THE BENDING AND CONTACT STRESS ANALYSIS OF SPUR GEARS

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Abstract

With the developing of science and technology the use of gears has become more common in all the upcoming industries. Accessory units used in aircraft provide the power for hydraulic, pneumatic and electrical systems in addition to providing various pumps and control systems for efficient engine operation.
The design of an effective and reliable gearing system is governed, amongst other factors, by its ability to withstand high root bending stress (RBS) and surface contact stress(SCS). A number of authors have utilized the finite element method to predict RBS and SCS in gears.

This paper presents the utilization of a three-dimensional (3D), finite element method (FEM) to conduct RBS and SCS calculations of a pair of spur gears. Firstly a pair of parallel spur gears without errors and tooth modifications is defined in a CAD system. Then, MES analysis is conducted. Using FEM to calculate the RBS the load is applied as a force to the tip of the gear, it is modeled as a linear force uniformly distributed along the face width and perpendicular to the tooth surface. The SCS is considered as nonlinear contact analysis, where the contact of the pair of gears is assumed on teeth flanks. The tooth RBS and SCS of the same pair of gears are calculated and presented. Gears with wrought alloy steel are considered. The results are compared with the ISO standards, Levis formula and Hertz equation. The analysis was made for applied moment equal: 600[Nm], 1200[Nm] and 1600[Nm]. It was found that the calculated results are comparable. Finally, an example of calculating a pair of spur gears is given in the paper.

Keywords*: spur gear, FEM, contact stress, bending stress*

1. Introduction

Gears have been manufactured for a number of years with extensive ongoing research related to their efficiency, operational quality, and durability. They are relatively complex and there are a number of design parameters involved in gear design. The design of gears requires an iterative approach to optimize design parameters. Due to the complex combinations of these parameters, conventional design office practice tends to become complicated and time consuming. It involves selection of appropriate information from a large amount of engineering standards data available in engineering catalogues and design handbooks [6]. To overcome the difficulties and limitations associated with the use of this semi-empirical formula, currently used by designers, a number of authors have utilized the finite element method to predict root bending stress and surface contact stress in gears [1, 2, 9].

In this paper a comparison for two gears was undertaken. A drive gear is a full body model; a driven gear is thin rimmed (see Figure 1). The true bending stress at the tooth root has different trends and values, and the designer must be aware of this difference, especially for light gears with narrow ribs and rims [2]. Contact problem is non-linear issue. Finding the best choice for contact elements, element option, solver, and solution option was the purpose of the article.

2. Geometry of considered gears

The geometrical parameters of the full-body and the thin-rimmed gears analysed in this paper are presented in Table 1.

Tab. 1. Parameters of gears

	Drive gear (full-body)	Driven gear (thin rimmed)
Number of teeth	23	46
Tip diameter [mm]	150	288
Pitch diameter [mm]	138	276
Root /Rim diameter [mm]	122.8	260.8 / 237
Face width [mm]	50.8	50.8
Normal Pressure angle [^{0}]	20	20

3. The Lewis Formula

The classic method of estimating the bending stresses in a gear tooth is the Lewis equation. It models a gear tooth taking the full load at its tip as a simple cantilever beam.

$$\sigma_F = \frac{F_t P}{bY}, \tag{1}$$

where: $Y = \frac{2xP}{3}$, $x = \frac{t^2}{4l}$.

In above equation: σ_F is the maximum bending stress, b is the face width, P diametric pitch, Y Lewis form factor, F_t is the tangential load (lbs). The form factor, Y is a function of the number of teeth, pressure angle, and involutes depth of the gear. Assuming that maximum bending stress by similar triangles we get x. It accounts for the geometry of the tooth, but does not include stress concentration [8]. When considering the Lewis stress model of drive gear (full-body) and driven gear (thin rimmed) results for applied moments are presented in Table 2.

