ASSESMENT OF OPERATION OF SHIP MAIN DIESEL ENGINE USING THE THEORY OF SEMI-MARKOVIAN AND MARKOV PROCESSES

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Abstract

Operation of ship propulsion system is associated with realization of definite operational goals. If to elements of the system the operational reliability strategy could be applied, the situation would be very simple as existing conditions would unambiguously determine application of means being on hand. However decision to reject application of the strategy (even if hypothetical) becomes obvious with a view of necessity of ensuring an acceptable level of safety to ship and environment as well as presence of associated formal and legal limitations. Therefore complexity of operational reality makes that means intended for operation may be used in various ways. Objectivity and rationality in making decision, assumed optimal in given conditions, forces to apply an evaluating (quantitative) approach to the problem, hence to search for such their parameters (indices) which, in a given decision situation, can be deemed most adequate.

To precisely determine the task it is necessary to specify also its duration time, apart from conditions in which it will be realized. When considering propulsion engine, i.e. the main element of ship propulsion system, especially important becomes not only the problem which amount of energy could be at one's disposal but also within which time interval it could be delivered. Therefore apart from applying the commonly used reliability indices, it seems sensible to consider the operation in such evaluating approach as it could be determined by energy and time simultaneously.

Keywords: *operations, ship power plant, diesel engine, semi-Markov processes, Markov processes*

1. Introduction

Operation of any power systems is associated with realization of determined operational goals. In the case of ship power systems the goals are specified in relatively various ways, depending on ship function, and cover first of all the following:

- realization of assumed values of ship speed within a determined range and direction of operation,
- ensurance of appropriate living conditions for ship crew (and passengers),
- ensurance of appropriate values of stability and unsinkability parameters for ship,
- surveillance in the area of fire safety and ensurance of suitable conditions to undertake fire fighting action in the case of fire occurrence etc.

Making far-going generalization one can state that operation of every ship power system is aimed at the providing and appropriate distributing of energy of various kinds (media) necessary for correct functioning the ship.

Means intended for operation may be used in various ways because of the complexity of operational reality, which results mainly from:

- lack of a precise theory and operational procedures in the area of operation of complex technical systems (e.g. such as ship power plant),
- aiming at minimization of operational cost (profit index),
- uncertainty and incompleteness of information on current technical state of objects being in use,

- consequence of making erroneous decision.

Hence the problem of making a single decision, out of many possible, arises.

Process of selection of such decision which could be deemed the best in given conditions, should be based on as-complete-as- possible data available in the conditions in question. However even in the case (practically unrealistic) of having complete data about an object under operation, essence of the decision-making process is to select a criterion which makes it possible to assess and compare consequences of making different decisions.

In the case of complex technical systems which realize especially important tasks connected on one hand with expected profit and on the other hand with generation of a significant thread to environment, two equivalent criteria are at one's disposal:

- maximization of amount of profits (minimization of losses),
- minimization of a hazard to safety, possible to occur.

Since natural contradiction appears between the presented criteria - as the increasing of safety level leads to the lowering of amount of profits, and in reverse - and present formal safety regulations determine only a certain required state of the matters, deemed minimum, realization of the decision making process respective to the same objects used in similar conditions is somewhat different and often of an intuitive character.

The described state of matters may be transferred - in a special way - to the situation in which ship power systems (especially ship power plant and ship propulsion system) are used.

Thread to people, ship and the environment results in that during operation direct and indirect users of power plant make permanently decisions which concern the using and maintaining of particular devices (first of all ship propulsion system), aiming at ensurance of normal situation, i.e. that in which their safe operation is possible. Hence it seems that more and more indispensable become the systems which would be able to automatically elaborate a decision proposal on the basis of information gathered in real time [5].

Possibility of practical realization of the systems which make it possible to effectively provide support in the area of operational control, requires a quantifying approach to be applied to operation and decision making process, hence this is connected with application of mathematical models of the systems.

In the system a superior role is to be played by decision-making processes for which values of parameters obtained on the basis of operation process models will constitute a group of input quantities.

Objectivity and rationality in making decision assumed optimal in given conditions, forces to apply an evaluating (quantitative) approach to the problem, hence to search for such their parameters (indices) which can be deemed most adequate to a given decision situation.

In practice, predictions dealing with tasks under realization are usually based on the broad-understood notion of operational reliability of an object or system. And, when the notion of reliability of power devices (e.g. ship diesel engines) is considered it should be observed that from the user's point of view the most important problem is quality of realization of a given task (in the extreme case - its non-realization). Hence the notion of reliability is closely associated with unambiguous definition of the task in question.

