APPLICATION OF UNCERTAINTY ANALYSIS IN STRUCTURAL DYNAMICS

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Abstract

The paper deals with the application of uncertainty analysis in structural dynamics. The usage of soft computing method, namely Genetic Algorithms (GA), is presented to show effective computational technique that allows for assessing the propagation of defined uncertainties in modelled mechanical structure. Tested method is capable of finding variation of eigenfrequencies of Finite Element models and is based on scanning of interval global system matrices. During this process combination of values of input design parameters are found for which extremes of frequencies of vibration appear. FE model of windscreen, made available by Renault Technocentre, has been analysed. Assumed uncertainties have taken into account variability of material properties, geometrical characteristics and environmental conditions. Fuzzy theory together with alpha-cut strategy has been applied for modelling uncertain parameters. Sensitivity analysis has been performed to investigate the contribution effect introduced by each assumed input parameter to studied frequency of vibration. Obtained results have been presented and discussed within the context of referential results yielded from Monte Carlo simulation and GA used directly for the search of frequency extremes. Finally, observed time savings have been mentioned to justify the usage of tested computational method.

Keywords: *structural dynamics, uncertainty analysis, eigenproblem, Finite Element Method, Genetic Algorithms*

1. Introduction

The process of designing a mechanical system is usually connected with the necessity of assigning its dynamical properties. Existing tools for numerical analyses in engineering allow an effective computation of dynamical properties of mechanical structures by means of virtual prototyping and simulation. It has to be noticed however that those simulations are of a deterministic nature i.e. calculations are performed for nominal or mean values of parameters. This type of simulations fails if we are looking not only for response values, but also for their scatter and statistical parameters. Uncertainty analysis aims at filling this gap. It allows for the definition of input parameters in terms of their uncertainty model and gives response values together with their statistical characterization.

In the paper the main uncertainty sources in mechanical systems have been discussed. Methods applied in uncertainty analysis, probabilistic and non-probabilistic approaches have been presented. Moreover a numerical technique allowing calculation of extreme values of eigenfrequencies considering uncertain input parameters have been presented. It is based on Finite Element Method (FEM) as a way to discretize the solution domain and Genetic Algorithms (GA) [1, 2]. It allows reducing the number of required eigensolutions, substituting part of them with the process of assembling global mass and stiffness matrices.

Present work considers uncertainty analysis in structural dynamics. As the test model a windshield of a passenger car provided by Renault Technocentre has been chosen. According to manufacturer's specification, uncertain parameters of the material properties (Young's modulus, Poisson's ratio, density), windshield geometry (thicknesses of glass and polymer layers) and environmental conditions (temperature) have been defined. Uncertain parameters have been defined as triangular fuzzy numbers. Sorting of the parameters by their influence on the first natural frequency has been performed by means of the sensitivity analysis. Alpha-cut strategy was chosen as the solution technique. An application searching global mass and stiffness matrices based on the GA was used to determine the intervals matching appropriate α-cuts. Reference results have been obtained using Monte Carlo (MC) method with Latin Hypercube sampling (LHS) to improve the coverage of the parameters domain. GA used directly to search the extreme values of selected eigenfrequency were additionally applied. Numerical results have been verified with experimental test results made by the windshield manufacturer.

2. Uncertainties in mechanical systems

2.1. Definition and sources of uncertainties

Uncertainties describe the variation of parameters characterizing the properties of a system or its model and the environmental conditions. The following basic sources of uncertainties can be specified [6]:

- incomplete knowledge of the design parameters,
- variety of methods to model the system and constituent phenomena,
- inability to manufacture identical copies of a product,
- variability of environmental conditions,
- change of parameters with time.

Uncertainties concern all stages of product's life cycle and they should be identified and exploited in uncertainty analysis to find their influence on system's characteristics.

2.2. Classification of uncertainties

Considering the similarities of features, uncertainties can be divided among other things by the product's life cycle and possibility of their reduction. Uncertainties exist in the design stage, manufacturing and operation of the product.

In the design stage uncertainties are present in:

- the choice of solution concept,
- topology and number of structural elements (e.g. dampers, springs),
- incomplete information on the material properties (material constants, constitutive model),
- variety of methods to model physical phenomena, approximation, numerical errors.

In the manufacturing stage there exists a variation of the physical properties of particular copies of the product and it concerns:

- variable parameters of the production process,
- change in the quality of manufacturing of components and a final product,

- differences in the geometry due to manufacturing tolerances,
- wearing and aging of tools,
- imprecision of measuring devices,
- quality of joints.

In the operation stage uncertainties are manifested in:
- the environmental conditions (temperature, humidity, pressure),
- variable loading conditions,
- aging, wearing and deterioration of original parameter values (corrosion, fatigue, micro cracks).

In close relation with the above classification there are possibilities to reduce uncertainties (Fig. 1). Uncertainties in the design stage are usually susceptible to reduction. In the process of gathering new information about a product some of them can be reduced and some of them can be even eliminated. In case of the manufacturing stage it is impossible to eliminate manufacturing tolerances and the quality level of the processes used. The operation stage as the least controllable is also the least susceptible to the reduction of uncertainties.

