APPLICATION OF THE CORRELATION FUNCTION AND FOURIER TRANSFORMATION TO EVALUATION OF TECHNICAL CONDITION DEMONSTRATED BY BLADES OF A ROTOR MACHINE DURING THE OPERATION PROCESS

Paweł Lindstedt, Edward Rokicki, Henryk Borowczyk, Paweł Majewski

Air Force Institute of Technology (AFIT)
Ksiecia Boleslawa Street 6, 01-494 Warsaw, Poland
tel.: +48 22 685-20-01, fax: +48 22 836-44-71
e-mail: pawel.lindstedt@itwl.pl, edward.rokicki@itwl.pl, henryk.borowczyk@itwl.pl
pawel.majewski@itwl.pl

Abstract

The paper presents fundamentals of a new method for monitoring of technical condition demonstrated by blades of rotor machines during operation thereof.

The method involves the diagnostic model expressed as a quotient of the amplitude amplification for the diagnostic signal y(t) that results from the blade impact and the signal x(t) produced by the blade surrounding when the blade tip approaches the sensor, divided by the amplitude amplification of these signals when the blade tip has already passed the sensor position and moves away.

The adopted diagnostic model takes account for current surrounding of the blade x(t) with no need of measurements of its parameters [12, 14]. Thus, the model is sensitive to alterations of the technical condition demonstrated by the blade but very little vulnerable to variations in the blade surroundings. Consequently, the proposed method may be able to play an important role for diagnostics of rotor blades during operation thereof.

The presented method applicable to monitoring of changes in technical condition of a blade is an innovative approach to carry out diagnostic of the blade in its natural environment with no need to measure signals from that environment.

Keywords: diagnostics of blades, correlation function, power spectral density (PSD) function, amplitude amplification

1. Introduction

Monitoring (on-line assessment) of technical condition demonstrated by crucial components of rotor machinery, such as bearings and blades, is an essential problem associated with operation (use) of such equipment. Practical experience shows that ripping off only a single blade (of a several dozen or even several hundred ones) nearly always leads to a very serious breakdown (and consequently, very expensive) breakdown of the entire rotary machine (an axial compressor, a turbine). It is why methods for on-line monitoring of technical condition demonstrated by rotor blades during regular operation of the equipment is the issue of constantly growing interest.

Currently, a number of various methods for monitoring of technical condition demonstrated by rotor blades are in use. These methods is based on so called 'contactless' measurements of values for temporary displacements of a blade tip at moments when it passes the zone just below the measuring sensor. A huge number of 'contactless' measuring methods have already been developed (with dedicated computer software) that can be used to record and process signals acquired from working blades during operation of a rotor machine. These include commonly known and used measuring systems offered by such companies as Hood, Agilis, Prat&Whithey (USA), Rolls Royce (UK), ABB Turbocharges (Swiss), MTU (Germany), AFIT (Poland) and also other suppliers from Russia, China and India [1, 3-11, 16, 18-20].

The already existing methods for monitoring of rotor blade technical condition base exclusively on processing of measurement signals acquired from working blades without sufficient

account for varying environment of measurements. This probably results from the fact that measurements of fast changing environment of blades (with such variables as temperature, pressure, centrifugal forces, bending and twisting moments, clamping forces that fix the blade in its holder) are extremely difficult or even unfeasible. Therefore one can state that all these methods fail to fully adhere the major rule of the diagnostic technology that imposes the need to examine and analyze technical condition of the investigated facilities with due consideration to the effect of their environment and, consequently, all these methods are not enough accurate. It is why the need to develop a new method to monitor technical condition of a blade during its operation with account to its environment but with no need to measure signals from its environment as they are frequently unavailable or immeasurable appeared.

