DETERMINING OF MODEL SIMILARITY FOR FLEXSPLINE OF HARMONIC DRIVE WITH THE USE OF FEM AND EXTENSOMETER METHOD

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Abstract

This paper presents an analysis of the feasibility of using the model similarity method in engineering design. The analysis applies to virtual and physical models of the flexspline body of harmonic gear drive. The task consisted in determining stress distribution in the virtual models of the flexspline by means of the finite element method (FEM) and next its verification during bench tests by the extensometer method. The values of stresses calculated by the FEM method and the extensometer method were compared with each other with the aim of defining model similarity.

The numerical calculations were made by means of the ADINA application which uses FEM with the use of contact elements. Stress distribution calculations applied to simplified virtual models of the flexspline. By the simplification of models a great number of finite elements used for the discretisation process were limited and because of that the time of calculations was shortened. The simplified model was precise enough to determine the stresses in the flexspline body. The subject of this analysis was the impact of different values of the torque moment T2 on the stresses in the flexspline body of the harmonic drive.

The results of numerical calculations were checked against the standing tests of the real drive by applying the extensometer method. During the work of the gear, the values of relative elongation were measured and, based on them, the values of stresses were determined and compared to the results of the numerical calculations by FEM in the analogous cross-sections of the flexspline. The comparability of the results received by both methods proves both that the models used in the numerical calculations were properly designed and that the assumptions as well as the calculation were properly made. The determining of model similarity for the examined harmonic drive will make it possible to carry out analyses of stress calculations by FEM on virtual models without the necessity to verify them by laboratory standing tests each time.

Keywords: *mechanics, harmonic drives, flexspline, FEM, extensometers*

1. Introduction

A characteristic feature of harmonic drives is the way of transmitting motion from the driver to the follower as a result of the cyclic deformation of the flexspline. With the harmonic drive at work, the flexspline with an external toothed wheel rim is deformed by the generator and mates with the internally toothed circular spline [2, 6, 14]. In harmonic drives hermeticity can be ensured thanks to the employment of the flexspline having the shape of a thin-walled sleeve with a toothed wheel rim in the middle. At the one end of the flexspline is a flange used to attach it to the baffle, at the other – a bottom separating the hermetic space from the surroundings [1, 9]. Work on the optimization of the flexible sleeve shape requires that a good number of flexspline prototypes be constructed and examined, which involves considerable expenses. The finite element method (FEM) enables the determination of stress values for any shape of the flexspline with different loads thus allowing the definition of stress distribution for a large number of similar construction solutions of the flexspline quickly and easily without incurring additional costs [3-4, 7].

However, proper application of the FEM numerical method in engineering design requires a considerable knowledge and experience, and the results received should be verified by means of another analytical or experimental method. As part of this publication, the process of determining stress distribution with the use of FEM has been presented together with the verification of the results by the extensometer method on the physical model. A comparison of the received results will make it possible to evaluate the feasibility of the employment of model similarity of virtual and physical models of the hermetic harmonic drive in engineering design.

2. FEM numerical calculations

The virtual model of the flexspline designed to determine stress distribution in the ADINA application has been constructed on the basis of geometrical calculations in the Mechanical Desktop application. The model of the flexspline was subjected to the discretisation process through a division into hexagonal, eight-node finite elements [11]. The calculations were made for a flexspline model whose neutral layer under the toothing was deformed to the shape it obtains under the impact of cam generator with a torque moment of T_2 = 150 Nm.

To ensure that the calculations correspond to the real character of work, contact elements have been used between the models under analysis, adopting the generator as the driver, and the flexspline as the follower.

What is obtained as a result of the calculations in the ADINA application on the computer screen is the distribution of stress displayed as coloured contour lines on the model. However, such a graphic representation of results does not allow one to determine precisely the value of stress corresponding to a specific node point or a point of the model. In order to facilitate the analysis of stress in the flexspline body, graphs have been produced showing stress values in selected points of the model (Fig. 1). It was also necessary to determine stress values in selected sections because of the verification technique of the received results through tests with the use of the extensometer methods.

