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# OUTLINE OF A MODEL FOR FLOW OF INHOMOGENEOUS WORKING FLUIDS VIA AN INTERSTICE

**Leszek Ułanowicz**

*Air Force Institute of Technology*
*Księcia Bolesława St. 6, 01-494 Warsaw, Poland*
*e-mail: leszek.ulanowicz@itwl.pl*

### *Abstract*

*The paper presets flow characteristics of a hydraulic unit made up of a cylinder and a plunger, which is typical for most of precision pairs, as a function of contamination content in the working fluid. The consideration cover typical cases, including flow of contaminated fluid via an interstice (positive allowance), when the manifold plunger (spool) is in its initial position and the flow rate* $Q_z$ *of the working fluid at the manifold inlet is higher than the flow rate Q via the interstice, the manifold plunger (spool) is in its initial position and the flow rate* $Q_z$ *of the working fluid at the manifold inlet is lower than the flow rate Q via the interstice and when the manifold plunger moves against the manifold cylinder (body). Mathematical relationships, determined for all the above cases, define the following characteristics: volume of contaminants captured upstream the interstice, volume of contaminants captured in the interstice, volume of contaminants that flow out downstream the interstice, flow rate of the working flow downstream the interstice as a function of time, average velocity for the working fluid that flows out of the interstice as well as time interval that is necessary to fill in the interstice. Distribution of contaminants along the interstice length is described as well. The determined relationships served as a basis to find out interconnections between volumes of fluid that flows via an interstice and duration of the flow process.*

**Keywords:** *interstice, positive allowance, manifold plunger (spool), flow rate, volumetric content of contaminations, filtration, contamination concentration*

## 1. Introduction

Specific phenomena associated with flow of suspensions via an interstice (positive allowance) have not been investigated so far with sufficient thoroughness and only scarce number of studies dedicated to that phenomenon has been published. Most of these studies represent the attempt to explain the phenomenon of so called obliteration, i.e. gradual diminishment of the interstice active cross section, which results from adsorption of particles dispersed in the fluid and deposited on the interstice walls. As solids are considered the major component of contaminations that are present in the working fluid, a part of solid particles is captured in the interstice. Sedimentation of solids in interstices results from differences of dimensions between particles present in the working fluid and the interstice itself. As a consequence, reduction of the effective surface of the interstice occurs and concentration of suspension downstream the interstice is altered. Contamination particles, which are much larger than the positive allowance of the hydraulic node (a hydraulic precision pair) [1] or smaller than that clearance are not a reason for damages and consequential increase of friction in the hydraulic node. Working surfaces of the manifold body and the control plunger (spool) are vulnerable to particles of the same size as the positive allowance, as such particles pose the reason to growth of forces (torques) in control parts and then wear of hydraulic node surfaces in the hydraulic appliances. It results in many unfavourable effects, such increase of leaks, jamming of the slide (spool), postponing of the switchover time, changes in characteristic curves of flow, slowdown of movements executed by the driven parts or even total immobility of them, mispositioning and vibrations, deviations in speed adjustment, different speed of movements executed by driven parts depending on the movement directions, step movements of driven parts when the control signal changes in continuous (infinite) manner, decrease of the system stiffness and reduction of speed achieved by driven parts.

Therefore, there is a need to define relationships between flow parameters of contaminated working fluids passing the design clearances (interstices) and the contamination degree. These relationships are necessary to carry out theoretical calculations concerning flow of inhomogeneous fluids via design clearances, to estimate the effect of contamination onto reliable operation and growth of frictions in hydraulic nodes as well as to appropriate design of various hydraulic appliances. This paper describes relationships between various flow parameters of contaminated working fluids passing the design clearances (interstices) of a manifold unit. In case of manifold units, the interaction between the slide (spool) and the manifold cylinder (body) is considered as the hydraulic node, where a cylindrical plunger (spool) with ring-shaped recesses serves as a working component for such an appliance, where this plunger (spool) moves axially inside a cylinder (manifold body) having inlet and outlet ports to deliver and drain the working fluid.

