AN ATTEMPT OF AN EMPLOYMENT OF A CONTINUOUS WAVELET TRANSFORM FOR EVALUATION OF TEMPORARY COMFORT DISTURBANCES

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Abstract

This paper focuses on an attempt of an employment of a continuous wavelet transform as a tool of time-frequency analysis to identify and asses the comfort on discrete events. A short review of practical procedures for predicting vibration discomfort defined by ISO 2631-5 and BS 6041, ENV 12299 is enclosed. The results of research of tram temporary comfort disturbances are also described in the paper. The research enclosed the comfort on curve transitions and comfort on discrete events. Research was carried out in the old city centre as in this area there are a lot of curves and large number of excitations can be expected. An exemplary analysis of tram comfort using a continuous wavelet transform is presented. The signals in time intervals in which P_{CT} and P_{DE} indexes that have high values are analyzed. Moreover, the evaluation of comfort disturbances using wavelet transform based on Morlet mother wavelet is done. The continuous wavelet transform that was employed proves high usefulness of this method for assessment of temporary comfort disturbances. The results of Fourier transform of the signals measured are also presented in this article. Traditional methods based on the vibration dose value and frequency weighting methods for assessment of these vibrations are less useful in respect to non-stationary character of temporary disturbances.

Keywords: *ride comfort analysis, continuous wavelet transform, vibration discomfort, comfort on discrete events*

1. Introduction

Vehicle ride comfort is influenced by many factors both outer and inner ones. Some of the outer factors are the vibrations characterized by wide range parameters. Human responses to vibration are highly variable. The evaluation of vibration measurements with respect to human response requires knowledge of magnitude, frequencies, direction and duration of a vibration. The vibration magnitude, frequency, and direction vary with location in a vehicle. The procedures of measurement are defined by appropriate standards. Since vehicle vibration is often not in steady state and human reaction to a vibration depends on the duration over which vibration is felt, human response could be divided into two groups [1]:

- Mean feel; continuous vibrations with no shocks, long durations (a couple of minutes),
- Temporary feel; vibrations are not in steady state, sudden change of mean feeling caused by short duration of acceleration impulse, rolling of vehicle with high velocity.

It is assumed that when short duration of accelerations appears in a vehicle motion temporary comfort disturbances are taken into consideration.

2. Standards [2, 3, 4, 5]

International Standard 2631 and British Standard 6841 define the procedures for predicting vibration discomfort from measurement of vibration at the seat pan, the seat back, and the feet of seated person. If vibration is steady, state the *r.m.s.* value may provide a useful indication of the average severity of vibration.

$$r.m.s. = \left[\frac{1}{T} \int_{t=0}^{t=T} a_w^2(t) dt \right]^{\frac{1}{2}}. \tag{1}$$

The scale of vibration discomfort suggested in British Standard and International Standard using the *r.m.s.* value is shown in figure 1. On the bases of experimental studies it has been stated that doubling vibration magnitude requires a 16-fold reduction in duration to maintain equivalence comfort feeling. This led to fourth–power relationships between the acceleration magnitude and duration. It seems that *r.m.q* gives greater sensitivity to shock and other acceleration peaks.

$$r.m.q. = \left[\frac{1}{T} \int_{t=0}^{t=T} a_w^4(t) dt \right]^{\frac{1}{4}}. \tag{2}$$

