APPLICATION OF THE FINITE ELEMENT METHODS IN LONG-TERM SIMULATION OF THE MULTI-PHYSICS SYSTEMS WITH LARGE TRANSIENT RESPONSE DIFFERENCES

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Abstract

Application of the Finite Element Method (FEM) and the Multibody Dynamics Method (MBS) allows analyzing of complex physical systems. Complexity of the system could be related both to the geometry and the physical description of phenomenon. MBS is the excellent tool for analyzing statics or dynamics of the mechanical systems. MBS permits tracking of Multi Body System transient response for the long-term simulations and application of any arbitrary set of mechanical forcing functions. However, this method does not allow observing state of the continuum systems. Besides, applying forcing functions other than mechanical ones (i.e. general forces or displacements) is troublesome in MBS. In case of FEM most of algorithms encounter continuity conditions across the element boundaries , and thanks to this FEM is one of the most suitable calculation method for continua multi-physics systems (i.e. thermo-structural, electro-thermo-structural, magneto-thermo-structural, MEMS, etc.). Common problem with FEM is that there are major calculation difficulties when tong-term simulation results are required and/or large relative motions are present in the system. Drawbacks of FEM and MBS could be overcome with use of algorithm based on the modified Hybrid Finite Element Method presented further in this paper. Traditional Hybrid Finite Element Method model consists of rigid end deformable elements, system matrices derived for all compound elements are calculated concurrently. In this approach both advantages and disadvantages of FEM and MBS are transferred to the model. Proposed modified Hybrid Finite Element Method algorithm exploits two corresponding coupled discrete models, one containing FEM elements and the other MBS only. Both models are coupled by means of forcing functions. Such approach is applicable for the multi-physics systems with large transient response differences.

Keywords: *dynamics, thermo elasticity, FEM, MBS, simulation, multi-physics systems*

1. FEM formulation of coupled thermo-structural problem for rotor-stator system

Periodic frictional contact and/or changes in the thermo-dynamical state of working agent could generate transient temperature and strain fields in rotor-stator system. In general case temperature and strain fields should be considered as coupled. System matrices including coupling terms, and plasticity for this multiphysic problem could be presented as follows [5].

$$\begin{bmatrix} {}^{(MM)}K & {}^{(MT)}K \\ {}^{(TM)}K & {}^{(TT)}K \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{T} \end{bmatrix} = \begin{bmatrix} {}^{(M)}\dot{Q} \\ {}^{(T)}\dot{Q} \end{bmatrix}, \tag{1}$$

where:

${}^{(MM)}K$ - stiffness matrix with plasticity taken into account,

$^{(TT)}K$ - conductivity matrix,
$^{(MT)}K$ and $^{(TM)}K$ - coupling terms,
q and T - accordingly structural and thermal degrees of freedom,
$^{(M)}Q$ and $^{(T)}Q$ - accordingly structural and thermal load vector.

Variety of numerical methods for solving (1) could be found [2, 5], where algorithm which treats both groups of equations (1) uniformly, integration method for derivative equations with variable multipliers and unsymmetrical matrix coefficient is presented. If only elastic model should be taken into consideration, finite element matrix equation could be presented in the form, [3, 4]:

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \{\ddot{T}\} \end{Bmatrix} + \begin{bmatrix} [C^{(MM)}] & [0] \\ [C^{(TM)}] & [C^{(TT)}] \end{bmatrix} \begin{Bmatrix} \{\dot{q}\} \\ \{\dot{T}\} \end{Bmatrix} + \begin{bmatrix} [K^{(MM)}] & [K^{(MT)}] \\ [0] & [K^{(TT)}] \end{bmatrix} \begin{Bmatrix} \{q\} \\ \{T\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix}, \tag{2}$$

where:

[M] - mass matrix,
$[C^{(MM)}]$ - structural damping matrix,
$[C^{(TT)}]$ - thermal damping matrix,
$[C^{(TM)}]$ - thermo-elastic damping matrix,
$[K^{(MM)}]$ - structural stiffness matrix,
$[K^{(TT)}]$ - thermal conductivity matrix,
$[K^{(MT)}]$ - thermo-elastic stiffness matrix,
{F} - structural load vector,
{Q} - thermal load vector,
{T} - temperature vector,
{u} - displacement vector.
Sub matrices with mixed superscripts (TM, MT) are coupling terms.

