DETERMINATION OF THE INDICATED WORK FROM CRANKSHAFT ROTATIONAL SPEED MEASUREMENTS

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Abstract

The paper presents the results of investigation into the relationship between the value of the indicated work of individual cylinders and variations in the momentary crankshaft rotational speed of a multi-cylinder engine in selected angular positions. A multi-mass dynamic model for torsional vibrations of the crankshaft of an engine furnished with a torsional vibration damper and flexibly coupled with a generator is developed. The results of measurement of pressure in the cylinders of an eight-cylinder industrial engine are used to calculate crankshaft and generator rotational speeds. The simulation results are compared with the speed measurements. A clear correlation between the indicated work of individual cylinders, as calculated in 90 degree angular intervals, and the crankshaft rotational speed is found to exist. The indicated work of individual cylinders shows a little poorer correlation with the generator rotational speed. The condition for the reliability of this method of evaluating cylinder operation quality is, however, to measure these speeds with a high accuracy. For the front end of the engine shaft, the required accuracy is at a level of ±0.025%. For the front end of the generator, on the other hand, the diagnosis of the operation of individual cylinders is possible with even more accurate measurements, being at a level of ±0.01% and, in addition, it is less reliable compared to the measurement of the momentary rotational speed of the engine.

Keywords: *indicated work fluctuation, crankshaft speed fluctuation*

OKREŚLENIE PRACY INDYKOWANEJ NA PODSTAWIE POMIARU PRĘDKOŚCI OBROTOWEJ WAŁU KORBOWEGO

Streszczenie

W pracy przedstawiono wyniki badań związku między pracą indykowaną poszczególnych cylindrów silnika wielocylindrowego i chwilową prędkością obrotową silnika w wybranych położeniach kątowych. Sporządzono wielomasowy dynamiczny model drgań skrętnych wału korbowego silnika z tłumikiem drgań skrętnych i elastycznie sprzężonego z generatorem. Wyniki pomiaru ciśnień w cylindrach ośmio-cylindrowego silnika przemysłowego zostały wykorzystane do obliczeń prędkości obrotowych silnika i generatora. Wyniki symulacji zostały porównane z wynikami pomiarów prędkości. Stwierdzono istnienie wyraźnej korelacji między pracą indykowaną poszczególnych cylindrów obliczoną w 90 stopniowych przedziałach kątowych a prędkością obrotową wału korbowego. Praca indykowana poszczególnych cylindrów nieco gorzej koreluje z prędkością generatora . Warunkiem wiarygodności tej metody oceny jakości pracy cylindrów jest jednak pomiar tych prędkości z dużą dokładnością. W przypadku wolnego końca wału silnika wymagana dokładność jest na poziomie ±0,025%. Natomiast w przypadku wolnego końca generatora diagnozowanie pracy poszczególnych cylindrów możliwe jest przy jeszcze dokładniejszej pomiarach na poziomie ±0,01%, i dodatkowo jest mniej wiarygodne niż w przypadku pomiaru chwilowej prędkości obrotowej silnika.

Słowa kluczowe: *fluktuacje pracy indykowanej, fluktuacje prędkości obrotowej wału korbowego*

For the diagnostics of the operation of individual cylinders and, at the same time, the control of the multi-cylinder engine, attempts are being made to make use of readily available engine operation parameters, including information on variations in the momentary rotational speed of the engine. Researches aimed at establishing the relationship between fluctuations in the engine shaft rotational speed and the indicated work of cylinders have been conducted for many years. One of the two methods of determining the correlation between the measured rotational speed and the indicated work is mainly used. The first method is based on the modelling of crankshaft assembly dynamics and the frequency analysis of rotational speed variation [2, 6, 8, 9-11]. The second method relies on interrelations between cylinder pressures and various parameters of the rotational speed and geometry of the engine [4-5]. The present work makes an attempt to identify a correlation between the momentary rotational speed of the engine in selected angular positions and the indicated work of individual cylinders.

1. The multi-mass model of the generating set

The subject of testing was an eight-cylinder biogas internal combustion engine [3, 7] being the drive unit of a 650 kW-power industrial power generator. With the aim of making the evaluation of the effect of the operation of individual cylinders of the eight-cylinder internal combustion engine on the variation of the rotational speed of the engine shaft and, at the same time, the generator shaft, a multi-mass model of the 8A20G generating set under examination was developed. In the discrete model of the multi-mass mechanism performing an unsteady rotational motion, the equations of motion were set separately for the substitute moments of inertia of this mechanism, referred to as masses for short. The engine crank mechanism was assumed to be composed of sections of an ideally elastic and weightless shaft of a known torsional elasticity constant, k_i, and infinitely thin and non-deformable disks of a known dynamic moment of inertia, θ_i. It was also assumed that the vibration damping moments acting on individual disks were viscotic, i.e. the moments of damping forces were the products of damping coefficients, d_i, and angular velocity. The torques inducing torsional vibrations are the variable components of the moment M_i of tangential forces acting on the crank-pins of individual shaft cranks.