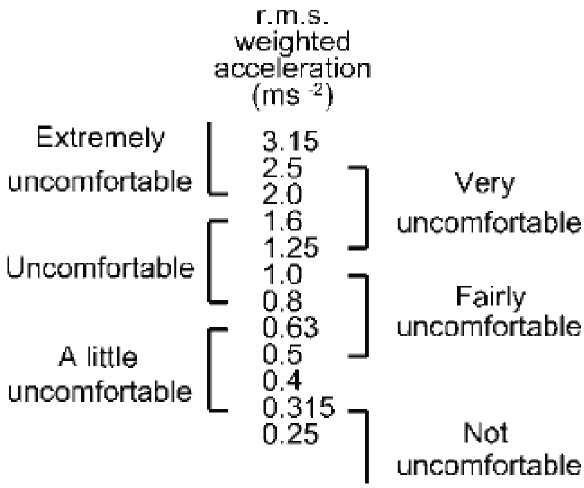

Fig. 1. Discomfort scale suggested by BS 6841:1987and ISO 2631:1997

Both *r.m.s.* and *r.m.q.* values are averages. They do not increase with increases of the duration of steady-state signal and decrease with increasing measurement duration if the signal is non stationary. Vibration of vehicle is often not statistically stationary and it is difficult to define the moment of starting and finishing the *r.m.s.* or *r.m.q* evaluation. The solution of this problem is Vibration Dose Value (VDV). This index comes from *r.m.q.* and is not divided by the exposure duration. In many items of bibliography this indicator is treated as better indicator than *r.m.s.*

$$VDV = \left[\int_{t=0}^{t=T} a_w^4(t) dt \right]^{\frac{1}{4}}, \tag{3}$$

where:

$a_w(t)$ - frequency weighted acceleration,

T - period during which a person is exposed to vibration.

There are other approaches to comfort assessment, that take into account vibration influence in three perpendicular directions as it was presented in CEN ENV12299 standard; where the N_{MV} (4) medium comfort is described by the sum of square powers. For example 95 percentile weighted acceleration for particular directions may be used. On the basis of this coefficient and description in Table 1 it is possible to define the comfort level.

$$N_{MV} = 6 \cdot \sqrt{(a_{XP95}^{W_d})^2 + (a_{YP95}^{W_d})^2 + (a_{ZP95}^{W_b})^2} \; . \tag{4}$$

Tab. 1. Values range of non-dimensional N_{MV} comfort index suggested by CEN ENV 12299

$N_{MV} < 1.5$	Very comfortable
$1.5 \le N_{MV} < 2.5$	Comfortable
$2.5 \le N_{MV} < 3.5$	Medium
$3.5 \le N_{MV} < 4.5$	Uncomfortable
$N_{MV} \le 4.5$	Very uncomfortable

In case of temporary comfort disturbance CEN standard gives detailed guidelines to define the discomfort level. The discomfort level is expressed by percent of passenger who feel uncomfortable or very uncomfortable. The coefficient of temporary discomfort is called P_{CT}(5) for transient railway curve and P_{DE}(6) for straight track, junctions and curves.

$$P_{CT} = 100\% \cdot \left\{ \max\left[\left(A \cdot \left| \ddot{y}_{1s} \right|_{\max} + B \cdot \left| \dddot{y}_{1s} \right|_{\max} - C \right); 0 \right] + \left(D \cdot \left| \dot{\varphi}_{1s} \right|_{\max} \right)^{E} \right\}, \tag{5}$$

where:

$\left| \ddot{y}_{1s} \right|_{\max}$ - The maximum absolute value of lateral acceleration in vehicle body, in the time interval between the beginning of transition curve and the end +1.6s expressed in m/s^2,

$\left| \dddot{y}_{1s} \right|_{\max}$ - The maximum absolute value of lateral jerk on the transition curve, in the time interval between 1 s before the beginning of transition curve and the end of the transition expressed in m/s^3,

$\left| \dot{\varphi}_{1s} \right|_{\max}$ - The maximum absolute value of roll velocity, in the time interval between the beginning of transition curve and the end of the transition expressed in rad/s.

A,B,C,D,E - constants for P_{CT} comfort index obtained on the base of experimental research.

$$P_{DE}(t) = 100\% \cdot \max\left[a \cdot \ddot{y}_{pp}(t) + b \cdot \left| \ddot{y}_{2s}(t) \right| - c; \; 0 \right], \tag{6}$$

where:

$\ddot{y}_{pp}(t)$ - maximum corresponding peak to peak lateral acceleration for 2 s interval expressed in m/s^2

$\left| \ddot{y}_{2s}(t) \right|$ - absolute value of mean value of lateral acceleration for 2 s interval, expressed in m/s^2

a, b, c - constant for P_{DE} comfort index.