The following cases shall be considered, where the contaminated working fluid passes an interstice:

a) the manifold plunger (spool) is immobile with regard to the cylinder (no slide/spool movements) and the flow rate $Q_z$ of the working fluid at the manifold inlet is higher than the flow rate Q via the interstice,

b) the manifold plunger (spool) is immobile with regard to the cylinder (no slide/spool movements) and the flow rate $Q_z$ of the working fluid at the manifold inlet is lower than the flow rate Q via the interstice,

c) the manifold plunger (spool) moves inside the cylinder (body).

## 2. Flow of contaminated working fluid via an interstice when manifold plunger (spool) is immobile with regard to the cylinder and the flow rate $Q_z$ of the working fluid at the manifold inlet is higher than the flow rate Q via the interstice

The volumetric degree of contamination for the working fluid $Z_V$ can be calculated with the formula [2]:

$$Z_V = \frac{V_1(t)+V_2(t)+V_3(t)}{V_1(t)+V_S(t)}, \tag{1}$$

where:

$V_1(t)$ - volume of contamination captured upstream the interstice inlet,

$V_2(t)$ - volume of contamination captured in the interstice,

$V_3(t)$ - volume of contamination that passes through the interstice,

$V_S(t)$ - volume of contaminated fluid that is fed to the interstice, i.e. $V_S = V_C + V_1(t) + V_2(t) + V_3(t)$, where $V_C$ is the volume of pure (uncontaminated) fluid.

Volume of contamination captured upstream (at the inlet of) the interstice depends on the flow rate of the fluid that is fed to the unit. In that sense the entire interstice can be considered as a filtering device. Based on the filtration theory [3] the assumption can be made that concentration of particles that are deposited both upstream and downstream the interstice is constant, which is expressed by the equation:

$$\frac{V_1(t)}{V_S(t)} = const\ , \ \frac{V_2(t)}{V_S(t)} = const\ . \tag{2}$$

If so, one can infer that concentration of contaminations captured upstream the interstice (at its inlet) $X_{V_1}$ as well as concentration of contaminations captured in the interstice $X_{V_2}$ is constant, which can be noted in the following manner:

$$\frac{V_1(t)}{V_S(t)-V_1(t)}=X_{V_1}=const\,, \tag{3}$$

$$\frac{V_2(t)}{V_S(t)-V_2(t)}=X_{V_2}=const\,. \tag{4}$$

Similarly, volumetric concentration of particles that pass through the interstice $X_{V_3}$ is also a constant value, which is expressed by the formula:

$$X_{V_3}=Z_V-\left(X_{V_1}+X_{V_2}\right)\left(1-Z_V\right)=const\,. \tag{5}$$

When the contaminated working fluid only starts flowing towards the interstice, i.e. at the moment $t = t_0$ and when no movements of the plunger (spool) inside the manifold cylinder (body) occur and flow rate $Q_z$ of the fluid at the manifold inlet is higher that the flow rate $Q$ via the interstice, the volume of contaminated fluid that flows into the interstice can be expressed in the following way:

$$V_S=V_{SZ}(0)=\pi d_{śr}h_0a_1=f_{SZ}(0)a_1\,,$$

where:

$V_{SZ}$ - volume of the interstice,

$d_{śr}$ - average diameter of the interstice, i.e. $d_{śr}=\frac{D+D_1}{2}$,

$D$ - cylinder diameter of the manifold,

$D_1$ - diameter of the working part of the plunger (spool),

$h_0$ - positive allowance,

$a_1$ - length of the working part of the plunger (spool),

$f_{SZ}(0)$ - cross-section of the interstice at the moment $t = 0$.

The equation (4) leads to the following:

$$V_2(t_0)=\frac{X_{V_2}V_2(t_0)}{1+X_{V_2}}=\frac{X_{V_2}}{1+X_{V_2}}f_{SZ}(0)a_1\,. \tag{6}$$

Particles that are deposited in an interstice with its volume $V_2(t)$ reduce its cross-section. It is why the average cross-section area $\overline{f_{SZ}}(t_0)$ at the moment of $t_0$ equals to:

$$\overline{f_{SZ}}(t_0)=\frac{V_{SZ}(0)-V_2(t_0)}{a_1}=\frac{V_{SZ}(0)}{a_1}\left(1-\frac{V_2(t_0)}{V_S(t_0)}\right).$$