3. Bending stress tests

For the purpose of the tests, all components of the harmonic drive have been constructed and the drive has been mounted on a specially prepared test stand [10]. A flexspline of a size corresponding to the dimensions of the model used previously for the numerical calculations was additionally equipped with extensometers [12]. As the extensometers allowed for unit elongation ε to be measured in one direction only, circumferential bending stresses σ_1 and longitudinal stresses σ_2, were being determined separately in the flexspline body [8, 13, 15]. The measurements of the unit elongation ε were taken in eight cross-sections, the same as for the numerical calculations along the length of the flexspline (Fig. 1).

Four extensometers have been mounted on each part of the flexspline, both the conical and the cylindrical ones, in such a way as to ensure that for each of the measuring directions one of the extensometers is located in the immediate proximity of the toothed wheel rim with the others evenly distributed on a given part of the flexspline body.

The measurements have been taken for a hermetic harmonic drive loaded with a torque moment of T_2=150 Nm, maintaining the same conditions of work and surroundings. The results of the measurements yielded values of unit elongation ε in a time function for specific points on the flexspline body in which extensometers had been stuck.

For each of the measurements, graphs have been prepared depicting bending stresses varying cyclically, whose pattern is connected with the nature of work performed by the generator deforming the flexspline. On account of the repetitive and symmetrical character of results, analysed were their fragments corresponding to the half of the flexspline limited by the minor axis of the generator.

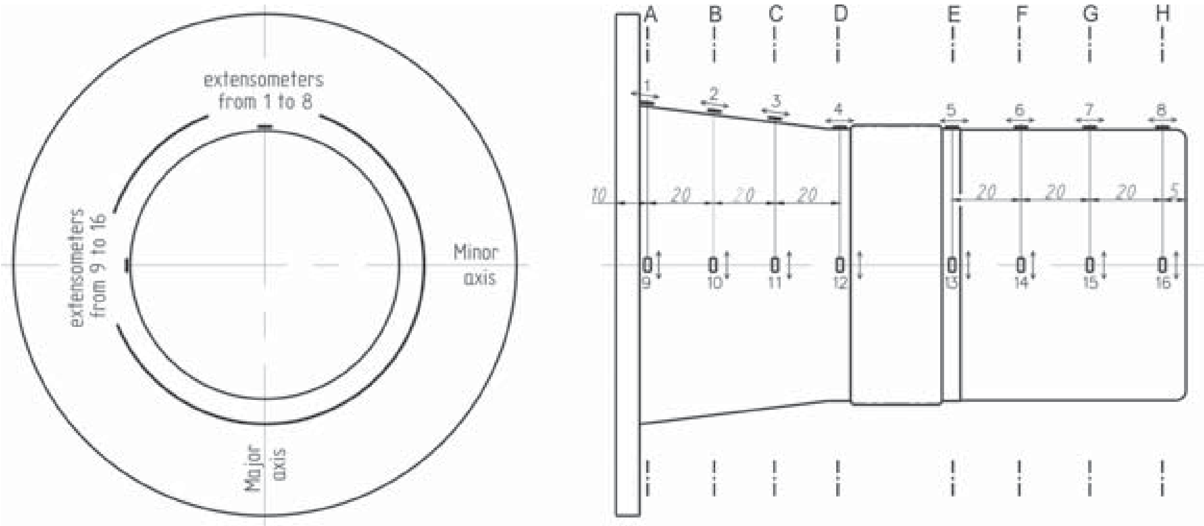

Fig. 1. A diagram of extensometers and measuring sections distribution on the flexspline. Marking: from 1-16 extensometers; A - H measuring sections

4. Model similarity

The law of model similarity can be used in analyses of physical and virtual models provided that Hoocke's law applies to them. Other prerequisites for determining the similarity constant include: the maintenance of geometric similarity as to shape and deformation as well as identical loads in terms of character and value. Then the state of stresses is dependent on material constants: Young's modulus E and the modulus of rigidity v. With the above-mentioned requirements fulfilled, it is possible to determine the similarity constant k, which is a ratio of physical, mechanical or strength quantities corresponding to one another [5].