The analyzed acceleration signal is a signal filtered by 2Hz low pass filter. The results of P_{CT} calculation is percent of discontented passenger for defined transition curve. In case of P_{DE} the value of comfort index is a time function of a railway section.
Only in case of the CEN ENV 12299 standard the additional procedures are given to evaluate the level of temporary discomfort since this standard refers only to rail-vehicle and only then it is possible to define a motion character. BS 6841 and ISO 2631 standards have more general procedures and do not define the kind of transport means.

3. Wavelet transform [6, 7, 8]

Vehicle vibration is often not statistically stationary and that is the cause of limitation of employing the Fourier transform for analysis. It seems that time-frequency analysis could be better in case of vehicle vibration analysis. One of the time-frequency analysis tools is continuous wavelet transform. $CWT_f(a,b)$ wavelet transform is defined as follows:

$$CWT_f(a,b) = \int_{-\infty}^{\infty} f(t) \cdot \psi_{a,b}(t)\, dt \,. \tag{7}$$

Where: $f(t)$ examined signal as time function, belongs to L^{2r} function space

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \cdot \psi\left(\frac{t-b}{a}\right) \quad a,b \in R,\ a \neq 0 \,. \tag{8}$$

Element 1/a is responsible for wavelet normalization

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0 \,. \tag{9}$$

An average value of a wavelet function in the time domain must be zero and therefore it must be oscillatory. $\psi(t)$ function is called mother wavelet or main wavelet and it has average value that equals zero and compact carrier. Some of the exemplary wavelets are shown in Figure 2.

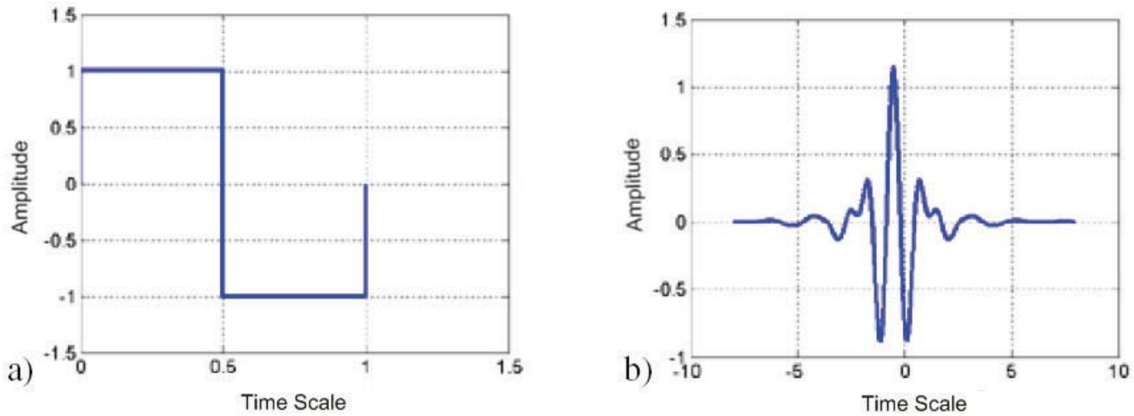

Fig. 2. The run of wavelet: a) Haar'a, b) Meyer'a

The wavelets are generated from a single basic wavelet, the so-called mother wavelet, by scaling and translation in equation (8) a is a scaling factor, b is the translation factor and the factor a1/2 is for energy normalization across the different scales.

Wavelet scaling corresponds to spectrum inverse shift. Equation (7) represents signal band-pass filtration using sequentially scaled wavelets. It means using sequential filters with various band-passes with centre frequency f_c. The $CWT_f(a,b)$ result are the coefficients $CWT_f(a,b)$ that are scale and translation factors. The algorithm of $CWT_f(a,b)$ is as follows: chosen mother wavelet is compared with the beginning of analyzed signal, calculated $CWT_f(a,b)$ coefficient defines the similarity of wavelet to analyzed part of a signal. Next successive part of a signal is chosen (by increasing b coefficient in equation 8) and compared once again with wavelet. This procedure is repeated until the full signal is covered. Afterwards the wavelet is scaled again (by increasing the a coefficient in equation 8) and the comparison procedure is repeated. For visualization the obtained $CWT_f(a,b)$ coefficients the system time-scale (time-frequency) is used in which the point lightness is proportional to value of $CWT_f(a,b)$ coefficient.