The above relationships make it possible to note that:

$$\overline{f_{SZ}}(t_0)=f_{SZ}(0)\frac{1}{1+X_{V_2}}\,. \tag{7}$$

When the contaminated working fluid passes via the interstice and starts to flow out of it, i.e. $t > t_0$ and when no movements of the plunger (spool) inside the manifold cylinder (body) occur and flow rate $Q_z$ of the fluid at the manifold inlet is higher that the flow rate $Q$ via the interstice, the equation (4) can be converted to the following form:

$$\frac{V_2(t-t_0)-V_2(t_0)}{V_2(t-t_0)}=X_{V_2}=\frac{\Delta V_2(t-t_0)}{Q(t-t_0)\Delta(t-t_0)}\,, \tag{8}$$

where:

$V(t-t_0)$ - volume of contaminated working fluid downstream the interstice,

$\Delta(t-t_0)$ - time differential,

$Q(t-t_0)$ - flow rate of the working fluid downstream the interstice,
$\Delta V_2(t-t_0)$ - increase of contamination volume in the interstice.

As the pressured drop across the interstice is constant and $h_0 \ll a_1$ the assumption can be made that the working fluid behaviour is governed by the rule of laminar flow. Flow rate of the working fluid via the interstice can be defined by the equation [4]:

$$Q_0 = \frac{\pi d_{śr} h_0^3 p_z}{12\mu a_1} = \frac{p_z}{12\mu a_1 (\pi d_{śr})^2}\, f_{SZ}^3(0).$$

Based on the above relationship the flow rate of the contaminated working fluid downstream the interstice and at the moment of $(t-t_0)$ can be expressed as:

$$Q(t-t_0) = \frac{Q_0}{f_{SZ}^3(0)}\, \overline{f}^{\,3}_{SZ}(t-t_0), \tag{9}$$

whilst increase of contamination volume in the interstice:

$$\Delta V_2(t-t_0) = \left[\overline{f_{SZ}}(t-t_0) - \overline{f_{SZ}}(t_0)\right] a_1 = \Delta \overline{f_{SZ}}(t-t_0)\, a_1 . \tag{10}$$

Substitution of equations (9) and (10) to the formula (8) leads to the following:

$$\begin{gathered} \frac{\Delta \overline{f_{SZ}}(t-t_0)}{\overline{f}^{\,3}_{SZ}(t-t_0)} = \frac{X_{V_2} Q_0}{a_1 f_{SZ}^3(0)}\, \Delta(t-t_0), \\ \text{or} \\ \frac{d\, \overline{f_{SZ}}(t-t_0)}{\overline{f}^{\,3}_{SZ}(t-t_0)} = \frac{X_{V_2} Q_0}{a_1 f_{SZ}^3(0)}\, d(t-t_0). \end{gathered} \tag{11}$$

After resolving the equation (11) with the initial condition $\overline{f_{SZ}}(t-t_0)\big|_{t-t_0} = \overline{f}_{SZ}(t_0)$ the expression for the cross-section area is obtained:

$$\overline{f_{SZ}}(t-t_0) = \frac{\overline{f_{SZ}}(t_0)}{\sqrt{1 + \dfrac{2X_{V_2} Q_0 \overline{f}^{\,2}_{SZ}(t_0)(t-t_0)}{a_1 f_{SZ}^3(0)}}} .$$

As $\overline{f_{SZ}}(t_0) = f_{SZ}(0)\dfrac{1}{1+X_{V_2}}$, the average cross-section area of the interstice shall reach at the moment $(t-t_0)$ the value:

$$\overline{f_{SZ}}(t-t_0) = \frac{f_{SZ}(0)\dfrac{1}{1+X_{V_2}}}{\sqrt{1 + \dfrac{2X_{V_2} Q_0 (t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}} . \tag{12}$$

On the other hand, the equation (10) makes it possible to derive the relationship for increase of contamination volume in the interstice:

$$\Delta V_2(t-t_0) = a_1 f_{SZ}(0)\frac{1}{1+X_{V_2}}\left(1 - \frac{1}{\sqrt{1 + \dfrac{2X_{V_2} Q_0 (t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right). \tag{13}$$

The volume of contaminations captured in the interstice is defined by the formula:

$$V_2(t-t_0)=\frac{a_1 f_{SZ}(0)}{1+X_{V_2}}\left(1+X_{V_2}-\frac{1}{\sqrt{1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right). \tag{14}$$