In the case of the flexspline being analysed, model similarity has been determined by the comparison of distribution of stress in corresponding cross-sections obtained by means of the extensometer method for a physical flexspline and a model used for calculations by FEM. In each of the aforementioned sections of the flexspline patterns of circumferential bending stresses σ_l and longitudinal bending stresses σ_2 for a harmonic drive loaded with an identical torque moment T_2.

Fig. 2-5 depict stress patterns calculated by the numerical method and determined on the basis of elongations measured with the aid of extensometers stuck on a physical gear fixed inside a hermetic harmonic drive. For easy comparison, stress graphs for the same sections of the flexspline have been juxtaposed with each other.

Fig. 2 presents the values of circumferential stresses and Fig. 3 shows longitudinal stresses occurring in the first four sections of the model. A conclusion may be drawn while comparing the graphs that in both cases, for analogous sections, the stress pattern is similar.

The most significant differences can be noticed in the fourth section, located closest to the toothed wheel rim. In both cases the highest values of circumferential stresses σ_1 occur on the right-hand side of the major axis of the generator, which is a result of the gear being loaded with a torque moment T_2. Differences in stress values occurred in the minor axis of the generator, which is caused by the way of fixing edge elements on the virtual model in the course of numerical calculations and these fragments of the graphs were not being considered while determining the constant k.

The values of stresses in corresponding sections differ from one another by a maximum of 10%. It is worth emphasizing that the higher values appear in the case of calculations by the FEM method.

The graphs of circumferential and longitudinal stresses presented on the subsequent figures of the flexspline show a considerable convergence of stress values in the sections from V to VIII. The similarity concerns both the shape of the graphs and the values of stresses in corresponding sections for both methods. Like in the case of circumferential and longitudinal stresses occurring in the conical part, the greatest differences in stress values can be observed in the minor axis of the generator, which is caused by the limitations of the FEM model.

a) b)

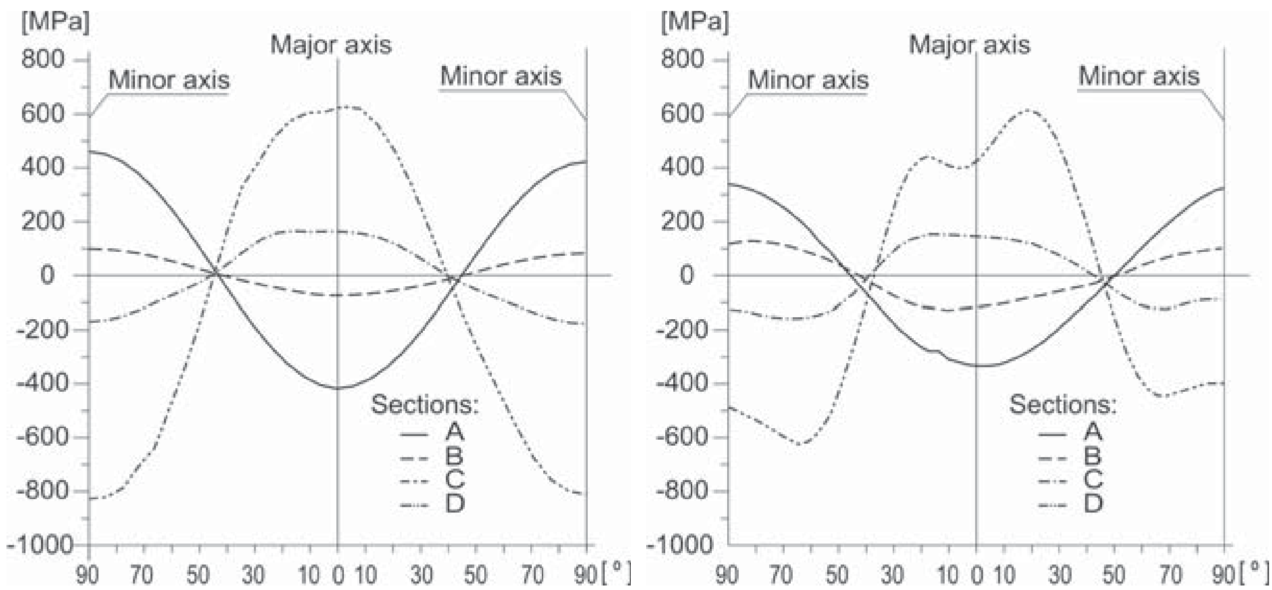

Fig. 2. Circumferential stresses σ_1 in specific sections of the conical part of the flexspline loaded with a torque moment of T_2=150 Nm: a)determined by means of FEM b) determined by means of the extensometer method

a) b)