2. Operation of a blade in a variable environment

The blade that is operated in a variable environment is shown in Fig. 1.
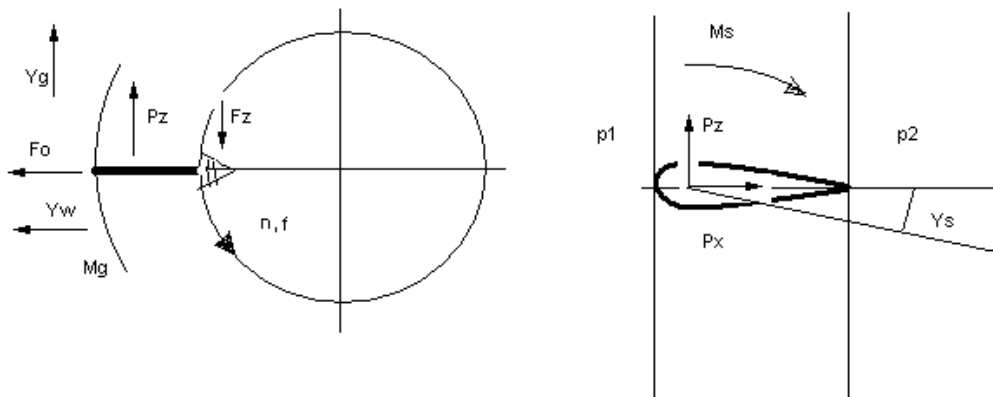

Fig. 1. *The blade of a rotor machine in a variable environment: F_o – centrifugal force; F_z – camping force of the lock; n – rotation speed; P_z – aerodynamic lift of the blade; P_x- resisting force; M_s- twisting moment; M_g – bending moment; p_1- gas pressure at the inlet of the rotor rim; p_2 - gas pressure at the outlet of the rotor rim; Y_g – blade deflection; Y_s – torsion angle of the blade; Y_w- longitudinal displacement of the blade; f- signal of various vibration forms (bending, twisting, longitudinal)*

Fig. 1 exhibits that the blade is a rather simple technical item in terms of its design but its operation represents a sophisticated system placed in a multidimensional variable environment. Current status of the blade operation is defined by the measurable signal *y(t)* of the blade tip displacement that is the resulting function of the signals (Y_g, Y_w, Y_s *f*) whilst the blade environment is determined by the multidimensional signal *x(t)* that is, in turn, the result of combined effect exerted by the signals *(n, F_0, P_z, P_x, p_1, p_2, F_z, f)*. Technical condition of the blade *A(Θ)* results from mutual relationship between the signals *y(t)* and *x(t)* at the current moment $Θ_1$ of the diagnostic time interval and the moment $Θ_0$ at the beginning of this time interval. Hence, the following equation is true [13,15]:

$$A(Q) = f(y(t)_{Q0}, x(t)_{Q0}, y(t)_{Q1}, x(t)_{Q1}, Q),$$ (1)

where:

$A(Q)$ - matrix of parameters that define technical condition of the blade,
t - time in terms of the Newton's definition (for diagnostic examinations),
Θ - time in terms of the Bergson's definition (for diagnostic inference).

Parameters of the technical condition $A(Q)$ as well as signals $y(t)$ and $x(t)$ occur in the status equation:

$$\frac{dy(t)_\Theta}{dt} = A(\Theta)y(t)_\Theta + B(\Theta)x(t)_\Theta, \tag{2}$$

where:

B(Q) - coefficients that define how intensely the environment affects the blade.

The coefficients that reflect technical condition of the blade and effect of the environment occur also in the transmittance equation:

$$G = \frac{Y(s)}{X(s)} = \frac{b_n s^m + \ldots\ldots b_2 s^2 + b_1 s + b_0}{a_n s^n + \ldots\ldots a_2 s^2 + a_1 s + a_0} \tag{3}$$

as well as in the function that suits for calculation of the square for the amplitude amplification W^2 of the $y(t)$ signal related to the environment signal $x(t)$:

$$W^2 = \left| G(jw) \right|^2 = \left| \frac{b_n(jw)^m + \ldots\ldots b_2(jw)^2 + b_1(jw) + b_0}{a_n(jw)^n + \ldots\ldots a_2(jw)^2 + a_1(jw) + a_0} \right|^2 = \frac{S_{yy}}{S_{xx}}, \tag{4}$$

where:

$G(j\omega)$ - spectral transmittance,

$(j\omega)$ - imaginary part of the complex variable S,

S_{yy}, S_{xx} - functions for spectral density of signals $y(t)$ and $x(t)$.