Flow rate of the working fluid downstream the interstice at the moment ($t-t_0$) is expressed in the following way:

$$Q(t-t_0)=\frac{Q_0}{f_{SZ}^3(0)}\,\overline{f}_{SZ}^{\,3}(t-t_0)=Q_0\frac{\frac{1}{\left(1+X_{V_2}\right)^3}}{\left(1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{3/2}}. \tag{15}$$

Whereas the average velocity of the working fluid flow outside of the interstice is described by the relationship:

$$\overline{\upsilon}(t-t_0)=\frac{Q(t-t_0)}{\overline{f}_{SZ}(t-t_0)}=\frac{Q_0}{f_{SZ}(0)}\frac{\frac{1}{\left(1+X_{V_2}\right)^2}}{\left(1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{2}}. \tag{16}$$

The volume of contaminated working fluid downstream the interstice is the following:

$$V(t-t_0)=\frac{V_2(t-t_0)}{X_{V_2}}=\frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1-\frac{1}{\sqrt{1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right), \tag{17}$$

whilst the volume of contaminations that flow out of the interstice is defined by the formula:

$$V_3(t-t_0)=X_{V_3}V(t-t_0)=X_{V_3}\frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1-\frac{1}{\sqrt{1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right). \tag{18}$$

Volume of contaminations captured upstream the interstice can be calculated as:

$$V_1(t-t_0)=\frac{a_1 f_{SZ}(0)}{1+X_{V_2}}\left(\frac{Z_V-X_{V_3}}{1-Z_V}-X_{V_2}\right)\left[1+\frac{1}{X_{V_2}}\left(1-\frac{1}{\sqrt{1+\frac{2X_{V_2}Q_0(t-t_0)}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right)\right]. \tag{19}$$

Flow rate of working fluid upstream the interstice at the moment of $t_0$ is the following:

$$Q(t_0)=\frac{Q_0}{f_{SZ}^3(0)}\overline{f}_{SZ}^{\,3}(t_0).$$

The time that is necessary to fill in the interstice with the working fluid (until the interstice is completely filled) can be calculated in the following way:

$$t_0=\frac{a_1}{\dfrac{Q(t_0)}{\overline{f_{SZ}}(t_0)}}=\frac{a_1 f_{SZ}(0)}{Q_0}\left(1+X_{V_2}\right)^2. \tag{20}$$

## 3. Flow of contaminated working fluid via an interstice when manifold plunger (spool) is immobile with regard to the cylinder and the flow rate Qz of the working fluid at the manifold inlet is less or equal than the flow rate Q via the interstice

It is also the case when contaminations, captured at the inlet to the interstice, lead to gradual clogging of the same. In accordance to the basic equation for the filtering process with gradual clogging of filtering components the supply of contaminated working fluid to the interstice of a slide (spool) manifold at the moment of $t = t_0$ shall be defined by the following general equation:

$$kV_S(t)=Q_0-Q(t), \tag{21}$$

where:

$k$ - constant coefficient,

$V_S(t)$ - volume of contaminated fluid that is supplied to the interstice,

$Q(t)$ - flow rate of the contaminated fluid that is supplied to the interstice.

The time $t_{01}$ that is necessary to fill in the interstices shall be the following:

$$t_{01}=\frac{a_1}{\dfrac{Q(t_{01})}{\overline{f_{SZ}}(t_{01})}}=\frac{a_1 f_{SZ}(0)}{Q_0-kV_{SZ}(0)}\frac{1}{1+X_{V_2}}. \tag{22}$$

Gradual clogging of the interstice inlet results in decrease of the flow rate into the interstice. The volume of contaminated working fluid downstream the interstice at the moment $(t - t_{01})$ is defined by the following equation:

$$V(t-t_{01})=\frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1-\frac{1}{\sqrt{1+\dfrac{2X_{V_2}\left[Q_0-kV(t-t_{01})\right](t-t_{01})}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right), \tag{23}$$

whereas the volume of contaminations that flow out of the interstice at the moment *$(t - t_{01})$* is the following:

$$V_3(t-t_{01})=X_{V_3}\frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1-\frac{1}{\sqrt{1+\dfrac{2X_{V_2}\left[Q_0-kV(t-t_{01})\right](t-t_{01})}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right). \tag{24}$$