For the building of the model, the factory specifications of the 8A20G generating set and the results of torsional vibration measurements made for this generating set by the manufacturer [1] were used. According to these data, for the calculation of torsional vibrations occurring in the crank mechanism of the eight-cylinder in-line engine coupled with the generator rotors by a flexible coupling, the real system of masses and elastic elements was substituted with a 14-mass model consisting of 13 elastic elements connecting individual masses.

As a result, the model is composed of 14 ordinary differential equations of the second order having the following form:

$$\theta_i \frac{d^2\varphi_i}{dt^2} = -d_i \frac{d\varphi_i}{dt} - b_{i-1}\left(\frac{d\varphi_i}{dt} - \frac{d\varphi_{i-1}}{dt}\right) - b_i\left(\frac{d\varphi_i}{dt} - \frac{d\varphi_{i+1}}{dt}\right) - k_{i-1}(\varphi_i - \varphi_{i-1}) - k_i(\varphi_i - \varphi_{i+1}) + M_i \quad (1)$$
$$(i = 1, 2, \dots, 13, 14)$$

For the both extreme masses, this equation reduces, because for the first mass *(i = 1)*, elements no. *(i-1)* do not occur, and for the last mass *(i = 14)*, no elements no. *(i+1)* occur.

In this system of ordinary differential equations describing the composite unsteady motion of the 14-mass generating set model, only the moments M_i for 8 engine cylinders *(i = 3, 4, ... , 9, 10)* are non-zero. These are variable momentary indicated moments for each working cylinder. Also for the generator rotor (mass 14), the moment M_i is non-zero, has a negative sign, and is a generator resistance moment.

For solving this system of equations, the Runge-Kutty-Fehlberg method with automatic integration step correction was employed.

1. 2. Numerical values of the characteristic parameters of the generating set model

The torsional elasticity of the vibration damper was adopted according to the data of [1] concerning the dynamic rigidity of the damper for three different deflections of its springs, φ_t, and approximated with a 2-degree polynomial. The torsional elasticity of the flexible coupling was assumed as a function of the moment m_s transferred by the coupling in the form of a 4-degree polynomial, as given in reference [1].

The external damping acting on the substitute masses of the movable engine parts was assumed to be the same for all of the eight cylinders and equal to 1/8 of the total engine damping. For the remaining engine and coupling rotating masses, on the other hand, the assumption was made that the damping was zero.

For the generator, instead of the external damping coefficient, the generator resistance moment was included in the right-hand side of the equation of motion, which, in the oversynchronous range of asynchronous machine rotation frequency, depends approximately linearly on the rotor rotation frequency:

$$M_{14} = -M_{nom} \cdot \frac{\omega_{14} - \omega_{syn}}{\omega_{nom} - \omega_{syn}}. \tag{2}$$

The internal damping occurring in the elastic elements connecting individual mass moments of inertia is caused by the material hysteresis phenomenon that causes, e.g., a cyclically deformed material to heat up. The viscotic internal damping occurs mainly in the torsional vibration damper and in the flexible coupling connecting the engine with the generator. The coefficient of the substitute viscotic internal damping occurring in the vibration damper was adopted according to the data provided in [1] for the damping coefficient of the damper for three different deflections of its springs, φ_t, and approximated with a 2-degree polynomial. The coefficient of the substitute viscotic internal damping occurring in the flexible coupling connecting the engine with the generator was assumed based on the equation below:

$$b_{11} = 2 \cdot D \cdot \sqrt{k_{11} \cdot \theta_{14}}, \tag{3}$$

where:

θ_{14} - moment of inertia of the generator rotor,

k_{11} - torsional elasticity of the coupling.

Assuming the damping degree value of $D = 0.175$, as given in [1], the value of the coefficient of internal damping occurring in the flexible coupling, as calculated from the above equation, depends on the torsional elasticity of the coupling, k_{11}.

