}{w|_{\eta_0 = 0}} \approx 2{,}5 \quad .$$

In the example, with the assumed friction the value of vertical deflections increases by about 20%, whereas radial deflections by about 150% in relation to the deflections without friction force.

3.4 The influence of connection between pavement layers

The limit conditions at the layers boundary:
- with full connection between layers:

$$\begin{aligned} u_i &= u_{i+1} \\ w_i &= w_{i+1} \\ \sigma_z^i &= \sigma_z^{i+1} \, , \\ \sigma_{rz}^i &= \sigma_{rz}^{i+1} \end{aligned} \tag{10}$$

- with lack of connection between layers:

$$\begin{aligned} \sigma_{zz}^i &= \sigma_{zz}^{i+1} \\ \sigma_{rz}^i &= 0 \\ \sigma_{rz}^{i+1} &= 0 \quad , \\ u_i &= u^{i+1} \end{aligned} \tag{11}$$

- with partial connection between layers and taking into account friction:

$$\begin{aligned} \sigma_{zz}^i &= \sigma_{zz}^{i+1} \\ \sigma_{rz}^i &= \sigma_{rz}^{i+1} = \eta_{i,i+1} \, \sigma_{zz}^i \quad , \\ u_i &= u^{i+1} \end{aligned} \tag{12}$$

$\eta_{i,i+1}$ - coefficient of friction between layers i and $i+1$.

The formulas determining influence of connection between layers defined by the parameter η on the values of displacements of pavement and subsoil, which are used in calculations of bearing capacity of the structure result from conditions (11) and (12).

4. Analysis of the influence of thermal impacts

Thermal phenomena (especially in flexible and semi-rigid structures) cause significant changes in behaviour of the pavement under loading conditions.
Thermal stresses are proportional to the temperature difference and are expressed by the formula (13):

$$\sigma_T = -\lambda K (T - T_0) \, , \tag{13}$$

where: λ - thermal expansion coefficient,

K - modulus of volumetric stiffness,
T - pavement temperature,
T_0 - ambient temperature

Distribution of temperatures in a layer and subsoil depend on many factors, among others such as:
- temperature changes cycle (its amplitude and period),
- thermal matching of layers,
- thickness of layers,

which are expressed with the following non-dimensional numbers:

$$\beta = \frac{aa}{k\sqrt{\omega}} \text{ - Biot's number;} \tag{14}$$

$$r = \frac{k_p a}{k a_p} \text{ - coefficient of thermal matching of layers;} \tag{15}$$

$$\gamma = \frac{h\sqrt{\omega}}{a} \text{ - Fourier's number;} \tag{16}$$

$$\delta = \frac{a}{a_p} \text{ - coefficient of diffusion matching of layers; } a = \frac{K}{\rho c}, \tag{17}$$

where: k and k_p - coefficient of thermal conductivity (internal) in the material of a pavement layer and of a subsoil,
a, a_p - coefficient of diffusion of layers,
a - coefficient of heat exchange (external);
h - thickness of layer;
ω - frequency of temperature changes; $\omega = \frac{2\pi}{\tau}$; τ - period of temperature changes.

The results of calculations of temperature distribution in the layer and subsoil have been presented in Fig. 6 and 8. T_a is the temperature amplitude at the pavement surface.

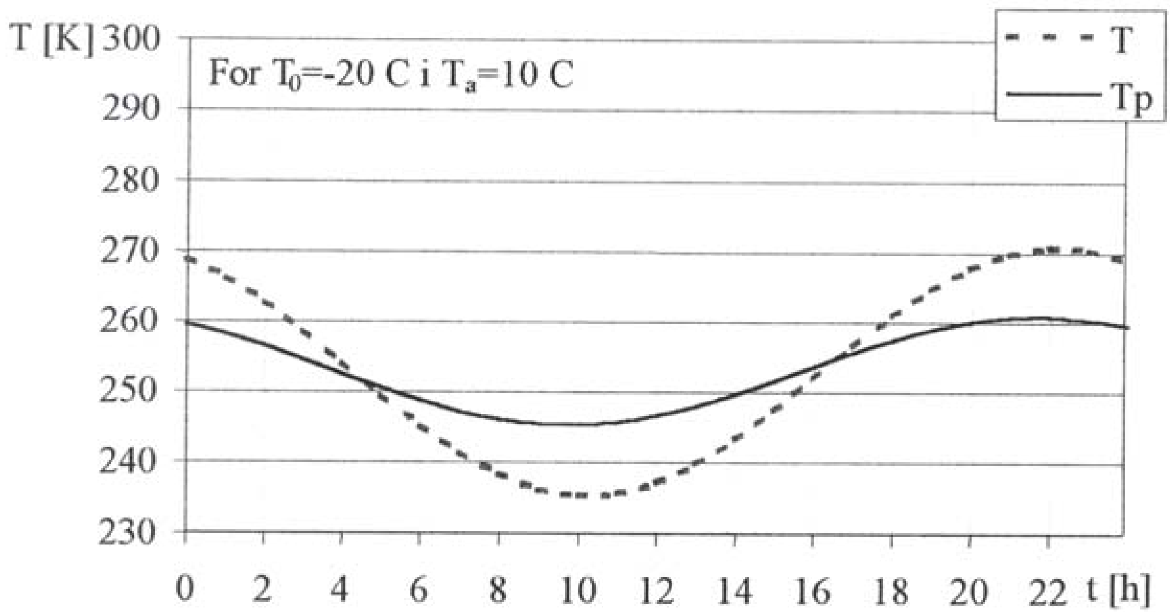

Fig. 6. The temperature changes in a pavement and subsoil with ambient temperature T_0 = - 20°C