And, to precisely define the task it is necessary to specify, apart from conditions in which it will be realized, also its duration time. The problem is specially important in such domains e.g. as sea shipping where specificity of tasks is usually connected with necessity of long-lasting functioning of crucial mechanisms and devices (e.g. ship).

Therefore specially important becomes not only the problem which amount of energy could be at disposal of operator of a given power device but also within which time interval it could be delivered. Apart from applying the commonly used reliability indices, it seems sensible to consider engine's operation (its functional subsystems) by using such evaluating approach as it could be simultaneously determined by energy and time.

2. Value of operation

In this case the operation (*D*) in the time interval [0, t] can be interpreted as a physical quantity determined by the product of the time-variable energy $E = f(t)$ and time *t,* which can be generally expressed as follows [5]:

$$D = \int_0^t E(\tau) d\tau . \tag{1}$$

In the case of a general analysis of operation of self-ignition engine it can be considered that the energy produced due to combustion of fuel in engine cylinders makes it possible to generate torque of the engine. As a result of transmitting the torque from the engine to a consumer the work L_e is done, which can be determined, in this case, from the expression:

$$L_e = M_o \cdot 2\pi n \cdot \mathrm{t}, \tag{2}$$

where:
L_e – effective work,
M_o – mean torque of engine,
n – engine (rotational) speed.

In this case, as results from Eq. (1) and (2), the engine's operation can be determined by the following expression:

$$D = 2\pi \int_0^t M_0 \, n \, t \, dt . \tag{3}$$

Further by introducing the notions of:

- *the required operation* D_W, i.e. that necessary for realization of a task (e.g. transportation of a cargo by sea within a given time, which is equivalent to keeping a given mean speed of ship, hence also power output developed by main propulsion engine (-s),
- *the possible operation* D_M, i.e. that possible to be realized by an engine being in a given technical state and in given functioning conditions,

and as a result of satisfying the relation:

$$D_M \geq D_W, \tag{4}$$

the assessment criterion of degree of serviceability is obtained in accordance with the principles presented in detail in [5, 8].

During functioning the engine, and first of all as a result of degrading action of wear processes, its total efficiency defined e.g. as [10]:

$$\eta_e = \frac{1}{g_e \cdot w_d}, \tag{5}$$

where:
g_e – specific fuel oil consumption,
w_d – net calorific value of fuel oil,

decreases along with time, that obviously leads to changes in the above defined value of *the possible operation* D_M.

In the case of ship main engine, with a view of taking into account the so- called design sea margin as well as service power margin [5, 9, 10] for the engine operating under partial loads, the process of the decreasing of available power output (hence also of *the possible operation* D_M) will be realized in two phases:

- in the first phase only an increase of hourly fuel oil consumption will take place (at a relatively constant value of developed engine torque), hence operational cost will also increase,
- in the second phase a limitation of effective power developed by the engine will appear due to design limitations and lack of possible increasing fuel charge. The described phenomena are caused by controlling action of fuel apparatuses which will increase the instantaneous fuel charge $g_p^{i\%}$ ($g_p^{i\%}$ - fuel charge for *i%* load of engine in serviceability technical state, under assumption that the maximum engine load amounts to 110% of its rated load, i.e. $i < 110$) within a determined time interval until its maximum value G_{pmax} is reached. Every subsequent decrease of total efficiency of engine will result in a noticeable drop of M_o.

If partial engine load is assumed constant the phenomenon can be interpreted as follows:

- in the first phase the time-variable drop of total engine efficiency results first of all in increasing its hourly fuel oil consumption (increase of specific fuel oil consumption). It can be described as a series of the recordable events F consisting in increasing the fuel charge $g_p^{i\%}$ by the increment Δg_p at a relatively constant value of the torque M_o (appropriate to a given state of engine load). This way an increase of operational cost of it is generated, however without any limitations imposed on ship motion parameters, in principle,
- gradual degradation processes during further engine operation cause the recordable events U consisting in decreasing the engine torque M_o at the constant fuel consumption g_p (i.e. $g_p = G_{p\ max}$) to occur. Further long-lasting operation of the engine results in significant worsening its characteristics which impose serious limitations on ship moving with an assumed speed or course. In heavy weather conditions such situation will obviously lead to producing a hazard to ship safety.

In the context of the above mentioned quality of task realization in making decisions with the use of a probabilistic decision-making process, to know the following data becomes important:

a) expected value of increased task realization cost resulting from increased fuel oil consumption,
b) value of occurrence probability of such number of F events which cause the fuel oil charge $g_p^{i\%}$ to increase up to the value $G_{p\ max}$, and ship motion limitations this way to occur.