Applying FEM approach series of simulations for rotor-stator system (Fig. 1) have been performed [1, 2, 3]. As a sample, in Fig. 2 results of thermo-structural simulation for rotor with central disk and frictional rotor-to-stator contact are presented. Initial rotor-to-stator contact arises due to the deformation caused by shaft's imbalance. Analyzed system's constants and parameters are presented in Tab. 1.

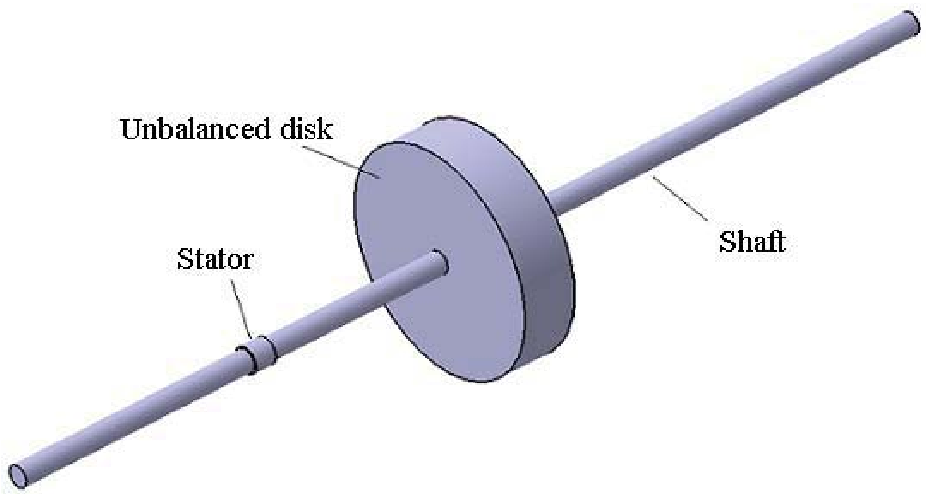

Fig. 1. The geometrical model of analysed rotor-stator system

Tab. 1. System constants and parameter. Values in parenthesis apply to stator

Young Modulus s/(r) $E = 2 \times 10^{11}/(2 \times 10^{11})$ [Pa]	Density s/(r) $\rho = 7890/(7900)$[kg /m^3]
Poisson's Ratio s/(r) $\upsilon = 0.3/(0.29)$	Linear Thermal Expansion s/(r) $\alpha_t = 1.7e10^{-6} /(1.23e10^{-5})$ K^{-1}
Heat Conduction Coeff. s/(r) $\lambda = 50 /(80)$ [W/mK]	Convection Coeff. $\alpha_k = 25$ [W/m^2 K]
Specific Heat s/(r) c = 419/(486) [J/kgK]	Ambient Temperature $T_o = 298$ [K]
Rotational Velocity $\omega = 374$ [RPM]	Unbalanced: m = 20 [g], r = 0.025 [m]
Diameters: shaft d = 0.01 [m], disk dd = 0.1 [m], stator inner diameter di = 0.01 [m], stator outer diameter do = 0.012 [m]	Dynamical Friction Coefficient $\mu = 0.1$