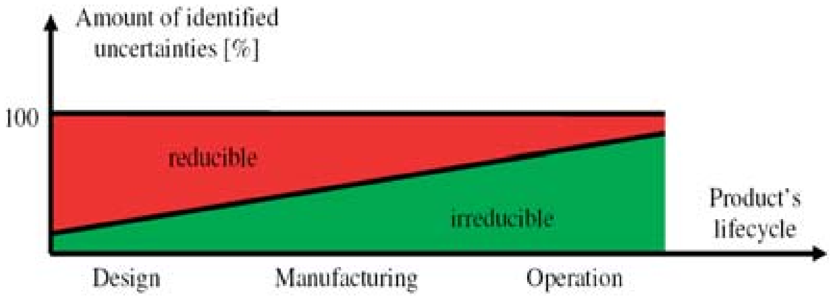

Fig. 1. Relation between reducible and irreducible uncertainties and the product's lifecycle

Summarizing the following types of uncertainties can be distinguished [7]:
- reducible uncertainty (epistemic, subjective uncertainty) – arises from potential deficiency of knowledge on design parameters; can be reduced as required information is gradually gathered; common sources: imprecision, inconsistency, lack of information; example: different possible techniques of modelling the same phenomenon, e.g. friction, damping,
- irreducible uncertainty (variability, aleatory uncertainty) – substantially connected with modelled mechanical system, expressing inevitable variability of its properties within time, subsequent items, changes of environmental conditions etc., example: manufacturing tolerances.

2.3. Modelling of uncertainty

Depending on the type of identified uncertainties input parameters of a real mechanical system or its model may be of the following types [3, 7]:
- random variables – defined by means of statistical moments (mean, standard deviation, covariance for dependent parameters etc.) and probability density functions, are utilized in probabilistic methods and allow to find statistics of output parameters,
- random fields – are an extension of random variables, and can be used to model spatial variation of parameters (e.g. roughness of a surface),
- intervals – used in cases when only limited information about the uncertainty of parameters is available. They are defined only by the extreme values of a parameter and are utilized in the possibilistic methods to model e.g. manufacturing tolerances,
- fuzzy sets – an alternative for random variables. They allow for a linguistic definition of parameters i.e. representing subjective, incomplete knowledge. They are used in possibilistic methods.

The type of uncertainty description determines the possibilities of application of particular numerical techniques. Fig. 2 shows relationships between respective applications.

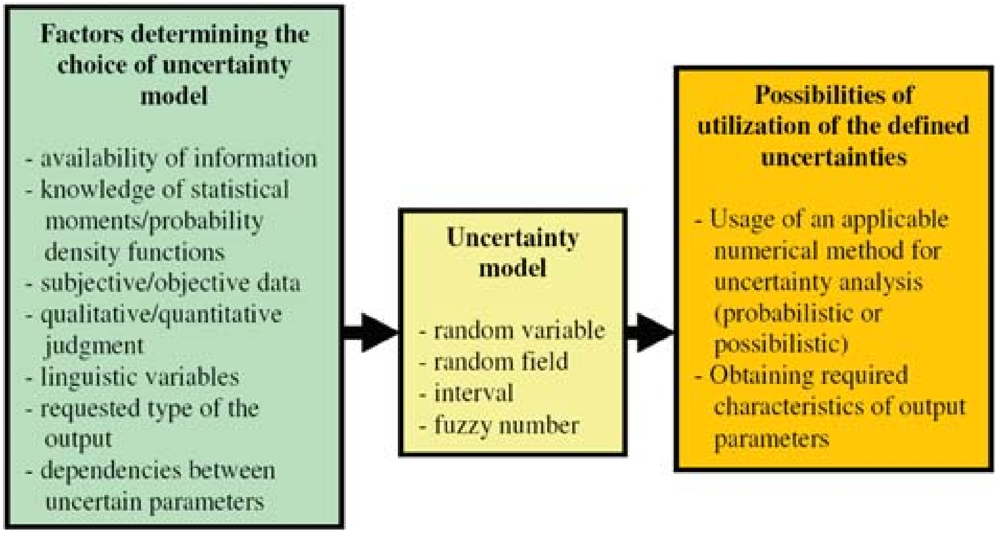

Fig. 2. Dependencies between the uncertainty model and the possibilities of their utilization for uncertainty analysis

Depending on the demands on the type of output parameters and considering the possibilities and limitations regarding gathering of data, probabilistic and possibilistic methods can be used. In the following sections, some of the available methods are described.

3. Uncertainty analysis

Uncertainty analyses allow approximating the behaviour a real system under operational loads with the results of numerical and experimental tests. They facilitate the interpretation of physical phenomena and variability of certain characteristics for different types of uncertainties.

3.1. Stages in uncertainty analysis

Consecutive stages in the application of the analysis of uncertainties are show in Fig. 3.