The mathematic expressions (2), (3), (4) may serve as diagnostic models for the examined object as they express mutual relationships between the signals $y(t)$ for the object operation and the environment signal $x(t)$ on one side and technical parameters of the examined facility $A(Q)$ on the other side.

The model (4) becomes the matter of particular importance as it enables to get rid (by means of special measures that are to be undertaken during examination and diagnostic inference) of environmental signals $x(t)$ expressed by the power signals S_{xx} and eliminate them from the diagnostic model.

It is initially assumed (Fig. 2) that for subsequent time intervals T_{01} and T_{12} with very short distance between them and assigned for recording of signals $y(t)$ and $x(t)$ the squares of amplitude amplifications of the signal $y(t)$ related to the signal $x(t)$ can be calculated:

$$W^2_{T01} = \frac{S^{T01}_{yy}}{S^{T01}_{xx}}, \tag{5}$$

$$W^2_{T12} = \frac{S^{T12}_{yy}}{S^{T12}_{xx}}, \tag{6}$$

where:

$S^{T01}_{xx}, S^{T01}_{yy}$ - power spectral densities (PSD) within the time interval T_{01},

$S^{T12}_{xx}, S^{T12}_{yy}$ - power spectral densities (PSD) within the time interval T_{12}.

The function $x(t)$ is immeasurable, hence neither S^{T01}_{xx} or S^{T12}_{xx} can be calculated. However, one can assume that the power spectral density (PSD) for the environment signal $x(t)$ within the time intervals T_{01} and T_{02} when the working parameters are recorded is exactly the same, i.e.

$$S^{T01}_{xx} = S^{T12}_{xx}. \tag{7}$$

This assumption is justified as the interval of measurements T_{01} and T_{02} are mutually biased by not more then several milliseconds.

Subsequently, based on the assumption (7) a new, virtual parameter $W^2_{T12,T01}$ is introduced in the form of a quotient for amplitude amplifications W^2_{T12} and W^2_{T01} [12, 14].

$$W^2_{T12,T01} = \frac{W^2_{T12}}{W^2_{T01}} = \frac{\dfrac{S^{T12}_{yy}}{S^{T12}_{xx}}}{\dfrac{S^{T01}_{yy}}{S^{T01}_{xx}}} = \frac{S^{T12}_{yy}}{S^{T01}_{yy}} \frac{S^{T01}_{xx}}{S^{T12}_{xx}} = \frac{S^{T12}_{yy}}{S^{T01}_{yy}} . \tag{8}$$

Therefore a new diagnostic model is obtained:

$$W^2_{T12T01} = \frac{W^2_{T12}}{W^2_{T01}} = \frac{S^{T12}_{yy}}{S^{T01}_{yy}} . \tag{9}$$

The function $W^2_{T12,T01}$ can be calculated exclusively on the basis of the measurable signal *y(t)*. Its characteristic feature is the fact that its takes account for environmental parameters *x(t)* with no need of measurement thereof.

3. The method for monitoring of technical parameters attributable to the blade based on recording of function $W^2_{T12,T01}$ parameters