1. 2. Verification and calibration of the model

The verification of the correctness of the model was made by comparing the variations of momentary engine and generator rotational speeds, as obtained from the integration of the equations of motion, with their counterparts recorded during the tests. These were velocity variations of the front end of the engine shaft and the generator rotor. In the calculation of velocity variations, the source of turning moments inducing the shaft motion were recorded variations of pressures in 8 engine cylinders.

In order to eliminate any velocity changes caused by transient processes connected with the lack of knowledge of the initial conditions of the system of equations being solved, the algorithm for the computation of velocity variations, implemented by the program, was as follows:

1. integration of the equations of motion from the first to the last recorded cycle of engine operation, with the following initial conditions: the angular position of all masses equal to $\varphi_i = 0$; the velocities of all masses equal to $\omega_i = \omega_{nom}$,
2. adoption of the final computation results from Stage 1, as the initial conditions for the computation proper,
3. integration of the equations of motion from the first to the last engine operation cycle.

Trial computation showed considerable differences to exist between the measured velocity variations and those obtained from simulation. Therefore, it was necessary to perform the calibration of those model parameters, whose values were burdened with the largest margin of uncertainty. These quantities are rigidities and the coefficients of damping of the flexible elements of the torsional vibration damper and the flexible coupling connecting the engine with the generator. Therefore, the examination of the effect of these model elements on the agreement of the model with the subject of testing was performed. The assessment of the correctness of the model was made by comparing the real and the simulated velocity variations - the measure of agreement was the calculated coefficient of correlation between the velocities in question. As a result of the examination, the following correction coefficients were established for the above-mentioned parameters:

- the torsional elasticity of the damper was reduced in the proportion of 6/10 - this has its justification in the factory specifications [1], where it is stated that „the damper used is much more flexible than assumed in the calculation",
- the coefficient of substitute internal viscotic damping of the damper was reduced in the proportion of 5/10,
- the torsional elasticity of the coupling was increased by a factor of 3,
- the coefficient of substitute viscotic damping of the coupling was increased by a factor of 17.

For so corrected model parameters, a very good agreement (improvement in the correlation coefficient from $r = 0.55$ *do* $r = 0.95$) was obtained between the real and the simulated variations of engine shaft front-end rotational speed (Fig. 1). The agreement between the real and the simulated variations of generator front-end rotational speed turned out to be slightly worse, though the improvement in the correlation coefficient is also considerable (from $r = 0.59$ *to* $r = 0.94$).

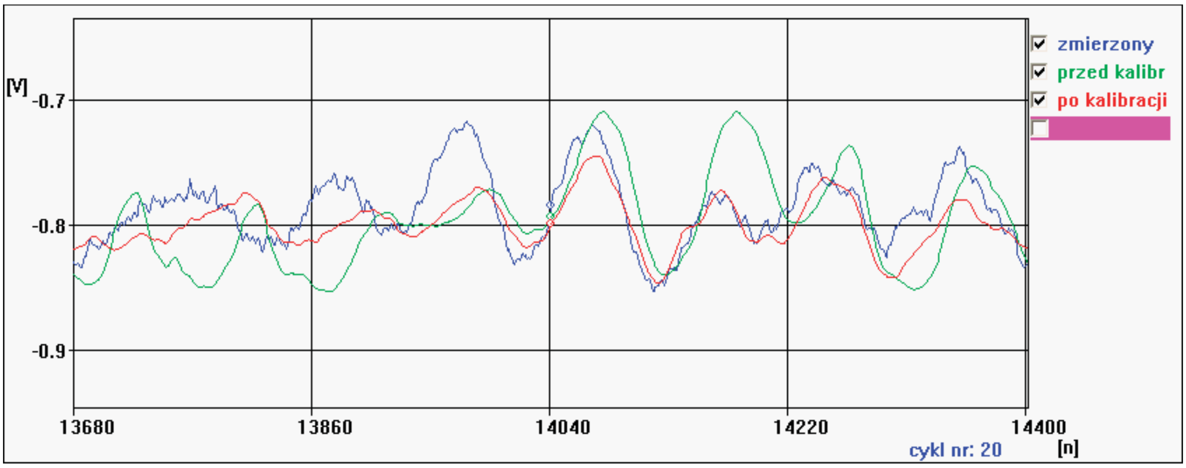

Fig. 1. Comparison of the measured and the calculated crankshaft front-end rotational speed

2. Examination of the relationship between the indicated work of individual cylinders and the momentary rotational speed of the engine shaft front end

The tests were aimed at the determination of the degree of correlation between the indicated work performed during the whole cycle by individual engine cylinders and the momentary rotational speed of the engine shaft front end. Due to the fact that the cycles of operation of individual cylinders in an 8-cylinder engine are offset by 90°CRA in relation to one another, their individual effect on the magnitude of the engine torque and also momentary speed should be noticeable most clearly in 90-degree angular intervals of engine shaft rotation, when the pressures in those cylinders are the highest - tests were also carried out on the relationship between the indicated work performed in the interval of 90°CRA (from 360° to 450°) of each cycle and the momentary rotational speed of the engine shaft front end.