Flow rate of the working fluid downstream the interstice at the moment of ($t - t_{01}$) can be calculated as:

$$Q(t-t_{01}) = \frac{Q_0 - kV(t-t_{01})}{f_{SZ}^3(0)} \bar{f}_{SZ}^{3}(t-t_{01}) = \left[Q_0 - kV(t-t_{01})\right] \frac{\dfrac{1}{\left(1+X_{V_2}\right)^3}}{\left(1+\dfrac{2X_{V_2}\left[Q_0 - kV(t-t_{01})\right](t-t_{01})}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{3/2}}, \qquad (25)$$

whereas the average velocity of the working fluid that flows out of the interstice is defined by the relationship:

$$\bar{\upsilon}(t-t_{01}) = \frac{Q_0 - kV(t-t_{01})}{\bar{f}_{SZ}(t-t_{01})} = \frac{Q_0}{f_{SZ}(0)} \frac{\dfrac{1}{\left(1+X_{V_2}\right)^2}}{\left(1+\dfrac{2X_{V_2}\left[Q_0 - kV(t-t_{01})\right](t-t_{01})}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{2}}. \qquad (26)$$

Volume of contaminations captured upstream the interstice at the moment ($t_{01}$) is calculated in the following way:

$$V_1(t-t_{01}) = \frac{a_1 f_{SZ}(0)}{1+X_{V_2}} \left(\frac{Z_V - X_{V_3}}{1-Z_V} - X_{V_2}\right) \left[1+\frac{1}{X_{V_2}}\left(1-\frac{1}{\sqrt{1+\dfrac{2X_{V_2}\left[Q_0 - kV(t-t_{01})\right](t-t_{01})}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right)\right]. \qquad (27)$$

## 4. Flow of contaminated working fluid via an interstice when manifold plunger (spool) moves inside the cylinder and the flow rate $Q_z$ of the working fluid at the manifold inlet is higher than the flow rate Q via the interstice

The equilibrium state for the flow of liquid via the interstice shall be reached within the time of $t_r$ after the plunger (spool) starts its movement, which is denoted as:

$$t_r = t_0 + t_1,$$

where $t_1$ stands for the time when the particles are stopped in the interstice during abrasion and immersion into the interstice surface.

The time when particles remain in the interstice during the processes of abrasion (undercutting) and immersion into the interstice surface is defined by the relationship:

$$t_1 = t_{1\acute{s}} + t_{1w},$$

where $t_{1\acute{s}}$, $t_{1w}$ - the time for abrasion and immersion into the interstice surface.

The respective times for abrasion $t_{1\acute{s}}$ and for immersion into the interstice surfaces $t_{1w}$ are expressed by the equations:

$$t_{1\acute{s}} = n_1 \frac{\varphi}{\omega} = \frac{4n_1\sqrt{l_{\max}^2 - h_0^2}}{\omega D_1}, \qquad t_{1w} = \frac{\varphi_1}{\omega} n_2,$$

where:

$n_1$, $n_2$ - multiplicity for participation of particles in abrasion and immersion into the interstice surface,

$l_{\max}$ - maximum size of particles,

$\varphi$, $\varphi_1$ - translocation angles of the plunger (spool) during abrasion and immersion into the interstice surface,

$D_1$ - working diameter of the plunger (spool) part,
$h_0$ - positive allowance.

The time $t_r$ that expires from the moment when the plunger (spool) starts moving until the equilibrium of the fluid flow via the interstice is reached shall be calculated in the following way:

$$t_r = \frac{a_1 f_{SZ}(0)}{Q_0}\left(1+X_{V_2}\right)^2 + \frac{1}{\omega}\left(n_2\varphi_1 + \frac{4n_1\sqrt{l_{\max}^2 - h_0^2}}{D_1}\right).$$

For the case, when the contaminated working fluid starts flowing towards the interstice and the flow rate $Q_z$ at the manifold inlet is higher that the flow rate $Q$ via the interstice, volume of particles captured in the interstice at the moment $t_r$ is defined by the relationship:

$$V_2(t_r) = \frac{a_1 f_{SZ}(0)}{1+X_{V_2}}\left(1+X_{V_2} - \frac{1}{\sqrt{1+\dfrac{2X_{V_2}Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right), \tag{28}$$

whereas flow rate of the working fluid downstream the interstice at the moment $t_r$ is calculated by the formula:

$$Q(t_r) = Q_0\frac{\dfrac{1}{\left(1+X_{V_2}\right)^3}}{\left(1+\dfrac{2X_{V_2}Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{3/2}}. \tag{29}$$