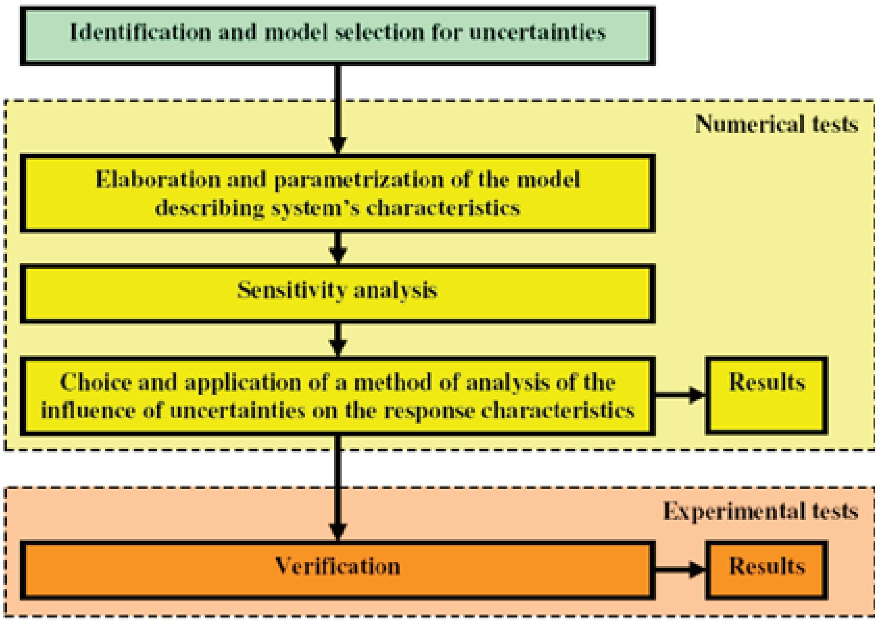

Fig. 3. Stages in uncertainty analysis

At the stage of identification and modelling of uncertainties a set of uncertain parameters is prepared. It is then considered in the following stages (see point 2). Subsequently a numerical or analytical model is prepared to simulate the behaviour of the real system by determining requested responses. In case of analytical models symbolic solutions of mechanical problems can be utilized e.g. eigenvalues and eigenvectors of a discrete systems with a finite number of degrees of freedom. For numerical methods, we could include the following:

- Finite Difference Method (FDM) – Richardson 1910 [21],
- Finite Element Method (FEM) – Turner 1950 s [20, 21],
- Rigid Finite Element Method (RFEM) – Kruszewski 1975 [22],
- Boundary Element Method (BEM) – Brebbia 1978 [23].

The model of a mechanical system is parameterized in such a way to allow for an easy change of the values of uncertain parameters. Fig. 4 shows a schematic application in case of a Finite Element Model.

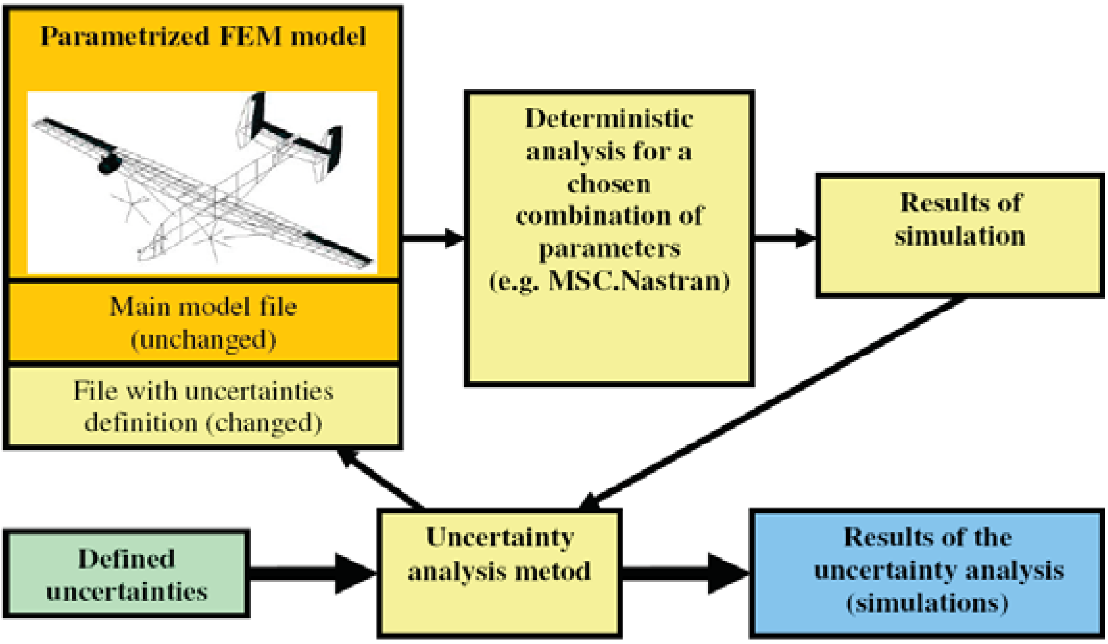

Fig. 4. Uncertainty analysis for a parametrized FEM model

After the preparation of a model the sensitivity analysis is performed. Sensitivity analysis requires the computation of partial derivatives of response values over the parameter values of the system [8]. These derivatives are the measure of influence of particular model parameters on the responses. There are several different methods for computation of sensitivities. The simplest case is approximation of the derivatives by Finite Difference Method. Sensitivity analysis is a way to sort the variables by their influence on the response. Then usually an arbitrary partition of the parameters is made:

- influential parameters – parameters which significantly influence the responses,
- non influential parameters – parameters which influence the responses only in a very limited way; these are omitted in further stages of the analysis.

Common practice in the sensitivity analysis is the use of the Design of Experiment (DOE) method. Results of the experiments allow not only identifying particular parameters or combinations of parameters, but also allowing building of metamodels. Metamodels, being the models (approximations) of the simulated models allow for shortening the time required for computations in the subsequent stages of uncertainty analysis. Time consuming calculations are substituted with simple models by the application of the Response Surface Method (RSM) [9, 10]. Fig. 5 presents sample results of the sensitivity analysis in the form of the Pareto plot and the

Lorenz curve. Pareto plot is a sorted bar plot of the influence values of particular parameters on the response. Lorenz curve represents the summary effect of input parameters' influences.