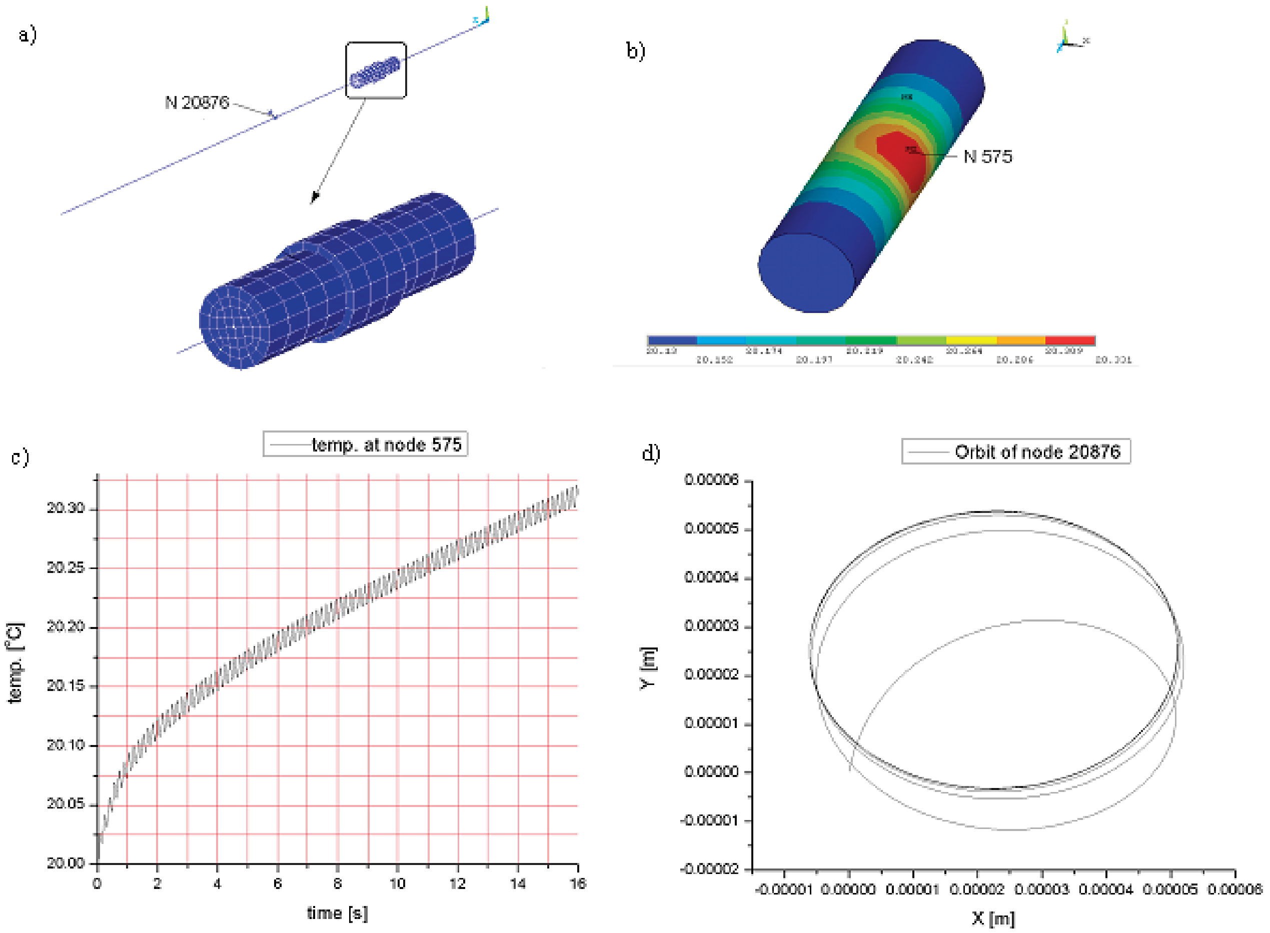

Fig. 2. Sample results of thermo-structural simulation for rotor with central disk and frictional rotor-to-stator contact: a) FEM model, b) temperature field after 16 seconds of simulation, c) temperature changes at node 575, d) orbit of node 20876

2. Difficulties in long-term simulation base no coupled FEM

In the analysis of compound problem concerning different engineering disciplines described by coupled equation, duration of allowable integration time step is defined by the fastest phenomenon. As a result part of the model is calculated with needless accuracy. This could significantly affect numerical efficiency, especially for long-term transient simulation. For previously presented thermo-structural problem it is evident that structural part (dynamics of rotating shaft) requires much shorter time steps than thermal part. As rule of thumb approximate time step in structural dynamics should be one-twentieth of last mode of interest period that contributes to the motion. For example for structural part (Fig. 2), assuming that only synchronous frequency is the last of interest (very rough approximation), integration time is 0.0086 sec. Required integration time for thermal part to this model is 0.026 sec (calculated as depth of element divided by four times thermal diffusivity [7]). Some advantages could be taken due to the physics of the phenomenon. For the analyzed case, applying to rotating machines modelling, assumption of elastic system's behaviour is acceptable. It also could be stated that heat generated due to the elastic strain velocity is relatively small in comparison to other heat sources (i.e. friction). With this supposition (1) and (2) could be accordingly simplified:

$$\begin{bmatrix} {}^{(MM)}K & 0 \\ 0 & {}^{(TT)}K \end{bmatrix}\begin{bmatrix} \dot{q} \\ \dot{T} \end{bmatrix} = \begin{bmatrix} {}^{(M)}\dot{Q} \\ {}^{(T)}\dot{Q} \end{bmatrix} - \begin{bmatrix} 0 & {}^{(MT)}K \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q} \\ \dot{T} \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \ddot{T} \end{Bmatrix} + \begin{bmatrix} [C^{(MM)}] & [0] \\ [0] & [C^{(TT)}] \end{bmatrix}\begin{Bmatrix} \dot{q} \\ \dot{T} \end{Bmatrix} + \begin{bmatrix} [K^{(MM)}] & [0] \\ [0] & [K^{(TT)}] \end{bmatrix}\begin{Bmatrix} q \\ T \end{Bmatrix} = \begin{Bmatrix} F + F^{T} \\ Q \end{Bmatrix}. \tag{4}$$

Equation (3) and (4) are coupled by mean of forcing functions only. Change in dynamical state of rotating machine caused by thermal deformation is relatively slow process. Therefore with use of (3) and (4) both phenomena could be solved separately. Practically it means that thermal phenomenon is one to be solved constantly, and only correction from dynamical analysis at certain times is needed (i.e. frictional heat rate changes, encountering thermal deformation caused by temperatures from the thermal step just before correction). Additional problems affecting solution time for FEM formulation could arise if analysed system is subjected to large relative motion. Large relative motion forces re-establishing of system matrices at each iteration step. Also it should be noted that if the contact elements are to be used on non-planar areas subjected to large relative motion slow converging solution is guaranteed. This reasons make FEM formulation feasible for short time simulations only (i.e. for presented rotor-stator problem it means few hundreds revolutions).

3. Multibody System Dynamics approach

Alternatively to the FEM formulation, structural dynamics of structures subjected to large relative motion could be solved with use of MBS. In this method structure is divided into subsystems. Each subsystem could undergo large relative motion. Subsystems consist of rigid bodies connected by massless spring-damper elements. To encounter large relative motion, four types of reference system are introduced: global reference frame – in which large motion of subsystem is described; subsystem's local reference frame – in which positions of all subsystem's rigid elements are described; rigid body's local reference frame – unique for particular rigid element; spring-damper's local reference frame – unique for particular spring-damper element [6, 8]. In Fig. 3 sample reference systems for rotor-stator problem is shown.

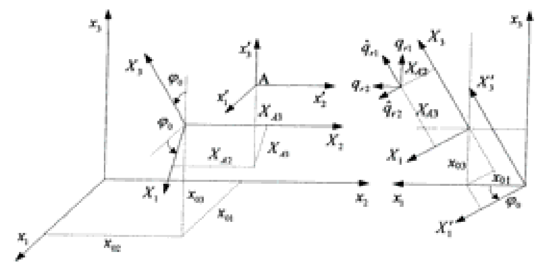

Fig. 3. Reference systems for the rotor-stator problem

For this particular case, concerning large rotation about shaft axis, coordinate transformation is written as:

$$\begin{bmatrix} X'_{A1} \\ X'_{A2} \\ X'_{A3} \\ X'_{A4} \\ X'_{A5} \\ X'_{A6} \end{bmatrix} = \begin{bmatrix} \cos\varphi_0 & 0 & -\sin\varphi_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin\varphi_0 & 0 & \cos\varphi_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{01} \\ x_{02} \\ x_{03} \\ 0 \\ \varphi_0 \\ 0 \end{bmatrix} + \begin{bmatrix} X_{A1} \\ X_{A2} \\ X_{A3} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{q}_{r1} \\ \hat{q}_{r2} \\ \hat{q}_{r3} \\ \hat{q}_{r4} \\ \hat{q}_{r5} \\ \hat{q}_{r6} \end{bmatrix}. \tag{5}$$

In general case equations of motion are derived with use of Lagrange's equations of the second type:

$$\sum_{r=1}^{m_c} \left[\frac{d}{dt} \left(\frac{\partial [T_r]}{\partial [\dot{q}_c]} \right) - \frac{\partial [T_r]}{\partial [q_c]} \right] + \sum_{k=1}^{l_c} \left(\frac{\partial [U_k]}{\partial [q_c]} + \frac{\partial [R_k]}{\partial [\dot{q}_c]} \right) = [f_c], \tag{6}$$

where:

$\hat{q}_c$ - generalized coordinates vector of unrestrained system,
m_C - number of rigid elements,
f_C - generalized forces vector,
T - kinetic energy,
U - potential energy,
l_C - number of spring-damper elements,
R - dissipation function.