The average velocity of the working fluid that flows out of the interstice at the moment of $t_r$ is expressed by the equation:

$$\bar{\upsilon}(t_r) = \frac{Q(t_r)}{\bar{f}_{SZ} t_r} = \frac{Q_0}{f_{SZ}(0)}\frac{\dfrac{1}{\left(1+X_{V_2}\right)^2}}{\left(1+\dfrac{2X_{V_2}Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^2}. \tag{30}$$

Volume of the working fluid downstream the interstice at the moment of $t_r$:

$$V(t_r) = \frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1 - \frac{1}{\sqrt{1+\dfrac{2X_{V_2}Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right). \tag{31}$$

Volume of contaminations that flow out of the interstice at the moment of $t_r$ can be calculated in the following way:

$$V_3(t_r) = X_{V_3}\frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})}\left(1 - \frac{1}{\sqrt{1+\dfrac{2X_{V_2}Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right), \tag{32}$$

whereas the volume of contaminations captured at the moment of $t_r$ upstream the interstice is:

$$V_1(t_r) = \frac{a_1 f_{SZ}(0)}{1+X_{V_2}} \left( \frac{Z_V - X_{V_3}}{1-Z_V} - X_{V_2} \right) \left[ 1 + \frac{1}{X_{V_2}} \left( 1 - \frac{1}{\sqrt{1 + \frac{2X_{V_2} Q_0 t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}} \right) \right]. \tag{33}$$

## 5. Flow of contaminated working fluid via an interstice when manifold plunger (spool) moves against the cylinder and the flow rate Qz of the working fluid at the manifold inlet is lower or equal to the flow rate Q via the interstice

When feeding of the contaminated working fluid to the interstice is commenced and the flow rate $Q_z$ of the working fluid at the manifold inlet is less or equal to the flow rate $Q$ via the interstice, the volume of contaminations captured in the interstice at the moment of $t_r$ is defined by the following relationship:

$$V(t_r) = \frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})} \left( 1 - \frac{1}{\sqrt{1 + \frac{2X_{V_2}\left[Q_0 - kV(t_r)\right]t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}} \right). \tag{34}$$

After passing the interstice, the volume of contaminations that flows out from the interstice at the moments of $t_r$ is the following:

$$V_3(t_r) = X_{V_3} \frac{a_1 f_{SZ}(0)}{X_{V_2}(1+X_{V_2})} \left( 1 - \frac{1}{\sqrt{1 + \frac{2X_{V_2}\left[Q_0 - kV(t_r)\right]t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}} \right). \tag{35}$$

Therefore the flow rate of working fluid downstream the interstice of the moment of $t_r$ shall be calculated as:

$$Q(t_r) = \frac{Q_0 - kV(t_r)}{f_{SZ}^3(0)} \bar{f}_{SZ}^3(t_r) = \left[Q_0 - kV(t_r)\right] \frac{\frac{1}{\left(1+X_{V_2}\right)^3}}{\left(1 + \frac{2X_{V_2}\left[Q_0 - kV(t_r)\right]t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^{3/2}}, \tag{36}$$

with the average velocity of fluid that flows out the interstice:

$$\bar{\upsilon}(t_r) = \frac{Q_0 - kV(t_r)}{\bar{f}_{SZ}(t_r)} = \frac{Q_0}{f_{SZ}(0)} \frac{\frac{1}{\left(1+X_{V_2}\right)^2}}{\left(1 + \frac{2X_{V_2}\left[Q_0 - kV(t_r)\right]t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}\right)^2}. \tag{37}$$

Volume of contaminations that are captured upstream the interstice at the moment of $t_r$ shall be the following:

$$V_1(t_r)=\frac{a_1 f_{SZ}(0)}{1+X_{V_2}}\left(\frac{Z_V-X_{V_3}}{1-Z_V}-X_{V_2}\right)\left[1+\frac{1}{X_{V_2}}\left(1-\frac{1}{\sqrt{1+\frac{2X_{V_2}\left[Q_0-kV(t_r)\right]t_r}{a_1 f_{SZ}(0)\left[1+X_{V_2}\right]^2}}}\right)\right]. \qquad (38)$$

For the condition $\int_0^{a_1} f_1(x,t)dx=V_2(t)$ concentration of contaminations $Z_{psz}$ in the interstice is equal to:

$$Z_{psz}(x)=\frac{V_0(t)-f_1(x,t)}{V_S(t)-f_1(x,t)},$$

where:

$V_0(t)$ - volume of contaminations that penetrate into the interstice,

$V_S(t)$ - volume of contaminated fluid that flows into the interstice,

$f_1(x,t)$ - distribution of contaminations that are captured down the interstice length.