Tab. 2. Results of Lewis equation

Moments applied:	Drive gear [MPa]	Driven gear [MPa]
600[Nm]	100.74	72.26
1200 [Nm]	201.46	144.53
1600 [Nm]	268.63	192.70

4. ISO standards (root bending stress calculations)

The ISO published standards ISO 6336/1 and 6336/3. It can be used to calculate the RBS of a pair of spur gears and helical gears. Equation 2 is the formula used by ISO 6336/3 for the RBS calculation. The meanings of the symbols used in Equation 2 are described in Table 3. Since it is a very difficult thing to determine the values of $K_A, K_{V,} K_{F\beta,} K_{F\alpha}$ exactly, the gears used here are regarded as ideal gears. The values of $K_A, K_{V,} K_{F\beta,} K_{F\alpha}$ are equal 1. After the values of all the factors in Table 3 are given based on ISO standards, the RBS σ_F of the wheel can be calculated by substituting all the factors into Equation 2.

$$\sigma_F = \frac{F_t}{bm_n} Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} = \frac{F_t}{bm_n} Y_{FS} Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} . \tag{2}$$

Tab. 3. Meanings of the symbols used in Equation 2. For moments: 600 [Nm], 1200[Nm], 1600 [Nm]

		Drive gear (full-body)	Driven gear (thin rimmed)
Nominal tangential load (N) for moments: 600[Nm] 1200 [Nm] 1600 [Nm]	F_t	8 695.65 17 391.30 23 188.41	8 695.65 17 391.30 23 188.41
Face width (mm)	b	50.8	50.8
Normal module (mm)	m_n	6	6
Application factor	K_A	1	1
Dynamic factor	K_V	1	1
Face load factor	$K_{F\beta}$	1	1
Transverse load factor	$K_{F\alpha}$	1	1
Form factor	Y_{Fa}	1.99	1.41
Stress correction factor	Y_{Sa}	2.04	2.12
Contact ratio factor	Y_ε	0.89	0.87
Helix factor	Y_β	1	1
Tip factor, equal to $(Y_{Fa} \cdot Y_{Sa})$	Y_{FS}	4.06	3.00
Tooth-root stress (MPa) for moment: 600[Nm] 1200 [Nm] 1600 [Nm]	σ_F	103.00 206.01 274.68	74.48 148.97 198.63

5. The FEM analysis conducted to estimate root bending stress

A hexahedron solid element, which has eight nodes at the corners, is used. It is modeled as a linear force uniformly distributed along the face width and perpendicular to the tooth surface applied to the tip. The analysis was carried out for three teeth of the gears considered, which reduces the time needed for computation. The mesh was applied in a Catia v5 system. The material properties boundary conditions and the applied moments were defined and calculated in Ansys software. The pair of spur gears were made from alloy steel (Young module E=204782 N/mm^2, Poisson ratio v=0.26).

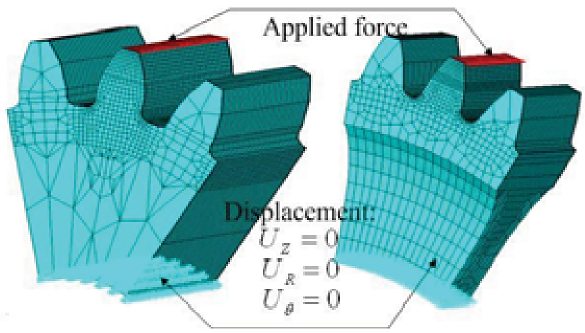


Figure1. Boundary condition applied for drive and driven gear.

For a drive and driven gear the 1st Principle average stress for the RBS achieved from the FEM model are presented in Figure 2.

The layout of stress along the face width at the tooth root is presented in Figure 3. In the layout of a full body gear (drive gear) two maxima points located close to edges can be noticed. In the layout of a thin rimmed gear (driven gear) stress has one maximum located almost in the centre of the face width.

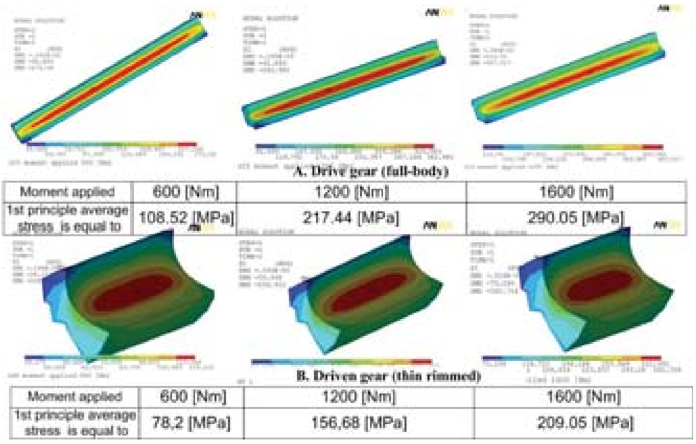


Fig. 2. 1st Principle stress for the drive and driven gear

Fig. 3. Comparison of 1st principle stress layout between full body and thin rimmed gear

6. The Hertz equation

Hertz contact between two cylinders is used for evaluating contact conditions at the major point of contact for meshing gears.