In the considered case of ship main diesel engine the problem defined in a) can be solved by making use of the assumption that the number $N_{\Delta gp}$ of repetitions of the event F within the time interval (0, t) is a random variable of non-negative integer values. Dependence of the random variable on time forms the $\{N(t) : t \geq 0\}$ stochastic process. Under assumptions on its stationarity [11], lack of consequences and flow singularity, the Poisson's homogeneous process can be applied to the process of increasing the fuel charge $g_p^{i\%}$ as a result of decreasing the engine's total efficiency η_e (in steady load conditions of the engine).

3. Application of semi-Markov processes to assessment of operation

In the presented aspect of operation analysis of main propulsion engine the successive problem is to determine value of occurrence probability of such number of the events F which would cause the fuel charge $g_p^{i\%}$ to increase up to the value $G_{p\ max}$, and this way ship motion limitations to occur. The probability can be expressed by the following relation:

$$P\left(N_{\Delta gp} = k\right) = \frac{\left(\lambda_f \cdot t\right)^k}{k!} \exp\left(-\lambda_f t\right); \quad k = 1,2,...,n. \tag{5}$$

However under the assumption that random variables, which describe duration time of remaining the engine in particular states in the period when the decreasing of its total efficiency occurs, have arbitrary probability distributions, a more versatile approach is to represent the process in the form of the semi-Markov process $\{X(t): t \ge 0\}$ or, Markov process [3] as its first approximation.

To apply such approach it is necessary first of all to positively verify the following hypothesis (which describes the Markov condition [6]): *„the process of decreasing total efficiency of engines (understood as a random function whose argument is time and values are random variables* [4]) *is such whose state considered in the arbitrary instant t_n ($n = 0, 1, ..., m$; $t_0 < t_1 < ... < t_m$) depends stochastically on the state directly preceding it and does not depend on the states which occurred earlier and on their duration periods".*

Consequences of the hypothesis are the following:

a) *the probabilities (p_{ij} ; $i \neq j$; $i, j \in N$) of passing the process $\{X(t): t \ge 0\}$ of engine from any state i in which it presently remains to the successive (arbitrary) state j, do not depend on the fact in which states the process has been earlier,*

b) *the unconditional duration time intervals of the particular states i of the process $\{X(t): t \ge 0\}$ are the stochastically independent random variables (T_i; $i \in N$),*

c) *the duration time intervals of every state possible to occur provided the next state is one of the remaining states of the process, are the stochastically independent random variables (T_{ij}; $i \neq j$; $i, j \in N$).*

The specified consequences reveal the probabilistic principle of changing the states of the process $\{X(t): t \ge 0\}$.

Transition graph of the considered process can be presented as follows:

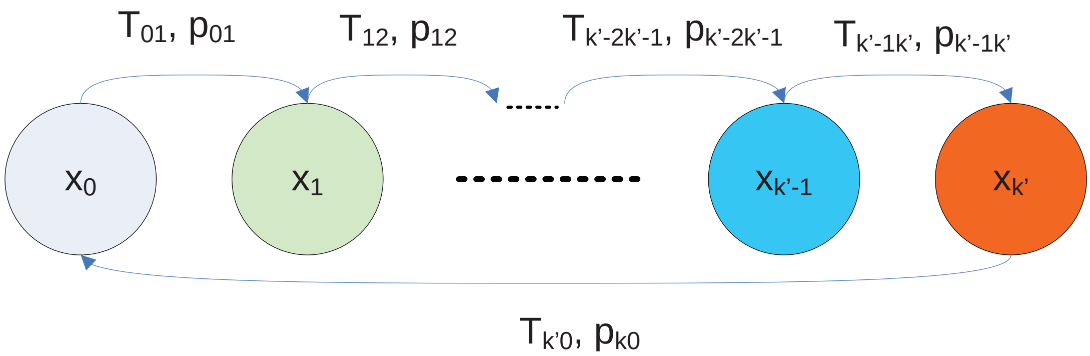

Fig. 1. Transition graph of the process $\{X(t): t \ge 0\}$

Functional matrix of the considered process $\{X(t); t \ge 0\}$ is of the form:

$$Q(t) = \begin{bmatrix} 0 & Q_{01}(t) & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & Q_{12}(t) & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & Q_{23}(t) & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{ii+1}(t) & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & Q_{k'-1k'}(t) \\ Q_{k'0}(t) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (6)$$

and the initial distribution of the process, under the assumption that in the instant of commencing realization of a given task engine's total efficiency is the greatest is:

$$p_0 = P\{X(0) = x_0\} = 1,$$

$$p_i = P\{X(0) = x_i\} = 0 \text{ dla } i = 1, 2, \ldots, k'. \tag{7}$$

This way the searched for semi-Markov process $\{X(t) : t \geq 0\}$ has been determined. The elaborated model makes it possible to determine occurrence probability values of such number of the events F which would cause the fuel charge $g_p^{i\%}$ to increase up to the value $G_{p\,max}$, hence also ship motion limitations to appear.