The function $f_1(x,t)$ is the relationship that depends on the distance from the interstice inlet and the duration of the working fluid flow throughout (see Fig. 1).

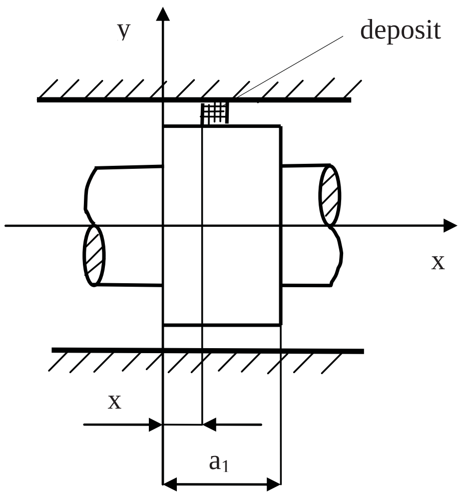

*Fig. 1. Illustrative sketch for distribution of contaminations down the interstice length*

Volume of contaminations that penetrate into the interstice can be calculated by the formula:

$$V_0(t)=Z_V\left[V_1(t)+V_S(t)\right]-V_1(t)=(Z_V-1)\frac{X_{V_2}}{1+X_{V_2}}V_S(t)+Z_V V_S(t).$$

On the other hand, the function $f_1(x,t)$ is determined from the relationship:

$$\frac{f_1(x,t)}{V_x(t)}=U_1(x,t),$$

where:

$V_x(t)$ - volume of contaminations along a small distance $dx$ from the interstice inlet,

$U_1(x,t)$ - capture factor for the entire volume of contaminations down the interstice length.

The factor $U_1(x,t)$ chiefly depends on the surface roughness. Under the assumption that the factor $U_1(x,t)$ is time-independent and linearly decreases with the distance of the contamination deposit inside the interstice from the interstice edge, i.e. $U_1(x)=1-kx$ where $k<\frac{1}{a_1}$, then

$$\int_0^{a_1} V_x(t)[1-kx]dx = V_x(t)a_1\left(1-\frac{ka_1^2}{2}\right)=V_2(t),$$

which leads to the equation:

$$V_x(t)=\frac{V_2(t)}{a_1\left(1-\frac{ka_1}{2}\right)}=V_S(t)\frac{X_{V_2}}{1+X_{V_2}}\frac{1}{a_1\left(1-\frac{ka_1}{2}\right)}.$$

After having the above equations substituted to the formula for concentration of contaminations $Z_{psz}$ along the interstice it is possible to obtain the following result:

$$Z_{psz}(x)=\frac{Z_V-(1-Z_V)\frac{X_{V_1}}{1+X_{V_2}}-\frac{X_{V_2}}{1+X_{V_2}}\frac{1}{a_1\left(1-\frac{ka_1}{2}\right)}(1-kx)}{1-\frac{X_{V_2}}{1+X_{V_2}}\frac{1}{a_1\left(1-\frac{ka_1}{2}\right)}(1-kx)}. \tag{39}$$

Distribution of contamination concentration $Z_{psz}$ along the interstice length as a function of the deposit distance from the interstice edge is exhibited in Fig. 2.