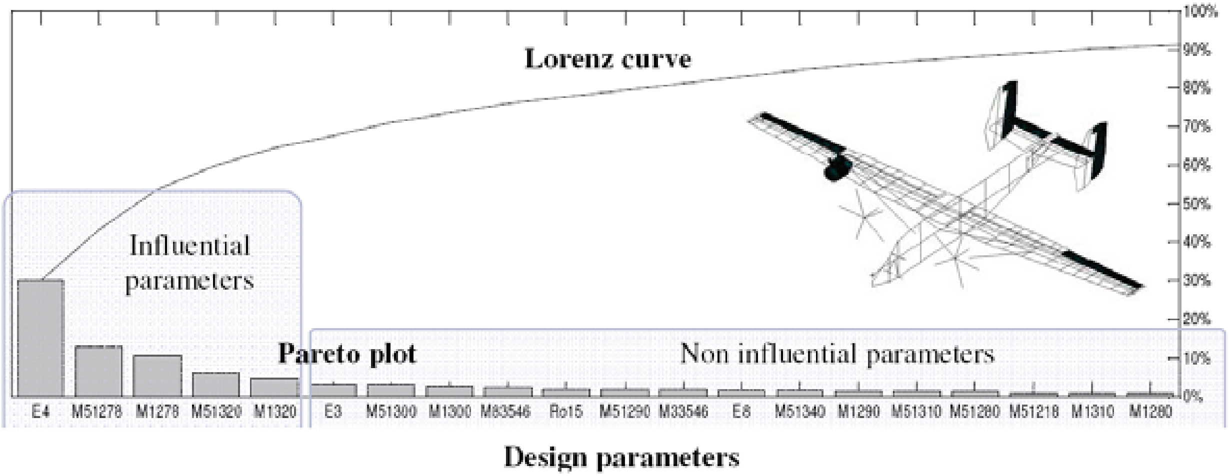

Fig. 5. Sample results of the sensitivity analysis [9]

The space of the design parameters identified as being influential is used to find the variability of selected responses for assumed uncertainties. The choice of the method of quantification of this variability takes into account the type of uncertainty definition and required form of the response.

The last stage of the uncertainty analysis is a verification performed with the use of the results obtained from experiments. Verification is performed for both single copies of a product, which allows finding bounds of variability of certain characteristics, and for a series of products, which allows finding statistics and histograms of the responses. In case when experimental results are unavailable, uncertainty analysis can be only numerical. In such cases the outcomes of the uncertainty analysis can be used to improve the performance of a virtual prototype and help in the selection of a specific construction type, before creating a physical prototype.

3.2. Methods of uncertainty analysis

Available methods for uncertainty analysis can be divided in the following categories [3, 7]:

- probabilistic methods – operate on random variables and random fields. They allow finding histograms and selected statistics characterizing specific responses. The most commonly used method among the probabilistic ones is MC method. It can be used in the most basic form as a crude MC simulation and with some more sophisticated methods of sampling the parameter space in order to reduce the required number of samples,
- possibilistic methods – major alternative for probabilistic methods. Most commonly used in the design stage when there is only a limited knowledge on the probability density functions and statistics of the parameters. The most common methods within the possibilistic framework are:
  - interval analysis [11],
  - vertex method [6],
  - fuzzy sets theory with Zadeh's extension principle [12, 13],
  - transformation method and its modifications [14].

Fig. 6 shows schematic presentations of available methods for uncertainty analysis.

Alpha-cut strategy was applied in present application as an alternative to the Zadeh's extension principle and the transformation method. Method is based on the principle of decomposition of input fuzzy sets into intervals (α-cuts) and application of interval analysis on those intervals. The task of finding an output fuzzy set is replaced with finding output intervals (α-cuts) which are used to assembly an output fuzzy set. Extreme values of the intervals can be found by means of interval analysis, vertex method, GA [15] or MC method. Tab. 1 presents base characteristics of the described methods.

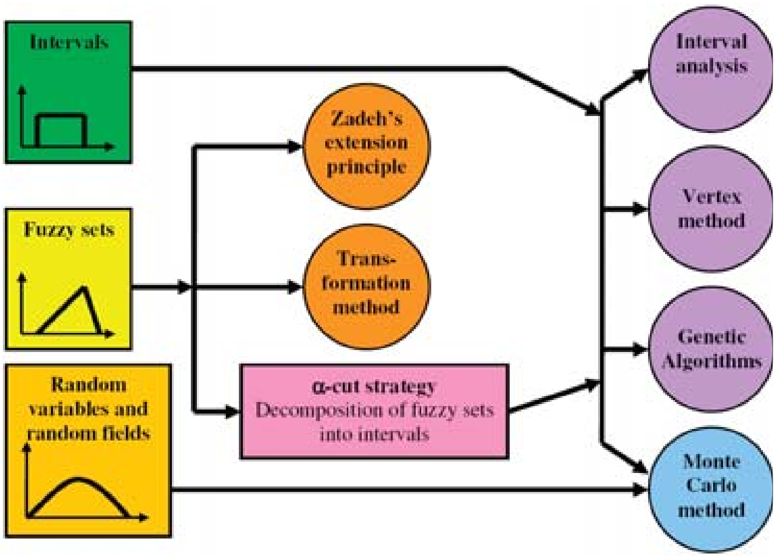

Fig. 6. Methods of uncertainty analysis

Tab. 1. Properties of the probabilistic and possibilistic methods of uncertainty analysis in structural dynamics

	Probabilistic methods	Possibilistic methods
Parameters	- random numbers, - random fields.	- intervals, - fuzzy sets.
Advantages	- generation of histograms of responses, - calculation of statistics of responses.	- lower demands on the definition of uncertain parameters.
Disadvantages	- for the crude MC method there is a need for a large number of samples in order to assure good quality of calculated statistics, allowing for reconstruction of the probability density function.	- interval analysis can produce conservative results, - risk to omit global extremes, - large number of iterations, usually with exponential relation to the number of uncertain parameters.
Possibilities for improvement of efficiency	- usage of the Response Surface Method or Neural Networks significantly reduces the time of computation, - usage of more advance methods of sampling the parameter space e.g. importance sampling, adaptive sampling, Latin Hypercube Sampling, - improvement of efficiency by parallel computing, - estimation error in MC method decreases with a square root of the number of iterations.	- possibility of substitution of the Zadeh's extension principle with the α-cut strategy requiring lower number of computations, - better search of extreme values of intervals with application of the GA or the MC method with an arbitrary choice of probability density functions, - usage of the Response Surface Method or Neural Networks significantly reduces the time of computation.