The method for monitoring the changes in technical condition attributable to the blade based on recording of function $W^2_{T12,T01}$ parameters needs to carry out relevant diagnostic investigations.
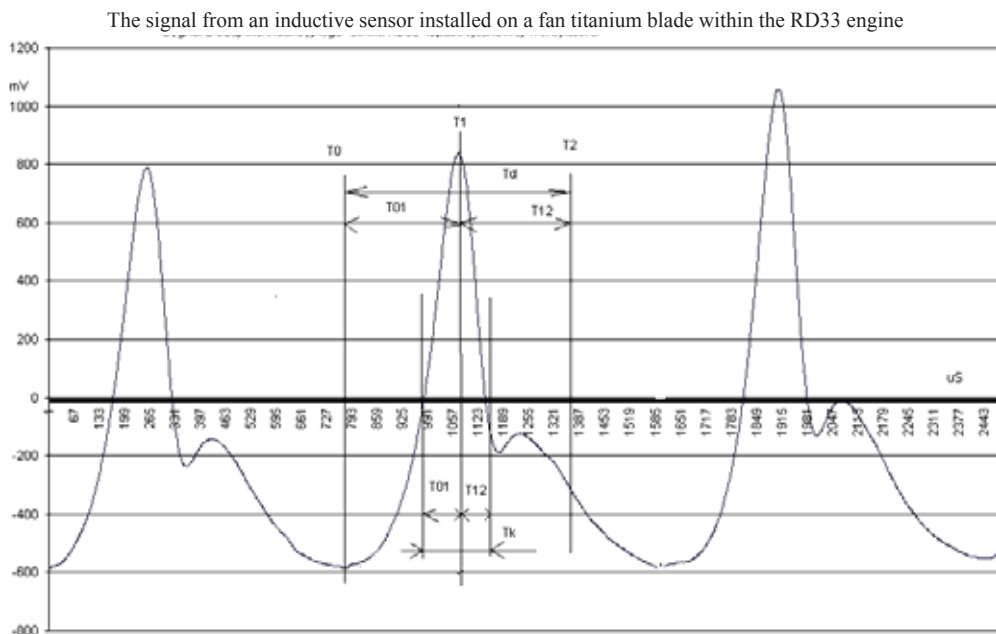

Fig . 2. The signal from an inductive sensor: T_d , T_k - respectively – long and short time periods of data recording when the blade tip remains within the vulnerability zone of the sensor, T_0 , T_1, T_2 - characteristic sub-periods of data recording when the blade tip remains exactly under the sensor, T_{01}, T_{12} – sub-periods of data recording for the blade tip, respectively for T_d and T_k, mV- signals for the blade tip displacements, μs – time intervals for the blade displacements

Another characteristic feature is the fact that the established time interval T (with the value of T_d or T_k) when the blade $y(t)$ moved below the sensor is split into two respective sub-intervals: T_{01} when the blade approaches the sensor and T_{12} when it has already passed the sensor and moves away (the moment T_1 corresponds to the position when the blade tip is exactly below the sensor – Fig. 2). Adoption of the long time T_d or the short time T_k of data recording results from the need to fulfil the (7) equation.

Then, for the displacement $y(t)$ during the assumed periods of data recording T_{01} and T_{12} it is possible to find out estimations for the autocorrelation functions R_{yy}^{*T01} and R_{yy}^{*T12} and to match them to appropriate analytic expressions [2, 12-14, 17]:

- for the time interval T_{01} within the recording time T_d or T_k,

$$R_{yy}^{T01} = \sum_{i=1}^{n} \alpha_{iT01} e^{-\beta_{iT01}\tau} \cos(\gamma_{iT01}\tau),$$ (10)

where:

$\alpha_{iT01}, \beta_{iT01}, \gamma_{iT01}$ - parameters for the analytic form of the correlation function,

τ - bias time (the space variable for the correlation function),

n - number of components for the correlation function.

- for the time interval T_{12} within the recording time T_d or T_k,

$$R_{yy}^{T12} = \sum_{j=1}^{m} \alpha_{jT12} e^{-\beta_{jT12}\tau} \cos(\gamma_{jT12}\tau),$$ (11)

where:

$\alpha_{iT01}, \beta_{iT01}, \gamma_{iT01}$ – parameters for the analytic form of the correlation function,

τ – bias time (different from the dynamic time variable t),

m – number of components for the correlation function.

The matching degree between the analytic form of the correlation function and their estimations should be higher than 0.9 and n should be equal to or higher than m. It can be achieved by assuming the necessary data recording time T_d or T_k as well as appropriate measuring window (of rectangular, Hamming, Hanning or other types) [13].

The switchover from physical signals $x(t)$ and $y(t)$ to correlation functions $R_{xx}(\tau)$ and $R_{yy}(\tau)$ defined for the bias time interval τ is justified by the fact that the function C in the space t is associated with the function C^2 in the space of the time of τ and, respectively, the function $C\sin(\omega t)$ is associated with the function $C^2\cos(\omega t)$.