It has been found that the indicated work of any cylinder is best correlated with the momentary rotational speed of the engine shaft in a different angular shaft position (Tab. 1), but in all cylinders this position is offset by approx. 40°CRA after TDC of the piston in a given cylinder.

Tab. 1. Correlation between the indicated work of individual cylinders and crankshaft front-end speed

Cylinder no.	Max. correlation angle [° CRA after TDC]	Correlation coefficient for L_{ind} (0° - 720°)	Correlation coefficient for L_{ind} (360° - 450°)
1	38	0.9285	0.9356
2	34	0.8923	0.8927
3	46	0.8601	0.8651
4	41	0.9513	0.9529
5	43	0.8915	0.8946
6	40	0.9103	0.9177
7	39	0.8407	0.8502
8	39	0.6937	0.6997

The maximum values of the correlation coefficient for all cylinders, summarized in Tab. 1, indicate a fairly large diversification of the effect of cylinder work on shaft front-end rotational speed. The strongest effect is shown by the work of cylinders 4, 1 and 6, while the weakest - the work of cylinders 8 and 7, i.e. the ones situated farthest away from the shaft front end. This diversification is likely to have two basic causes: the degree of distance of a particular cylinder from the shaft front end, and the different value of the mean indicated work of individual cylinders. The calculation shows also that the relationship of the momentary rotational speed of the engine shaft front end is slightly stronger for the indicated work performed in the interval of 90°CRA of the cycle compared to the indicated work of the whole cycle.

Fig. 2 illustrates differences between the model calculation results and the measurement results for one engine cylinder for angular shaft positions, in which the correlation coefficient is the greatest in both instances. A higher degree of correlation between the indicated work of a single cylinder and the measured momentary rotational speed of the engine shaft front end for the model (r = 0.935) compared to the real measurement results (r = 0.772) is visible.

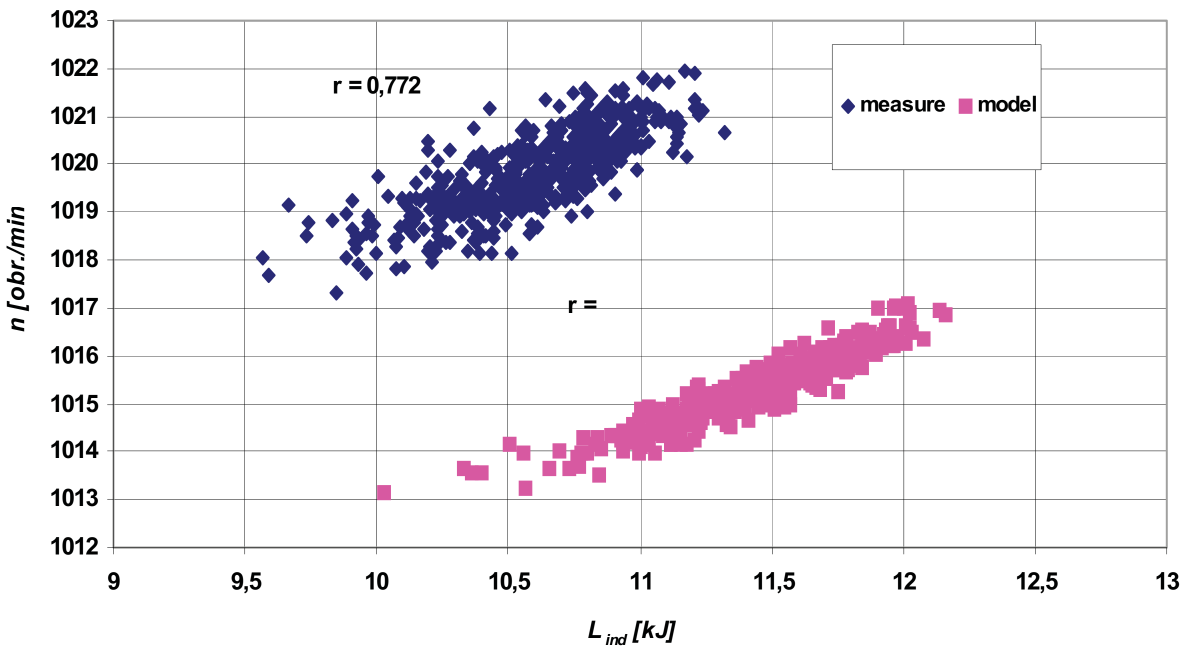

Fig. 2. Correlation between the indicated work of cylinder no.1 and the measured (27° after TDC) and the calculated (38° after TDC) crankshaft front-end speed