Improvement of the efficiency of uncertainty analyses for both probabilistic and possibilistic methods can be accomplished by:

- the reduction of the size of a problem:

  - using sensitivity analysis to eliminate the parameters that are not influential on the selected responses,
  - analysis of the input parameters and elimination of possible dependencies between them;
- lowering the number of required simulations by the usage of more effective sampling scheme in MC method,
- simplification of models e.g. lowering mesh density in Finite Element Method,
- parallel computing,
- hybrid methods e.g. combining GA to localize the extreme values of the result space with gradient based optimization to find exact values of the extremes,
- usage of metamodels by means of the Response Surface Method and Neural Networks.

3.3. Applied method of uncertainty analysis in structural dynamics

Natural frequencies of a given system are one of the most important parameters of interest in structural dynamics. Finite Element Method can be used to find those parameters. Natural frequencies of a system are found by solving the eigenproblem. The amount of computations required is high due to the fact that the global mass and stiffness matrices have to be inverted and an iterative procedure to find eigenvalues and eigenvectors has to be applied. In uncertainty analysis where usually a large number of simulations have to be performed there is a strong need to reduce the time of computations. In the problem of finding extreme values of natural frequencies there is a possibility to replace part of the eigensolutions with the process of assembling global mass and stiffness matrices only.

For structural dynamics applications we propose a modification of one of the aforementioned methods. The proposed method characterizes with:
- the usage of fuzzy sets to model uncertain parameters,
- the usage of α-cut strategy,
- the usage of GA to search different combinations of global mass (M) and stiffness (K) matrices in order to find extreme values of eigenfrequencies,
- the usage of the theorem defining the combinations of interval elements of the global mass matrix for the extreme eigenvalues [16],
- the usage of Finite Element Method,
- reduction of the computational effort by the reduction of the number of eigensolutions.

In more detail the proposed method can be described as follows. According to the aforementioned theorem [16] we can define global mass (M) and stiffness (K) matrices as the interval matrices:

$$\begin{aligned} K^I &= [\underline{K}, \overline{K}] \\ M^I &= [\underline{M}, \overline{M}] \end{aligned}, \tag{1}$$

where $\underline{K}$, $\overline{K}$, $\underline{M}$ and $\overline{M}$ are matrices with minimal and maximal values of parameters respectively. Global stiffness matrix K belonging to an interval $[\underline{K}, \overline{K}]$ should be real and symmetric. Global mass matrix M belonging to an interval $[\underline{M}, \overline{M}]$ should be additionally positive definite. According to the theorem an interval of eigenvalues can be defined as:

$$\lambda^I = [\underline{\lambda}, \overline{\lambda}], \tag{2}$$

where the extreme values are fund by solving the following equations:

$$\begin{aligned} \underline{K}\underline{u} &= \underline{\lambda}\overline{M}\underline{u} \\ \overline{K}\overline{u} &= \overline{\lambda}\underline{M}\overline{u} \end{aligned}, \tag{3}$$

Utilizing the aforementioned theorem, in the proposed method, a search of the global mass and stiffness matrices is performed in order to find the $\underline{K}^*$, $\overline{K}^*$, $\overline{M}^*$ and $\underline{M}^*$ matrices which will approximate the required $\underline{K}$, $\overline{K}$, $\overline{M}$ and $\underline{M}$ matrices for the defined parameter space. Two pairs of matrices $\underline{K}^*$ and $\overline{M}^*$ and $\overline{K}^*$ and $\underline{M}^*$ are fund by the minimization and maximization of the sum of their elements:

$$\begin{aligned}
&\exists\left(K=\underline{K}^*, M=\overline{M}^*\right): \min_{\text{Genetic Algorithms}}\left(\sum_{i,j} k_{ij}-\sum_{i,j} m_{ij}\right)=\sum_{i,j}\underline{k_{ij}}^*-\sum_{i,j}\overline{m_{ij}}^* \\
&\exists\left(K=\overline{K}^*, M=\underline{M}^*\right): \max_{\text{Genetic Algorithms}}\left(\sum_{i,j} k_{ij}-\sum_{i,j} m_{ij}\right)=\sum_{i,j}\overline{k_{ij}}^*-\sum_{i,j}\underline{m_{ij}}^*
\end{aligned}, \tag{4}$$

where:

$$\begin{aligned}
\underline{K}^* &= \left[\underline{k_{ij}}^*\right] & \overline{K}^* &= \left[\overline{k_{ij}}^*\right] \\
\underline{M}^* &= \left[\underline{m_{ij}}^*\right] & \overline{M}^* &= \left[\overline{m_{ij}}^*\right]
\end{aligned}. \tag{5}$$

Determined matrices are then used to calculate the extreme eigenvalues, by solving the following eigenproblems:

$$\begin{aligned}
\underline{K}^*\underline{u}^* &= \underline{\lambda}^*\overline{M}^*\underline{u}^* \\
\overline{K}^*\overline{u}^* &= \overline{\lambda}^*\underline{M}^*\overline{u}^*
\end{aligned}. \tag{6}$$

Presented procedure allows for an efficient search of the extreme eigenvalues only in cases when the uncertain parameters influence only one of the system's matrices (e.g. Young's modulus, mass density) [17]. There is a possibility to add a second stage to the proposed procedure which requires performing the solutions of the eigenproblem for the parameters that influence both mass and stiffness matrices (e.g. geometry of the model) [18]. The algorithm of the two stage method for finding the extreme values of eigenvalues is presented in Fig. 7.