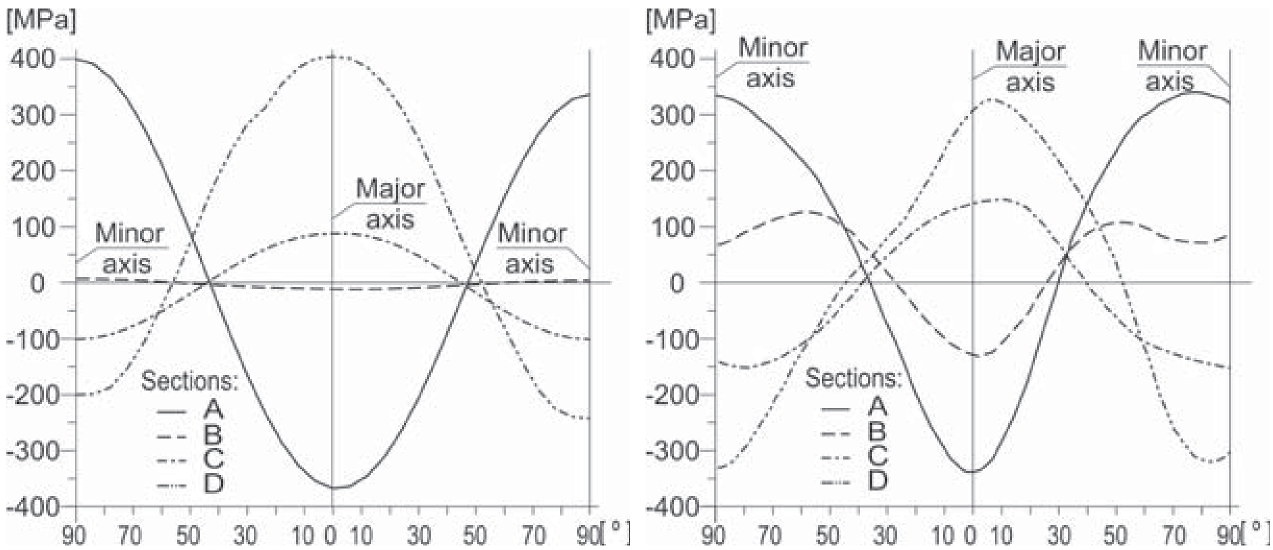

Fig. 3. Longitudinal stresses σ_2 in specific sections of the conical part of the flexspline loaded with a torque moment of T_2=150 N·m: a) determined by means of FEM b) determined by means of the extensometer method

Maximum values of longitudinal stresses σ_2 occur in the vicinity of the large and minor axis of the generator wheel. They are approximately 15% higher for calculations by the FEM method than in the case of extensometer measurements.

a) b)