Analytic forms for the autocorrelation functions R_{yy}^{T01} and R_{yy}^{T12} can be used to determine corresponding functions of power spectral densities (PSD) $S_{yy}^{T01}(\omega)$ and $S_{yy}^{T12}(\omega)$ by application of the Fourier transformation:

$$S_{yy}^{T01}(\omega) = F(R_{yy}^{T01}(\tau)),$$ (12)

$$S_{yy}^{T12}(\omega) = F(R_{yy}^{T12}(\tau)),$$ (13)

where: $F(.)$ - operation of the continuous Fourier transformation

Finally, the new virtual signal can be determined:

$$W_{T12T01}^2 = \frac{S_{yy}^{T12}}{S_{yy}^{T01}} = \frac{L_0 + L_1 s + L_2 s^2 + ...}{M_0 + M_1 s + M_2 s^2 + ...},$$ (14)

with parameters that bring information about technical condition of the examined blade.

The monitoring process for technical condition of a blade in a rotor machine looks like in the following way:

- for any moment of the machine operation Θ_0 (start of the monitoring process) one has to establish parameters of the $W^2_{T01,T12}$ function: $L_{00}, L_{10}, L_{20}, \ldots$ as well as $M_{00}, M_{10}, M_{20}, \ldots$,

- for a subsequent moment of the machine operation Θ_1 (when the blade is to be monitored for the next time) the new parameters of the function $W^2_{T01,T12}$ must be determined, namely $L_{01}, L_{11}, L_{21}, \ldots$ and $M_{01}, M_{11}, M_{21}, \ldots$,

- alteration of the technical condition is calculated by relative alterations of individual parameters:

$$\Delta \overline{L}_i = \frac{L_{i1} - L_{i0}}{L_{i0}} \;\; ; \;\; i = 1, \ldots, n \; ,$$

$$\Delta \overline{M}_i = \frac{M_{i1} - M_{i0}}{M_{i0}} \;\; ; \;\; i = 1, \ldots, m ,$$

- the diagnostic information that can be used for the monitoring process of technical condition demonstrated by the blade during operation of the rotor machine is assumed as alteration of the $\overline{L}_i$ and $\overline{M}_i$ parameters.

The assumption seems to be justified that a series of $\overline{L}_i$ and $\overline{M}_i$ parameters along with huge number of their mutual configurations shall enable to identify great many possible alterations of technical condition demonstrated by the blade under investigation.

4. Conclusions

The presented method applicable to monitoring of changes in technical condition of a blade is an innovative approach to carry out diagnostic of the blade in its natural environment with no need to measure signals from that environment.

The monitoring method that can be used to keep track on technical condition of the blade bases on the diagnostic model when the quotient of amplitude amplification W^2_{t12T01} is calculated for the output signal $y(t)$ related to the environment signal $x(t)$ for the data recording time T_{01} and T_{12}. The method consists in the fact that the time interval T when the blade tip $y(t)$ moves within the vulnerability zone of the sensor is split into two sub-intervals: T_{01} when the blade tip approaches the sensor position and T_{12} i.e. after passing this position and moving away.

The method for monitoring technical condition of an item is characterized in the fact that the virtual signal W^2_{t12T01} is calculated as a quotient of the power spectral density (PSD) S^{T12}_{yy} for the signal $y(t)$ that is monitored during a subsequent time period T_{12} divided by the power spectral density (PSD) S^{T01}_{yy} for the same signal $y(t)$ but monitored during the preceding period of time T_{01}. The time intervals T_{01} and T_{12} to keep tracks on the signal $y(t)$ are selected in such a way that the environment $x(t)$ for these time periods when the signal $y(t)$ is monitored can be considered as identical, i.e. $S^{T01}_{xx} = S^{T12}_{xx}$.

In addition, the method for monitoring technical condition demonstrated by a blade is characterized in the fact that the power spectral density (PSD) values S^{T01}_{yy} and S^{T12}_{yy} for the signal $y(t)$ is calculated from analytic forms of the autocorrelation functions R^{T01}_{yy} and R^{T12}_{yy} that are determined with the matching coefficient higher than 0.9. The required matching factor is achieved by assuming the necessary data recording time T_d or T_k as well as appropriate function of the measuring window (rectangular, Hamming, Hanning or other types).