Despite of introducing a second stage in the procedure the main advantage remains. There is a considerable reduction of computational time by means of finding the $\underline{K}^*$, $\overline{M}^*$ and $\overline{K}^*$, $\underline{M}^*$ matrices.

4. Application of uncertainty analysis

In the paper an application of uncertainty analysis with the use of selected numerical procedures has been presented. Uncertainty analysis of the dynamical parameters of a car windshield model has been performed.

4.1. Finite Element model

For the calculations a finite element model of a car windshield provided by Renault Technocentre within the work frame of a European Project MADUSE has been used. Fig. 8 presents the FE model together with the structure of the windshield and a deformation shape for the first natural frequency with free-free boundary conditions. Presently manufactured windshields have a layered structure. The external layers are made of glass, and the internal layers are made of different types of polymers.

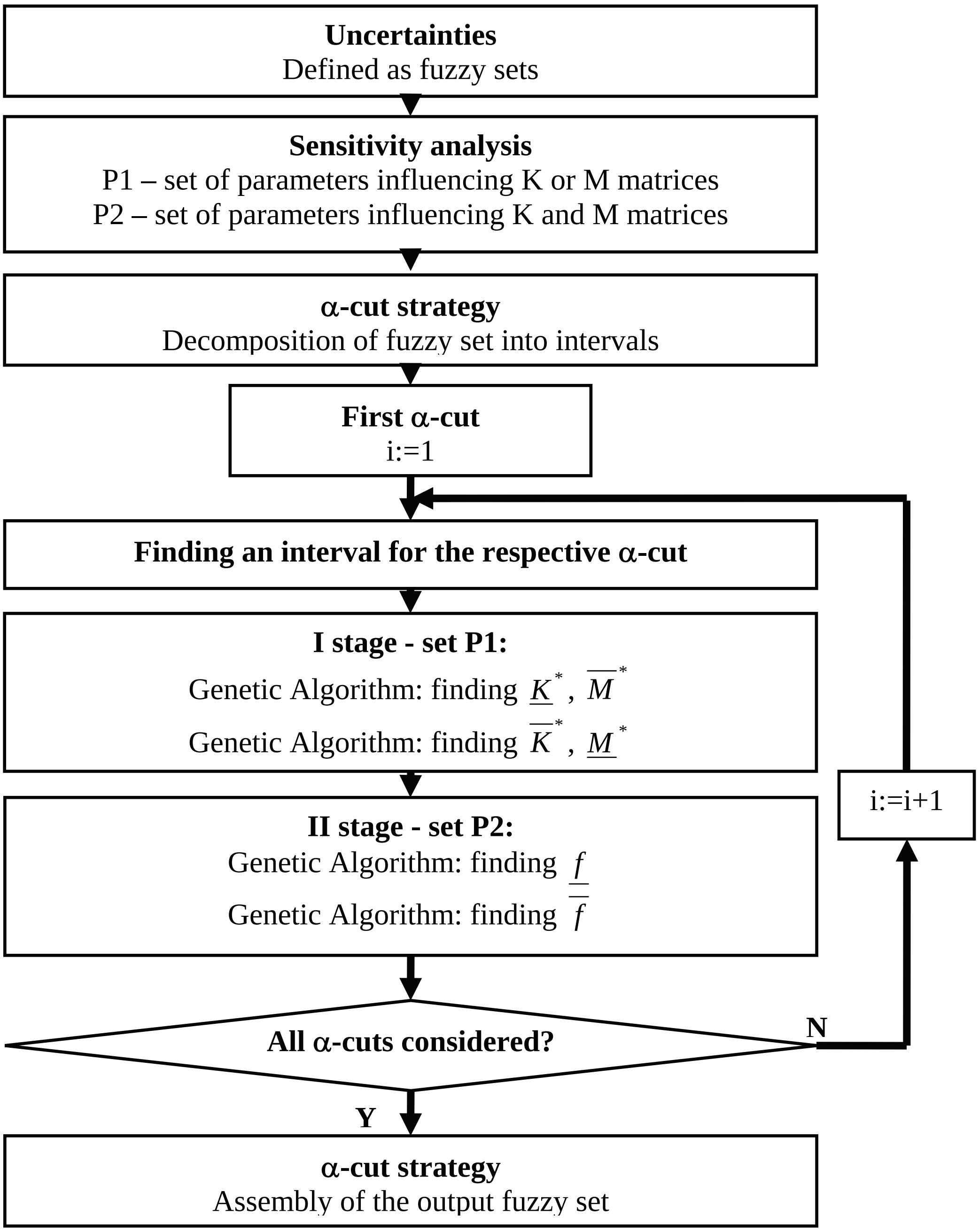

Fig. 7. 2-stage tested method for uncertainty analysis

MSC.Nastran FE solver have been used to find eigenvalues and eigenvectors of the model [19]. Times of solving the eigenproblem and assembling global mass and stiffness matrices were 4.15 s and 2.40 s respectively. Time reduction in the order of 42% allowed for an effective application of the method.