Evaluation of equations of motion divided in three stages [7, 8]: the first stage – equations of motion for all subsystems (with use of (5)); the second stage – equations of motion of unrestrained system; and in the last stage – equations of motion of system with restrains. In the last stage, comply vector of given variables and expressing vector of dependent variables with vector of independent variables by applying constrains equation. MBS allows solving dynamics of mechanical systems subjected to large relative motion including contact phenomenon in long-term simulations. This method is fast and efficient, but it does not allow solving for other degrees of freedom than displacements (i.e. temperature, and therefore thermal deformation in the rotor-stator problem).

4. Mixed MBS and FEM in algorithm for solving rotor-stator problem

The goal is to solve similar problem to that in Fig.1, but for long-term simulation. To improve solution efficiency advantage from presented methods will be taken. MBS is the best method to determine dynamical state of analyzed rotor-stator problem; however additional approach to state thermal part of the problem must be taken. Thermo-structural phenomenon is solved with use of FEM. As it was mentioned, in thermo-structural analysis, thermal and displacement fields could be solved separately; nevertheless correction at certain times should be made. Correction values are acquired from MBS (i.e. frictional heat calculated from friction force). Before determining friction force MBS model is updated with shaft axis deformation due to the temperature changes (thermal bow), and local thermal deformation (shape change of contact surfaces). In Fig.4 the algorithm chart-flow of mixed FEM-BEM approach for solving long-term simulation of the thermo-structural problem with large transient response differences is presented.

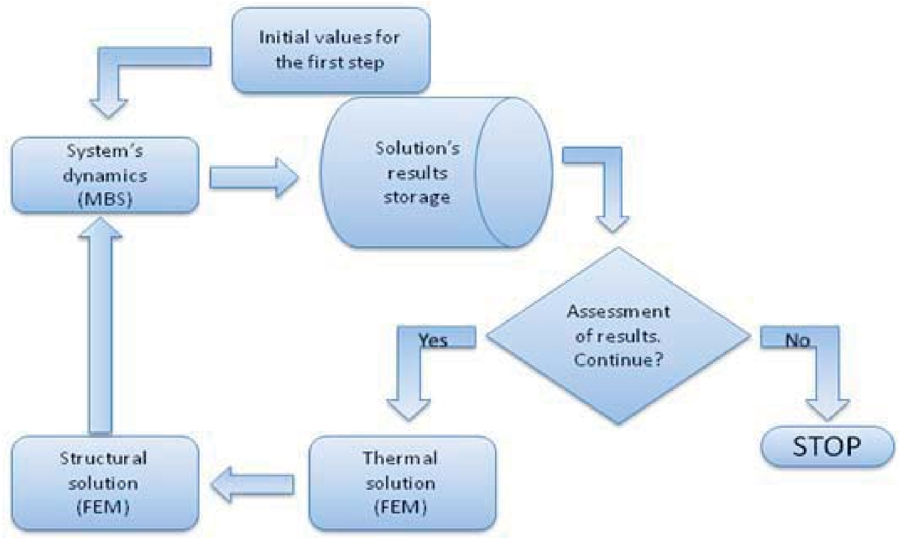

Fig. 4. The algorithm exploiting FEM and MBS in solving long-term simulation of the thermo-structural problem with large transient response differences