The probability can be determined on the basis of time distribution and parameters of the first transition of the considered process $\{X(t): t \geq 0\}$ to a separated subset of states [7].

On the basis of [7], if $N \subset X$ means a set of states imposing the above mentioned limitations and $N` = X - A$ - a set of states not imposing such limitations, then Laplace transform of the probability density of the random variable $\Xi_{iN}$ which describes the period passing between the instant of taking, by the process $\{X(t) : t \geq 0\}$, the value $i \in N`$ and that of taking, by it, any value from the subset of states N, will be the solution of the equation [7] :

$$\tilde{\varphi}_{N'}(s) = \left[I - \tilde{q}_{N'}(s)\right]^{-1} \cdot \tilde{b}(s), \tag{8}$$

where:

$$\tilde{q}_{N'}(s) = \left\lfloor \tilde{q}_{ij}(s) \right\rfloor_{wxw}, \qquad i, j \in N` = \{i_1, i_2, \ldots, i_w\}, \tag{9}$$

$$\tilde{\varphi}_{N'}(s) = \begin{bmatrix} \tilde{\varphi}_{i_1 N}(s) \\ \tilde{\varphi}_{i_2 N}(s) \\ \vdots \\ \tilde{\varphi}_{i_w N}(s) \end{bmatrix}, \quad \tilde{b}(s) = \begin{bmatrix} \sum_{j \in N} \tilde{q}_{i_1 j}(s) \\ \sum_{j \in N} \tilde{q}_{i_2 j}(s) \\ \vdots \\ \sum_{j \in N} \tilde{q}_{i_w j}(s) \end{bmatrix}. \tag{10}$$

The cumulative distribution functions $\Phi_{iN}(t)$ of the random variables Ξ_{iN} make it possible to determine the searched for probability. In the considered case the subset of states N is determined as follows:

$$N = \{x_{k'}\}. \tag{11}$$

In the situation the relations (8) ÷ (10) take the following form:

$$\tilde{q}_{N'}(s) = \begin{bmatrix} 0 & \tilde{q}_{01}(s) & 0 & 0 & \ldots & 0 \\ 0 & 0 & \tilde{q}_{12}(s) & 0 & \ldots & 0 \\ 0 & 0 & 0 & \tilde{q}_{23}(s) & \ldots & 0 \\ 0 & 0 & 0 & 0 & \tilde{q}_{ii+1}(s) & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, i, j \in N' = \{0, 1, 2, \ldots, k'-1\}, \tag{12}$$

$$\tilde{\varphi}_{N'}(s) = \begin{bmatrix} \tilde{\varphi}_{0k'}(s) \\ \tilde{\varphi}_{1k'}(s) \\ \vdots \\ \tilde{\varphi}_{k'-1k'}(s) \end{bmatrix}, \ \tilde{b}(s) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \tilde{q}_{k'-1k'}(s) \end{bmatrix}. \tag{13}$$

After performing the converse Laplace transform [2] the cumulative distribution function of the random variable $\Xi_{0k'}$ is expressed as follows:

$$\Phi_{0k'}(t) = \int_0^t [\varphi_{0k'}(t)]dt. \tag{14}$$

In the above presented cases one of the conditions of practical usefulness of the model elaborated in the form of semi-Markov processes is its functional matrix Q(t) of a possibly uncomplicated mathematical form - apart from a moderate complexity of the model in the sense of the possible lowest number of classes of distinguished states.

The condition is important in the case of calculating the instantaneous distribution of states of the process $p_k(t)$. The distribution can be calculated [6] if only initial distribution of the process and the function $p_{ij}(t)$ is known. Calculation of the probability $p_{ij}(t)$ consists in solving the set of Volterra equations of the second kind, in which the functions $Q_{ij}(t)$ being elementary elements of the process functional matrix Q(t), are known. In the case when number of states of the process is low and its functional matrix is uncomplicated, then the set can be solved by using Laplace transform operator calculus. However when number of states of the process is high or if its functional matrix (core of the process) is very complex, then only an approximate solution of the set of equations can be achieved.