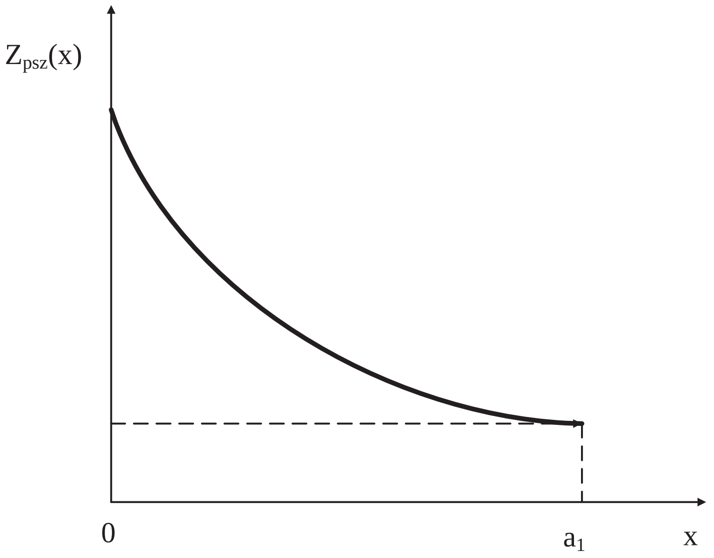

*Fig. 2. Distribution of contamination concentration $Z_{psz}$ along the interstice length as a function of the deposit distance from the interstice edge*

## 6. Summary

The foregoing deliberations demonstrate that only those particles that are larger than the interstice height can be captured during the initial phase of flow. Sedimentation of such particles results in reduction of active width of the interstice available for the flux by the total value of their characteristic dimensions. Increase of the particle amount (contaminations) captured at the interstice inlet increases the probability that the approaching particles shall be deposited as well. Any higher number of particles with their dimensions that meet the condition $x > h_0$ results in faster subdivision of the interstice width into the number of segments that is quite near to the number of captured particles. Mostly those particles penetrate into the interstice and are deposited therein that are of the size that is quite near the interstice height (with dimensions not much different from the clearance/

positive allowance value). This leads to increase of friction in a hydraulic node. As particles with larger sizes than the positive allowance $h_0$ also inflow to the interstice, they abrade surfaces of the plunger (spool) and the manifold cylinder. That effect results in increase of the momentary value of friction in a hydraulic node. After the plunger (spool) movement starts, nearly all particles with dimensions $x > h_0$ shall simultaneously abrade and immerse into the interstice surface. During movements of the plunger (spool) volume $V_2$ of the deposit is reduced, especially when working fluid passes the interstice. Even large particles that abrade the plunger (spool) and the manifold cylinder surfaces get blunt or are crushed, therefore they no more interact with the plunger (spool) and the manifold cylinder surfaces and are discharged from the interstice. If the interstice is flushed with fresh working fluid the degree of resistance to the plunger (spool) movements drops down to the value of viscous friction. However, if contaminated working fluid passes the interstice, the amount of deposit $V_2$ tends to a new, lower level.

In order to reduce friction that results from distribution of particles down the interstice length it is suggested to cut relief (ring-shaped) groves on perimeter surfaces of plungers (spools). Owing to application of relief grooves the interstice length $a_1$ is reduced (cf. Fig. 1) and distances of abrasive affect exerted by large particles as well as multiplicity of abrasion and immersion is reduced. Due to decreasing concentration of the deposit volume along the interstice length most of particles are deposited at the interstice inlet. Particles that are able to penetrate across the first lip on the plunger (spool) are then deposited in the groove where the velocity of lowing fluid rapidly decreases, therefore further migration of these particles into the interstice is difficult, the more that the groove volume is many times higher than the interstice volume. Provided that the particles are uniformly distributed in the fluid volume one can assume that only a small part of the deposit that penetrates into the interstice shall remain on lip surfaces. It is why the following measures are recommended:

a) make cuts within the shortest possible distance to the interstice inlet,
b) deploy grooves as close to one another as possible, within distances that are shorter that the working stroke of the plunger (spool),
c) make cuts with their depth not less than their width and with their bottoms as flat as possible,
d) maintain sharp edges of grooves,
e) cut grooves with the following dimensions: groove width 0.3-0.5 mm, groove depth 0.8-1.0 mm,
f) maintain the lowest possible positive allowance at the interstice inlet and in case of constant pressured drop across the interstice, shape the farther conical part of the cylinder with gradual decrease of he interstice width towards the lower pressure,
g) make the plunger (spool) longer than the cylinder (due to bevelling of the plunger/spool ends).

Experiments that were carried out by the author of this study have revealed that a single relief groove on the plunger (spool) surface reduces both static and dynamic friction forces from 45 N (with plain plunger/spool surfaces) to 19 N. In case of as many as three grooves the friction force decreases from 45 N to 3.2 N.

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