4.2. Parameter uncertainties

According to manufacturer's specification thirteen uncertain parameters have been defined. All of them were modelled by the triangular fuzzy numbers:

- geometrical parameters (±5% variation for the α0 cut):

- o thicknesses of all five layers: T1, T1, T3, T4, T5;
- material parameters (±5% variation for the $\alpha 0$ cut):
  - o glass: Young's modulus E1, Poisson's ratio ν , mass density ρ ,
  - o PVB polymer: Poisson's ratio $\nu 2$, mass density $\rho 2$,
  - o acoustic polymer AP: Poisson's ratio $\nu 3$, mass density $\rho 3$;
- environmental conditions:
  - o external temperature, Temp (in the range 5-33 degrees Celsius for the $\alpha 0$ cut) influencing the values of Young's modulus of the polymer layers.

Response quantity was the first natural frequency of vibration for the free-free boundary conditions.

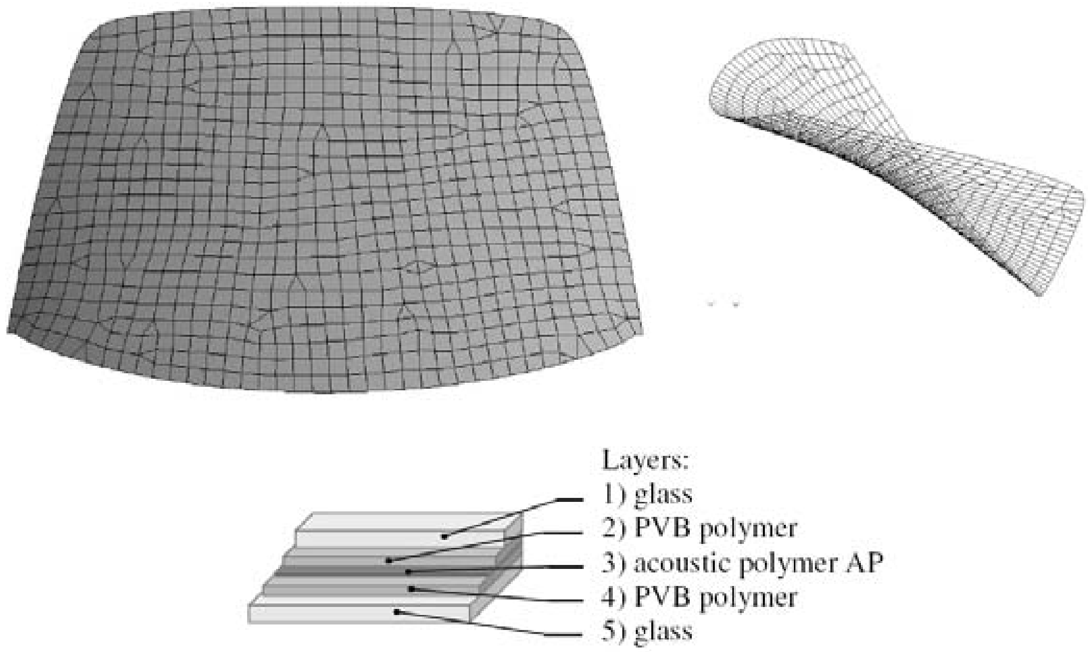

Fig. 8. FE model of the windshield, its structure and first natural frequency

4.3. Results

Uncertainty analysis was preceded by the sensitivity analysis giving the influence factors of the input parameters on the first natural frequency of vibration (Fig. 9) and on the sums of the elements of global system matrices (Fig. 10). According to the requirements of the method the parameters were divided in two groups:

P1 (parameters influencing K or M): E1, $\nu 1$, $\rho 1$, $\nu 2$, $\rho 2$, $\nu 3$, $\rho 3$, Temp,

P2 (Parameters influencing K and M): T1, T2, T3, T4, T5.

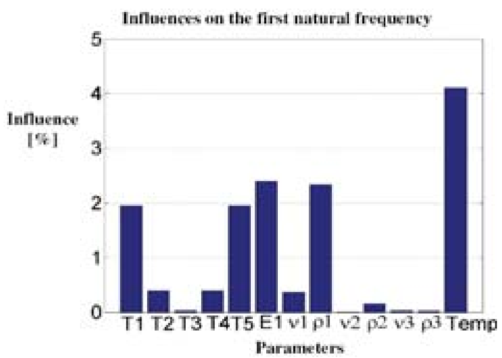

Fig. 9. Influence of the uncertain parameters on the first natural frequency

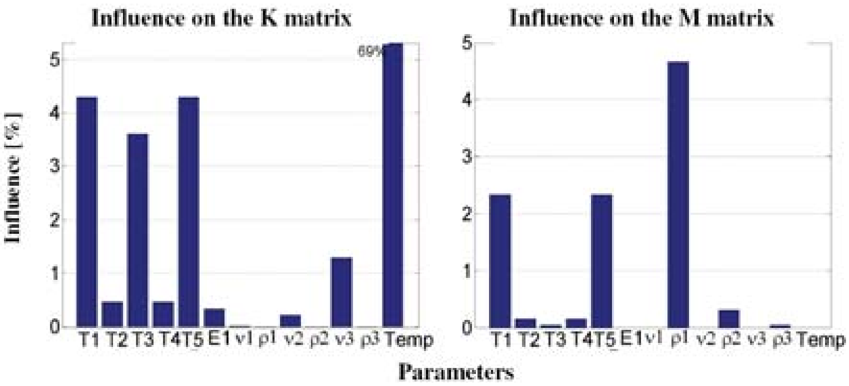

Fig 10. Influence of the uncertain parameters on the sums of the elements of K and M matrices

To perform the uncertainty analysis of the windshield model the following methods have been used (Tab. 2 presents details of the methods):

- GA1 – α-cut strategy with GA used to find extremes of eigenfrequency for the intervals created by the decomposition of input fuzzy sets,
- GA2 – α-cut strategy with GA used as in the proposed method,
- MC – α-cut strategy with the MC method used to find extremes of eigenfrequency for the intervals created by the decomposition of input fuzzy sets. Latin Hypercube Sampling was applied in order to increase the efficiency of the MC method.