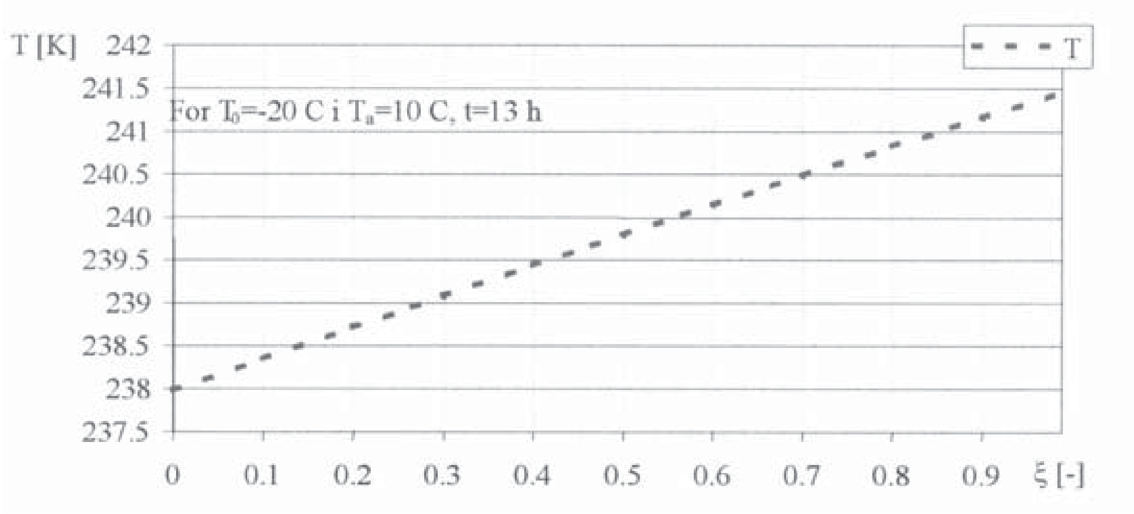

Fig. 7. The temperature change in a pavement with ambient temperature T_0 = - 20°C

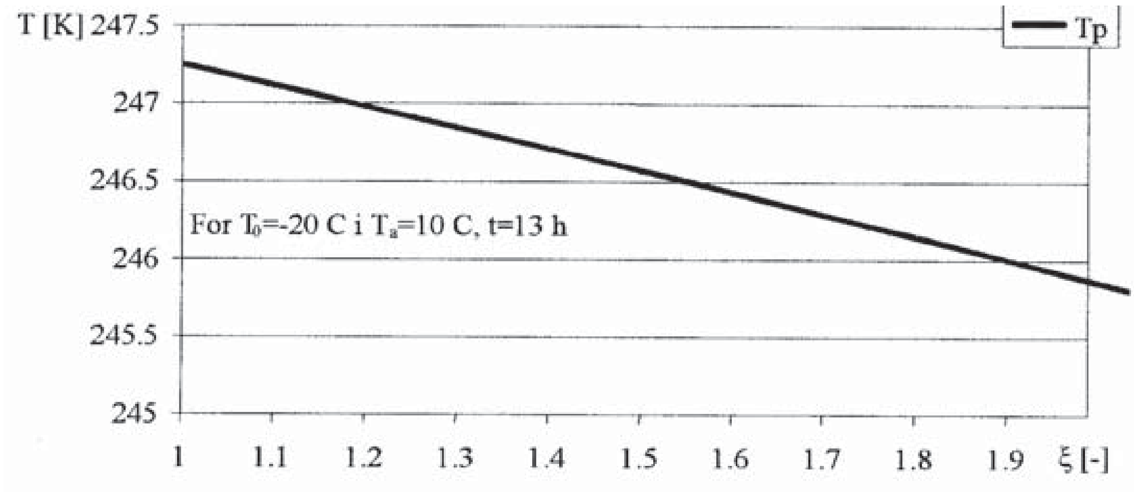

Fig. 8. The temperature change in a subsoil with ambient temperature T_0 = 20°C

5. Summary and conclusions

The presented results regarding modelling and influence of dynamic and thermal impacts indicate their considerable importance for the behaviour of the layered pavement in the process of its operation.

The factors, which characterize their qualitative and quantitative influence depend on a design method of the structure of the layered pavement, characteristics of used materials, technology of their production and also on appropriate construction of the pavement. Elaboration of new methods to design diagnostics and new construction technologies and their implementation require extensive research in the field of modelling, description and methods to solve the thermo-mechanical subjects in the layered pavements.

It is necessary to continue research into new materials for pavement construction, intentionally directed towards practical modification of not only the basic parameters as e.g. elastic moduli but also of material characteristics having also important significance, such as Poisson's ratio, velocity of waves inside the pavement (mass density), and also parameters characterizing thermal features of road construction materials. It allows constructing pavements with optimal features adjusted to real traffic loads, water -soil conditions and ambient conditions.

It is undoubtedly one of the ways of creating new, durable and at the same time not necessarily too expensive pavements.

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