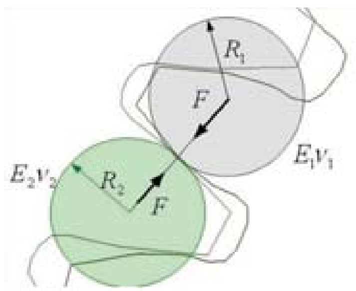


Fig. 4. Hertz theory for spur gears

According to Hertz theory the contact stress can be determined by:

$$\sigma_H = \frac{4F}{L\pi B}\ , \tag{3}$$

where: $B = \sqrt{\frac{16F(K_1+K_2)R_1R_2}{L(R_1+R_2)}}$, $K_1 = \frac{1-\nu_1^2}{\pi E_1}$, $K_2 = \frac{1-\nu_2^2}{\pi E_2}$,

where F is the normal contact force, ν denotes a Poisson's ratio, E is the modulus of elasticity, L is the face width, $R_1 R_2$ are radius of pinion and gear. A Hertz equation is used to calculate the SCS. In the considered model the Hertz stress for applied moments are presented in Table 4.

Tab. 4. Results of Hertz equation

Moments applied:	SCS calculated by Hertz equation [MPa]
600[Nm]	634.64
1200 [Nm]	898.52
1600 [Nm]	1036

7. ISO standards (the surface contact stress calculations)

The ISO standards 6336/1, 6336/2 can be used to calculate the SCS of a pair of spur gears and helical gears. Equation 4 is the formula used by ISO 63363/2 for the SCS calculation for a pair of spur gears and helical gears having contact ratios in the range 1< ε < 2. The meanings of the symbols used in Equation 4 are described in Table 5. The gears used here are regarded as ideal gears. The values of $K_A, K_{V,}$ $K_{H\beta,} K_{H\alpha}$ are equal 1. The SCS σ_H of the gear may be calculated by substituting all factors into Equation 4. All the factors are given in Table 5 [3,4,5].

$$\sigma_H = Z_D Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b}\frac{U+1}{U}}\sqrt{K_A K_V K_{H\beta} K_{H\alpha}}\ . \tag{4}$$

Tab. 5. Explanations of symbols used in Equation 4

Moment applied		600 Nm	1200 Nm	1600Nm
Nominal tangential load (N)	F_t	8695.65	17391.3	23188.41
Single pair tooth contact factor of the wheel	Z_D	1	1	1
Zone factor	Z_H	2.5	2.5	2.5
Elasticity factor	Z_E	186.96	186.96	186.96
Contact ratio factor	Z_ε	0.9704	0.9704	0.9704
Helix angle factor	Z_β	1	1	1
Reference diameter (mm)	d_1	138	138	138
Face width (mm)	b	50.8	50.8	50.8
Gear ratio (Z2/Z1)	U	2	2	2
Application factor	K_A	1	1	1
Dynamic factor	K_V	1	1	1
Face load factor	$K_{H\beta}$	1	1	1
Transverse load factor	$K_{H\alpha}$	1	1	1
Contact stress (MPa)	σ_H	618.71	874.99	1010.35

8. The FEM analysis conducted to estimate contact stress

For making the FEM model to calculate the SCS a hexahedron solid element, which has eight nodes at the corners, was used. The applied moment is equal to 600 [Nm], 1200 [Nm] and 1600 [Nm] [8]. The analysis was conducted for three teeth of the gears considered. The drive gear has axial and radial movement equal to 0, and rotational movement is allowed. The driven gear is clamped: $U_Z = U_R = U_\theta = 0$. Moment is applied to the node placed in axis of the drive gear. The node belongs to element Mass21, which is linked to the drive gear's edges by a stiff beam (element MPC184). Contact and target surface elements were applied on appropriate teeth flanges. FEM model is presented in Figure 5. The analysis does not include friction [7].