Application of the α-cut strategy assumes a decomposition of each of the input fuzzy sets into five intervals α0-α4 where, considering the triangular shape of the fuzzy set, the last interval α4 is degenerated to a point representing a nominal value of a parameter.

Tab 2. Characteristics of the applied methods of uncertainty analysis

Application	Characteristics	Number of eigensolutions	Number of times that system matrices have to be assembled	Total computational time [s]
GA1	25 individuals (80% of the individuals (20) changed in each iteration) Number of generations: 125 (GA1/GA2) + 30 (second stage for GA2)	20000	-	83000 s (23 h)
GA2		4800	20000	67920 s (18.9 h)
MC	5000 iterations for each α-cut	20000	-	83000 s (23 h)

The following decomposition of the fuzzy sets has been performed:

- for the α0 cut the intervals of ±5% of the nominal values of the geometrical parameters and material parameters have been assumed. For the temperature the α0 cut was 5-33 degrees Celsius,
- for the α1 cut the values of ±3.75% and 8,5-29.5 degrees Celsius have been assumed,
- for the α2 cut the values of ±2.5% and 12-26 degrees Celsius have been assumed,
- for the α3 cut the values of ±1.25% and 15.5-22.5 degrees Celsius have been assumed,
- α4 cut represents the nominal configuration of the parameters.

Fig. 11 presents the results of the uncertainty analysis for all three methods used. For comparison purposes the results of 15 experimental measurements, corresponding to the α0 cuts, have been superimposed on the numerical results. Tab. 3 presents the quantitative comparison of the results obtains with the three numerical methods.

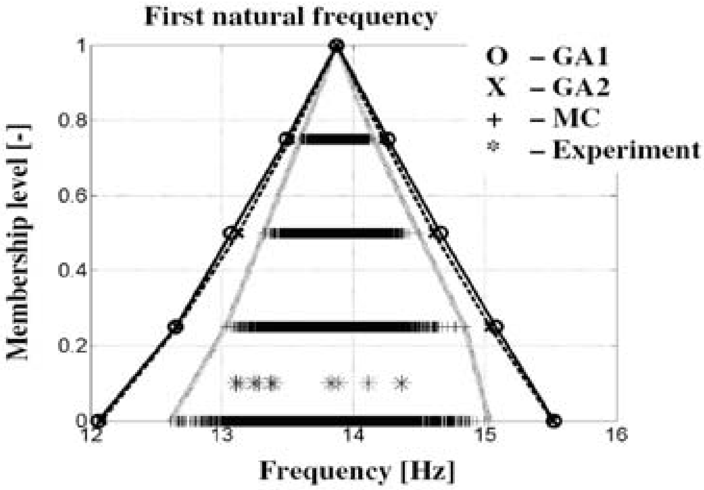

Fig. 11. Output fuzzy sets – comparison of the results

Tab. 3. Comparison of the numerical results

Parameter			α0 cut		α1 cut		α2 cut		α3 cut		α4 cut
			MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX	NOM
f_1[Hz]	GA1		12.059	15.523	12.648	15.082	13.069	14.661	13.489	14.261	13.873
	GA2		12.082	15.516	12.659	15.039	13.123	14.611	13.516	14.231	13.873
	MC		12.610	15.033	13.028	14.845	13.315	14.492	13.598	14.151	13.873
Error GA2 [%]	GA1	B	0.19	– 0.05	0.09	– 0.29	0.41	– 0.34	0.20	– 0.21	-
		P	0.66	– 0.20	0.45	– 1.77	3.39	– 3.14	3.50	– 3.89	-
	MC	B	– 4.19	3.21	– 2.83	1.31	– 1.44	0.82	– 0.60	0.57	-
		P	– 21.8	19.9	– 20.3	10.7	– 16.3	10.1	– 14.8	14.5	-

Values presented in Tab. 3 concerning the relative errors of GA2 application show the difference between the compared values referenced to the other two methods GA1 and MC (rows marked as ,,B”) and to the width of the intervals calculated for the reference value (rows marked as ,,P”). It should be noticed that the highest error appears comparing the application of GA2 with MC (maximal error of –4.19% for case „B” and –21.8% for „P”). It can be justified by the insufficient number of samples calculated in the MC method with respect to the number of uncertain parameters. The result is that the parameter space is not well covered, and some extremes could be omitted. When comparing GA1 and GA2 a good agreement of the results can be noticed – maximal error does not exceed 4% (–3.89% for α4 cut, for case „P”).

5. Summary and conclusions

Quality demands in the modern engineering analyses require the application of uncertainty analysis. Omitting the influence of parameters uncertainty may result in sub optimal performance or in extreme cases to a wrong design. Considering uncertainties allows for better understanding of the analyzed phenomena and allows for obtaining a broader view of the design. Numerical analyses are directed towards the improvement of the efficiency of computations. Reducing the time required for uncertainty analysis would allow for broader application of the methods especially in structural dynamics.

In the present paper an application of different types of uncertainty analysis has been presented. Finite Element model of a car windshield has been used as the analysis test case. A method based on the α-cut strategy with GA used to search global matrices of mass and stiffness has been described and applied on a test model. To verify the procedure two other methods have been applied and compared with the tested method. Uncertain parameters have been modelled with the triangular fuzzy numbers. Additionally the sensitivity analysis has been performed in order to get a better insight into the analyzed model.

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