The main cause of the lower correlation for the measured shaft rotational speeds is the insufficient measurement accuracy, which is additionally reduced by high-frequency noise. The estimated measurement uncertainty of the applied rotational speed measuring method is ±1 rpm, which is visible in Fig. 2 in the form of vertical measurement result scattering increased by approx. 2 rpm compared to the modelling results free from those inaccuracies.

The presented results illustrate the variation of engine work during the successive 360 cycles, with the engine operating with an average power of approx. 600 kW. It is interesting to compare the correlation between the indicated work of individual engine cylinders and the momentary rotational speed of the engine shaft front end for the engine load range from 60 kW to 600 kW (Fig. 3). In the case of cylinder no. 1, the correlation is much more pronounced - the graph can be approximated with a continuous line. While for the other cylinders, it is visible to a different extent that, for each load, the relationship between the indicated work and the speed is described by a different function.

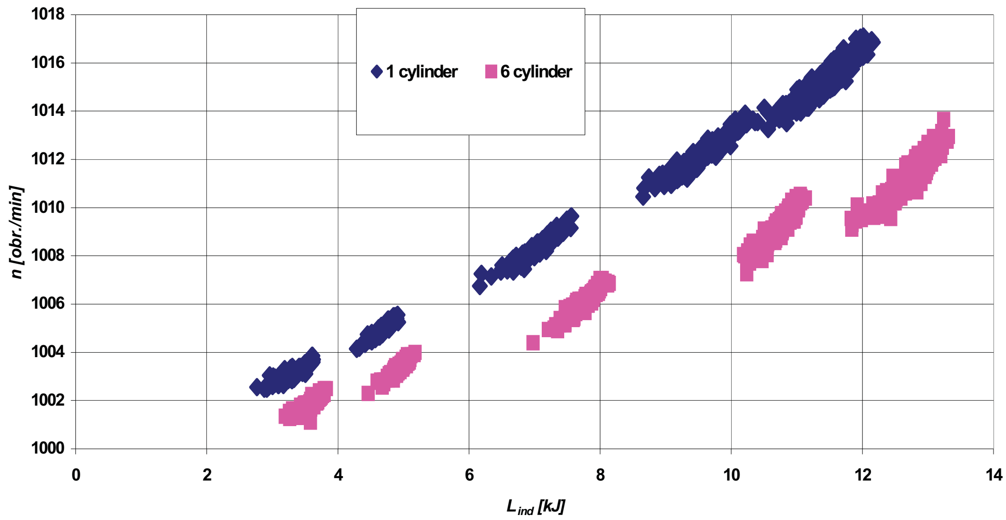

Fig. 3. Correlation between the indicated work of cylinder no.1 (38° after TDC) and cylinder no.6 (40° after TDC) and crankshaft front-end speed

3. Examination of the relationship between the indicated work of individual cylinders and the momentary rotational speed of the generator shaft front end

The tests were aimed at the determination of the degree of correlation between the indicated work performed during the whole cycle by individual engine cylinders and the momentary rotational speed of the generator shaft front end. Similarly as in the case of the engine shaft rotational speed, also in this case tests were carried out to examine the relationship between the indicated work performed in the range of 90°CRA (from 360° to 450°) of each cycle and a the momentary rotational speed of the generator front end.

It has been found that the indicated work of any cylinder is best correlated with the momentary rotational speed of the generator shaft in a different angular shaft position (Tab. 2), and in all cylinders this position is offset by an angle greater than in the case of the engine (from 49° to 93°CRA after TDC of the piston in a given cylinder) and increases with increasing distance from the generator.