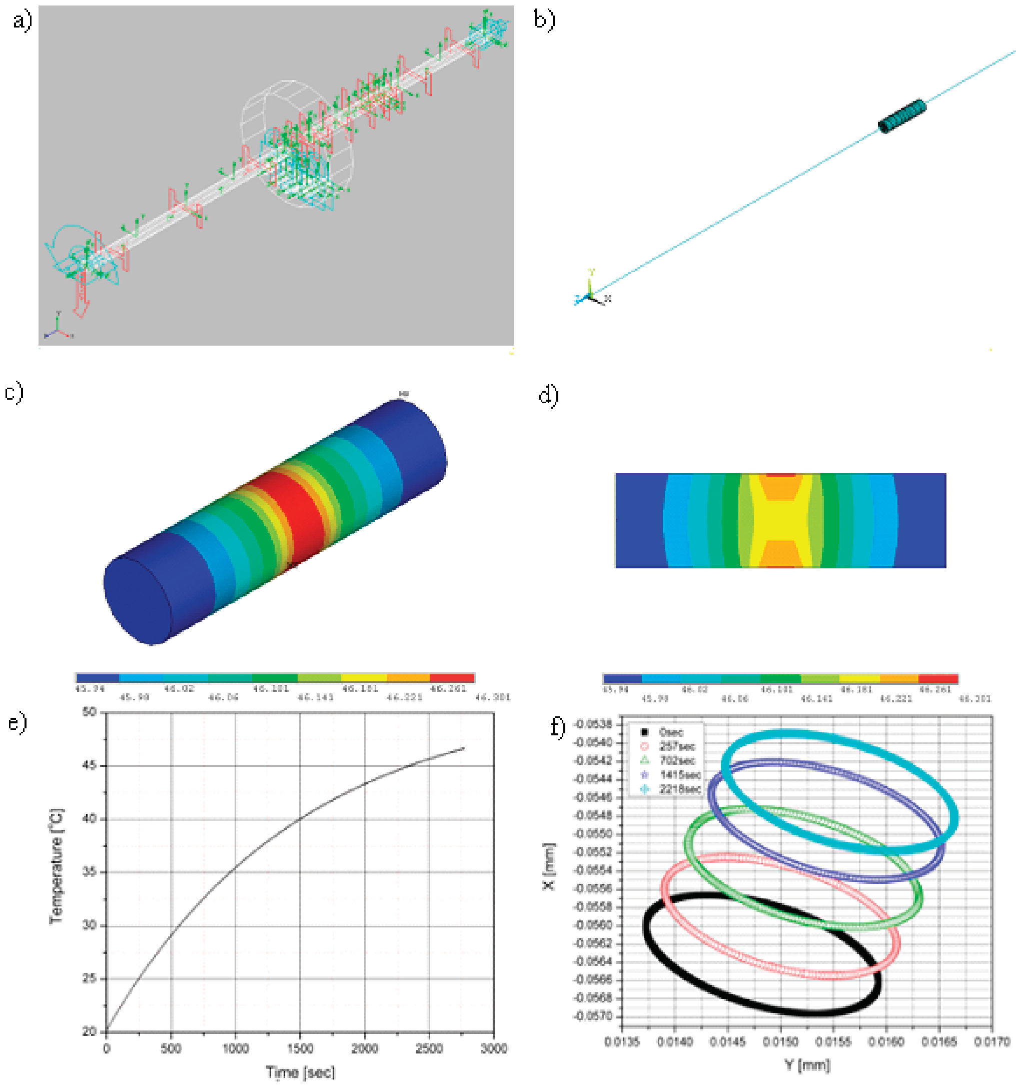

Fig. 5. Sample results of mixed FEM-MBS simulation for the rotor with central disk and frictional rotor-to-stator contact, a) MBS model, b) FEM model (both structural, and thermal), c) and d) temperature field after 2773 seconds of simulation, e) temperature changes at hottest spot, f) changes in the disk's centre point obits

With use of this algorithm similar problem like in Fig. 1 has been solved. Analyzed system's constants and parameters are presented in Tab. 1. Results of the simulation are shown in Fig. 5.

5. Conclusions

The finite element methods provide excellent tool for analyzing complex phenomenon. However, there are special cases when the direct application of only one of them is not feasible because of unacceptable solution runtimes. If evaluated problem has weak coupling nature, and there are large transient response differences, some advantages could be taken allowing to solve problems separately with correction at certain times (i.e. changing equations (1) and (2) into (3) and (4)). When equation's uncoupling is possible some parts of the problem could be solved with use of different methods. In this paper thermo-structural phenomenon concerning rotating machine has been analyzed. Uncoupling allows using MBS instead of FEM at some stages of simulation. Exploiting mixed FEM-MBS method, extensively improves calculation efficiency. After 114 hours of simulation runtime direct method reached end time of 16sec, while mixed method ended at 2773 sec (simulations have been performed in the same environment: the same OS and hardware).

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