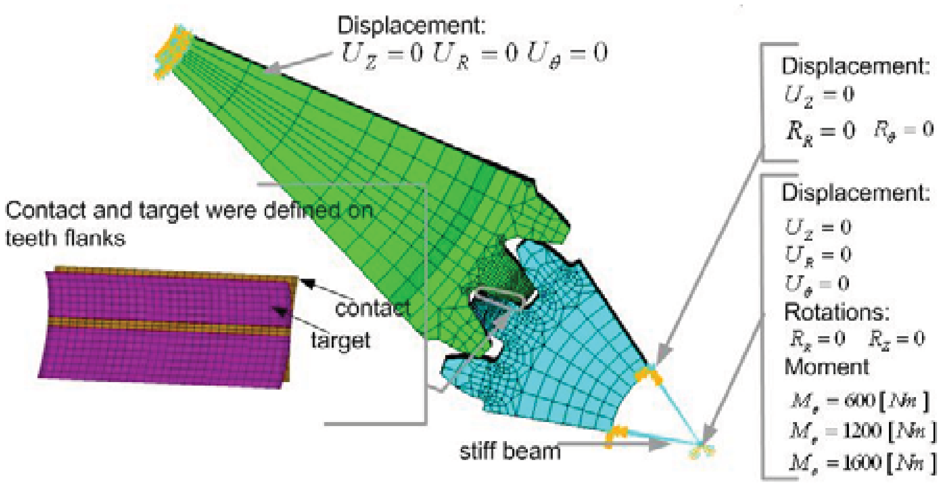
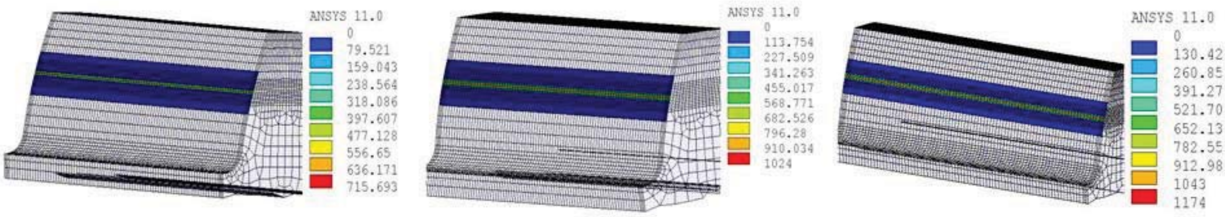


Fig. 5. Boundary condition applied

Contact and target were introduced on the flanks by surface-to-surface elements. Contact stress analysis was conducted for three loads (600 [Nm], 1200 [Nm], 1600 [Nm]). The results of contact stress achieved by the FEM are presented in Figure 6. Line contact is located on pitch diameter.



Moment applied	600 [Nm]	1200 [Nm]	1600 [Nm]
SCS calculated by FEM	715.69 [MPa]	1024 [MPa]	1174 [MPa]

Fig. 6. Contact total stress

Layout for contact stress along the face width for different load is presented in Figure 7.

9. Results and discussion

Comparing the results of the RBS it is noticeable that the Lewis equation can be used for a quick estimation of the stress on root diameter. The results are approximately 3% below the results achieved by the ISO standard. What is more, analyzing by the FEM value for the 1st principle average stress seems to be close to the ISO standard and the Lewis equation. The analysis was conducted for simple bending. The FEM results are approximately 5% above the results achieved in the ISO standard.