The maximum values of the correlation coefficient for all cylinders, summarized in Tab. 2, indicate a lower diversification of the effect of cylinder work on the momentary generator rotational speed, compared to the engine. The strongest effect is exhibited by the work of cylinders 1, 8 and 7, while the weakest - the work of cylinders 4 & 3 & 2, i.e. the ones situated farthest away from the generator shaft front end. This diversification is likely to have two basic causes: the distance of a given cylinder from the shaft front end and the different of the mean indicated work of individual cylinders - although the greatest effect of cylinder no. 1 which, besides cylinder no. 3, has the lowest indicated work value, is difficult to explain. The calculation shows also that the relationship of the momentary rotational speed of the generator shaft front end is slightly stronger for the indicated work performed in the interval of 90°CRA of each cycle compared to the indicated work of the whole cycle.

Tab. 2. Correlation between the indicated work of individual cylinders and generator speed

Cylinder no.	Max. correlation angle [° CRA after TDC]	Correlation coefficient for L_{ind} (0° - 720°)	Correlation coefficient for L_{ind} (360° - 450°)
1	93	0.9074	0.9121
2	92	0.7548	0.7574
3	91	0.6847	0.6857
4	84	0.7042	0.707
5	57	0.7401	0.7447
6	51	0.7628	0.7611
7	51	0.7595	0.7817
8	49	0.7838	0.7893

Fig. 4 illustrates differences between the model calculation results and the measurement results for one engine cylinder for angular shaft positions, in which the correlation coefficient is the greatest in both instances. A higher degree of correlation between the indicated work of a single cylinder and the measured momentary rotational speed of the generator shaft front end for the model (r = 0.912) compared to the real measurement results (r = 0.730) is visible. The main cause of the lower correlation for the measured shaft rotational speeds is the insufficient measurement accuracy, which is additionally reduced by high-frequency noise. In the case of the model, the determined speed is free from those inaccuracies.

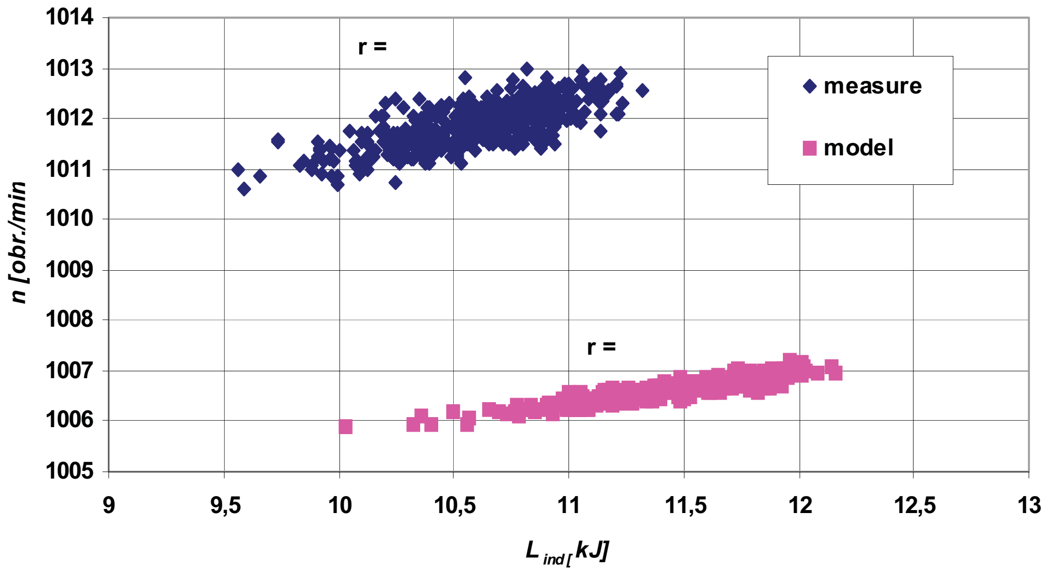

Fig. 4. Correlation between the indicated work of cylinder no. 1 and the measured (173° after TDC) and the calculated (93° after TDC) generator speed

4. Conclusions

The performed investigation of the relationship between the indicated work of individual cylinders and the momentary rotational speed of the engine and generator front ends confirms that the measurement of momentary rotational speeds can be a source of information on the indicated work of particular cylinders of an engine under examination. The condition for the reliability of this method of assessing the quality of cylinder working is to measure these speeds with a high accuracy. For the engine shaft front end, the required accuracy (the reduction of the vertical scatter of points in Fig. 4) is at a level of ± 0.25 rpm (± 0.025%). For the generator shaft front end, on the other hand, diagnosing of the work of individual cylinders is only possible with an even higher measurement accuracy, being at a level of ± 0.1 rpm (±0.01%) and, in addition, it is less reliable compared to the measurement of the momentary rotational speed of the engine.

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