4. Application of Markov processes to assessment of operation

In the case of a rather complex state-transition graph another possibility of estimating the searched for values of occurrence probability of such number of the events F which would cause the fuel charge $g_p^{i\%}$ to increase up to the value $G_{p\ max}$, appears by elaborating a model of the process $\{X(t): t \geq 0\}$ in the form of the Markov process $\{X'(t): t \geq 0\}$.

Such models constitute a simplification of semi-Markov models. This state of matters constrains application of the processes, however in the case of the above described difficulties in elaborating a semi-Markov process, or when considered random variables have unknown distributions, results obtained this way can be accepted as the first approximation.

The use of exponential distributions makes it possible to achieve very simple relations constituting distribution of the process in question.

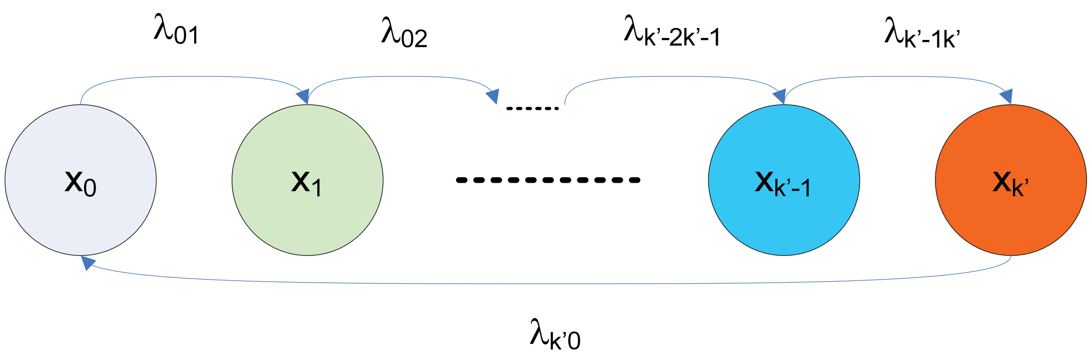

Fig. 2. Transition graph of the process $\{X'(t) : t \geq 0\}$

The state-transition graph of the considered process is presented in Fig. 2.
Its initial distribution, if only the assumptions of the process $\{X(t) : t \geq 0\}$ are satisfied, is described by the relation (15).

The form of the state-transition graph modified in relation to that presented in Fig. 2, results of course from the fact that determination of probability of remaining the engine in particular states necessitates form Kolmogorov equations for the assumed model of changes of technical states. To this end, instead of the transition probabilities p_{ij}, the transition intensities λ_{ij} (i, j = 1, 2, 3, ..., I; i ≠ j) of the following interpretation, are used [3]:

$$\lambda_{ij}(t) = \lim_{\tau \to 0} \frac{P\left(X'(t+\tau) = x'_j / X'(t) = x'_i\right)}{\tau}. \tag{15}$$

In the considered case the above mentioned set of equations in the general form [1]:

$$\frac{dP_i(t)}{dt} = -\left(\sum_j \lambda_{ij}\right) \cdot P_i(t) + \sum_j \left(\lambda_{ji} \cdot P_j(t)\right) \quad i \neq j, \tag{16}$$

is as follows :

$$\left.\begin{aligned}
\frac{dP_0(t)}{dt} &= -\lambda_{01} \cdot P_0(t) + \lambda_{k'0} \cdot P_{k'}(t),\\
\frac{dP_1(t)}{dt} &= -\lambda_{12} \cdot P_1(t) + \lambda_{01} \cdot P_0(t),\\
\frac{dP_2(t)}{dt} &= -\lambda_{23} \cdot P_2(t) + \lambda_{12} \cdot P_1(t),\\
&\dots\\
\frac{dP_{k'}(t)}{dt} &= -\lambda_{k'0} \cdot P_{k'}(t) + \lambda_{k'-1k'} \cdot P_{k'-1}(t),\\
P_0(t) + P_1(t) + \ldots + P_{k'}(t) &= 1.
\end{aligned}\right\} \tag{17}$$

The solving of the presented set of equations in which the initial distribution has been taken into account, makes it possible to find the distribution $P_{k'}(t)$ which can serve as an estimation of unknown occurrence probability of such number of the events F which would cause the fuel charge $g_p^{i\%}$ to increase up to the value $G_{p\,max}$, hence ship motion limitations to appear.

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