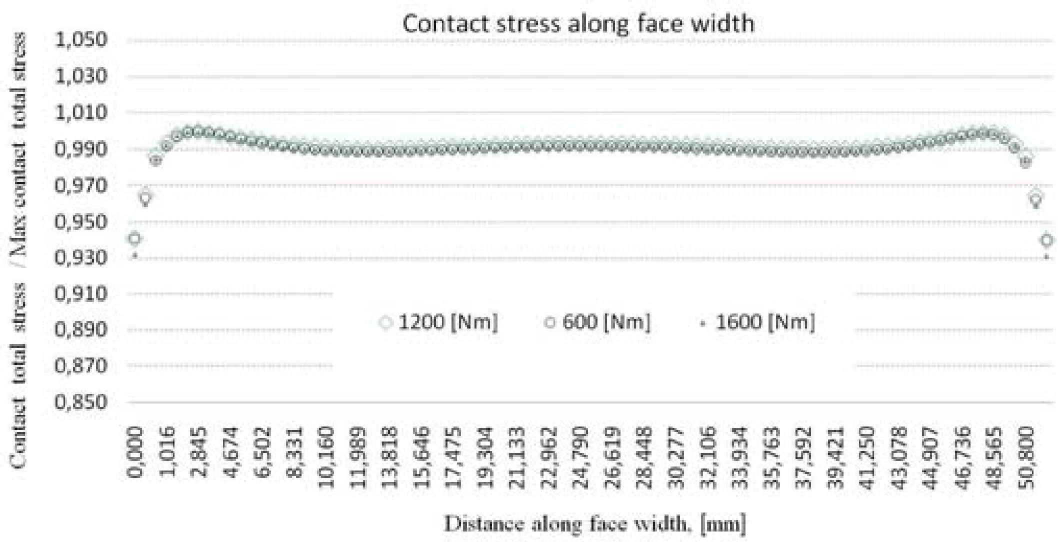


Fig. 7. Contact stress along face width

Table 6. Results summary of RBS

	Drive gear					
Moment applied	600[Nm]	Deviation%	1200[Nm]	Deviation%	1600[Nm]	Deviation%
ISO [MPa]	103.00	0	206.01	0	274.68	0
Lewis formula [MPa]	100.74	-2.2	201.47	-2.2	268.57	-2.2
1st Principle stress from FEM [MPa]	108.52	5.4	217.44	5.5	290.05	5.6
ISO [MPa]	74.48	0	148.97	0	198.63	0
Lewis formula [MPa]	72.26	-3.0	144.53	-3.0	193.7	-2.5
1st Principle stress [MPa]	78.20	5.0	156.68	5.2	209.0	5.2

A different layout of stress for the RBS for thin and full body gears can be observed. A thin rimmed gear has one maximum of the RBS placed almost in the middle of its face width. For a full body gear two max stresses of RGB can be observed close to the edges (see Figure 3).

This article presents a way of conducting the SCS analysis for spur teeth by using the FEM method. The results achieved of the SCS are correct according to the ISO standard and Hertz equation. The summary of results is presented in Table 7. If an assumption that the results calculated from the Herts equation are correct it can be noticed that results achieved by FEM contain approximately 13% deviation.

Table 7. Results for the SCS

Moment applied	600 [Nm]	Deviation %	1200 [Nm]	Deviation %	1600 [Nm]	Deviatio n %
Hertz equation [MPa]	634.64	0%	898.52	0%	1036.37	0%
ISO standard[MPa]	618.71	3%	874.99	3%	1010.35	3%
FEM SCS [MPa]	715.69	13%	1024	14%	1174	13%

10. Conclusions

The results achieved by FEM are acceptable, when compared to results with a well known method of SCS calculations (Herts, and ISO) and RBS (Lewis, and ISO). In recent years, lightweight and compact design size for gears has been required, so strictly that gear designers must predict the real SCS and RBS of a pair of gears. FEM used in linear analysis to calculate RBS show slight deviation. Contact problem used for SCS is highly nonlinear, achieved results have bigger deviation.

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References

[1] Cănănău, S., *3D contact stress analysis for spur gears*, National Tribology Conference, pp. 1221-4590, 2003.

[2] Conrado, E., Davoli, P., *The ”true” bending stress is spur gears*, Gear Technology, 8, 2007.

[3] International Standard ISO 6336/1, Calculation of load capacity of spur and helical gears – Part 1: basic principle, introduction and general influence factors, pp. 1–100, 1993.

[4] International Standard ISO 6336/2, Calculation of load capacity of spur and helical gears – Part 2: calculation of surface durability (pitting), pp. 1–28, 1993.

[5] International Standard ISO 6336/3, Calculation of load capacity of spur and helical gears – Part 3: Calculation of tooth strength, pp. 1–72, 1993.

[6] Kawalec, A., Wiktor, J., *Comparative Analysis of Tooth-Root Strength Using ISO and AGMA Standards in Spur and Helical Gears With FEM-based Verification*, Journal of Mechanical Design, Vol. 128, 2006.

[7] Kiełbasa, J., Stańco, K., *Konstrukcja walcowej przekładni wichrowej o zazębieniu wewnętrznym*, materiały XXIII Sympozjum Podstaw Konstrukcji Maszyn Rzeszów – Przemyśl, 2007.

[8] Sfakiotakis, V. G., Vaitsis, J., P., Anifantis, N., K., *Numerical simulation of conjugate spur gear action*, Computers and Structures, 79, pp. 1153-1160, 2001.

[9] Shuting, L., *Finite element analyses for contact strength and bending strength of a pair of spur gears with machining errors, assembly errors and tooth modifications*, Mechanism and Machine Theory, pp. 88-114, 42, 2007.