The characteristic feature of the model W_{t12T01}^2 is the advantage that it needs no measurements of signals from the environment, although indirectly they are taken into account by purposefully arranged diagnostic examinations (two time intervals for taking measurements, determination of the diagnostic model as a quotient of other diagnostic models that combine signals from the environment with parameters that reflect technical condition of the object).

References

[1] Bovishanskii, K. N. et. al., *A method for continuous monitoring the vibrational state of the rotating blades of turbomachines,* Thermal engineering (Teploenegetika), Vol. 47, No. 5, St. Petersburg 2000.

[2] Bendat, J. S., Piersol, A. G., *Metody analizy i pomiaru sygnałów losowych. (Methods for analysis and measurements of random signals)* PWN, Warszawa 1976 (in Polish).

[3] Duan, F., Fang, Z., Sun, Y., Ye, S., *Real–time vibration measurement technique based on tip - timing for rotating blades.* Opto-Electronic Energineering 2005, 30 (1) 29-31, htt://www.paper.edu.cn

[4] von Flotow, A., Mercadal, H., *Turbine rotor health management with blade-tip sensor*s; From Laboratory Tool To Fielded System, www hoodtech tlood River or USD 2000.

[5] *High Cycle Fatigue S & program 1997 Annual Report* http://stimet.dtic.mil

[6] *High Cycle Fatigue S & program 1998 Annual Report* http://stimet.dtic.mil

[7] *High Cycle Fatigue S & program 1999 Annual Report* http://stimet.dtic.mil

[8] *High Cycle Fatigue S & program 2000 Annual Report* http://stimet.dtic.mil

[9] *High Cycle Fatigue S & program 2001 Annual Report* http://stimet.dtic.mil

[10] *High Cycle Fatigue S & program 2002 Annual Repor*t http://stimet.dtic.mil

[11] Klein, B., *Non-Contact Vibration measurements Turbocharges Turbine and Compressor Blades,* Proceedings of 1st EVI-GTI International Conference on Gas Turbine Instrumentation, Barcelona 2004.

[12] Kotowski, A., Lindstedt, P., *The using of signals of impulse acoustic response in tests of rotor blades in stationary conditions*, The International Symposium on Stability Control of Rotating Machinery, ISCORMA 4, Calgary Alberta Canada 2007.

[13] Kurowski, W., *Podstawy teoretyczne komputerowego miernictwa systemów mechanicznych, (Theoretical fundamentals for computer-aided measurement techniques in mechanical systems),* Wyd. Politechniki Białostockiej, Białystok 1994.

[14] Lindstedt, P., Kotowski, A., *Basics for innovations in vibroacoustic diagnostics of transport machines rotor blades,* The Archives of Transport, Vd XVI, No. 4, 2004.

[15] Lindstedt, P., *Praktyczna diagnostyka maszyn i jej teoretyczne podstawy,* (*Practical diagnostics of machinery and its theoretical fundamentals*), Wyd. Naukowe ASKON, Warszawa 2002.

[16] Roberts, J. P., *Comparison of Tip Timing with strain gauges for rotor blade vibration measurement.* Proceedings of lecture series on Tip Timing an Tip Clearance Problems in Turbomachines Von Karman Institute, Belgium 2007.

[17] Szabatin, J., *Podstawy teorii sygnałów (Basics for signal theory*), WKŁ Warszawa 2000.

[18] Szczepanik, R., Przysowa, R., *Wykonanie badań i pomiarów drgań łopatek metodą bezstykową w odwirowni ALSTOM Power w Elblągu,* Sprawozdanie ITWL Nr 36/3/2004, (*Execution of tests and measurements of blade vibrations by means of a contactless methods on the rotation bench of ALSTOM Power in Elbląg. Report of AFIT No 36/3/2004),* Warszawa 2004

[19] Washburn, R. S., A *State of The Art PC based NSMS Analysis Software Package.* http://www.agilismeasurementsystems.com

[20] Zieliński, M., Ziller, G., *Non-contact Blade Vibration measurement system for aero engine application,* 17th International Symposium on Airbreathing Engines, No. ISABE - 2005-1220, http://www.mtn.de/channel/files/pdf noncontact blade vibration pdf, Munich Germany 2005.