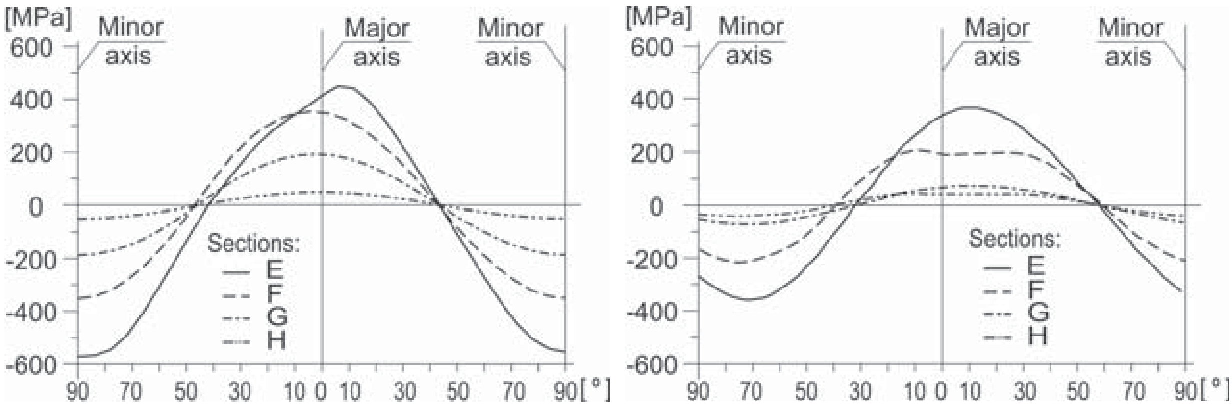

Fig. 4. Circumferential stresses σ_l in specific sections of the cylindrical part of the flexspline loaded with a torque moment of T_2=150 N·m: a) determined by means of FEM b) determined by means of the extensometer method

a) b)

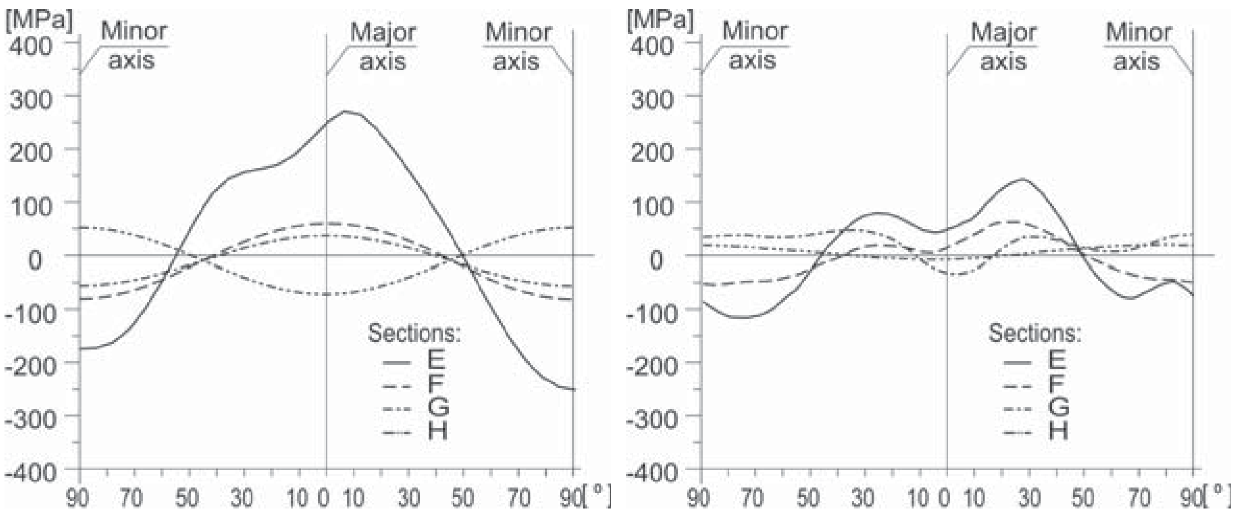

Fig. 5. Longitudinal stresses σ_l in specific sections of the cylindrical part of the flexspline loaded with a torque moment of T_2=150 N·m: a) determined by means of FEM b) determined by means of the extensometer method

The model similarity constant has been defined as a ratio of corresponding stress values determined by the numerical method to the values of stresses measured in the extensometer method. Partial constants were calculated for each of the sections, and then their average value for the whole flexspline model was determined k=1,17.

5. Summary

In the analysis of stress pattern presented above, in selected cross-sections of the flexspline, comparable results have been received for both methods. It should be noted that insignificantly higher stress values appear in calculations made by means of the FEM method. However, maintaining the similarity of shape, deformations and loads of the virtual and physical models, it is possible to observe model similarity and determine the constant k.

Comparing the results received by means of both methods, it may be stated that the virtual models of the flexspline and the models of generators used for numerical calculations were properly made, both in terms of discrete division into finite elements and mapping of the real way in which the construction was fixed and loaded. The precise results of the analysis prove the simplifications adopted in the numerical models to be correct for similar constructional solutions. Based on the analysis, it was proved that the FEM method as well as the extensometer measurements may be applied interchangeably in future analyses of flexsplines of harmonic drive, allowing for the difference of results determined with the model similarity constant.

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