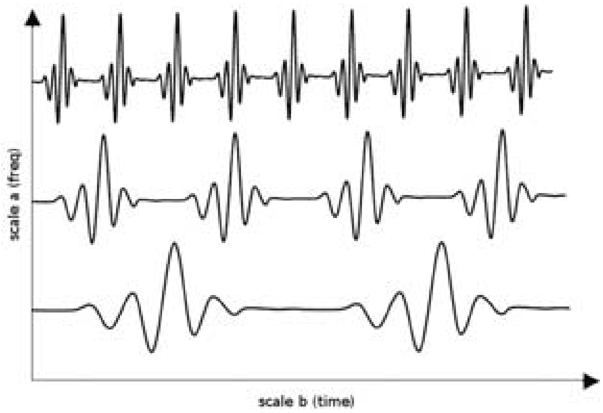

Fig. 3. The example of scaling and translation factors of mother wavelet

Wavelet transform has the properties that can make the approximation function possible. This procedure can be used to signal decomposition and to remove information, redundant in our opinion, from analyzed signal.

4. Interpretation of the wavelet transform results

The CWTf(a,b) coefficients of continuous wavelet are shown in a diagram with different lightness for individual frequency. On the diagram basis the time interval of particular frequency (Figure 4) can be defined.

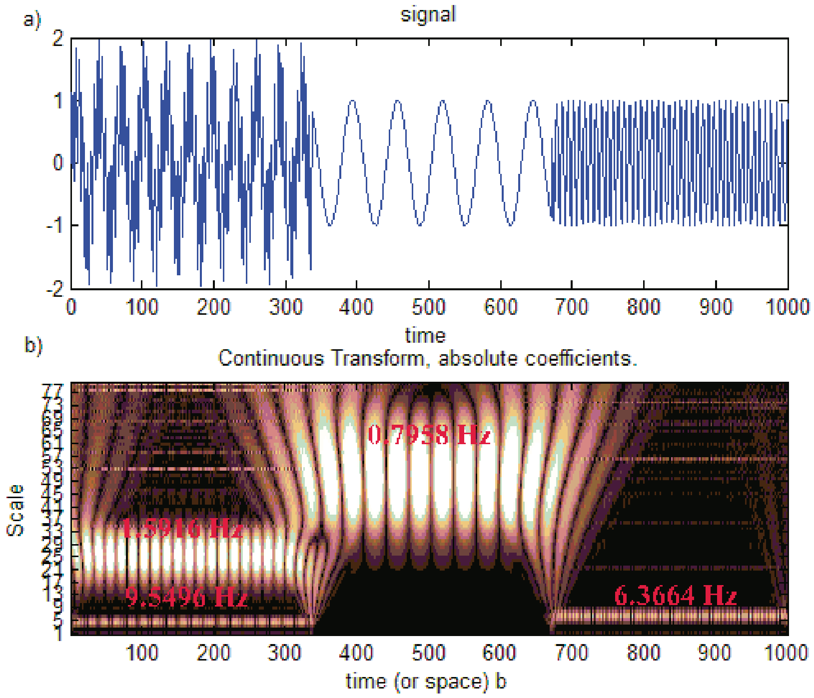

Fig. 4. The result of wavelet transform a) analyzed signal, b) diagram of wavelet coefficients in system time scale

Figure 4a shows the run of analyzed signal which consists of three parts with the same time interval. The first comprises two signal components, sin(10t),sin(60t), the second sin(5t), and the third sin(40t). Figure 4b shows the continuous wavelet result in which the Morlet mother wavelet for analysis was used. The frequency changes are in step manner.

Tab. 2. Frequency identification and relative errors calculated using r.m.q.

pattern of signal	scale a		frequency [Hz]				
	from scalogram	nearest true	from scalogram f_c	nearest true f_c	true	difference (6-4) [Hz]	relative error [%]
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
sin(10t) +sin(60t)	25	26	1.6250	1.5625	1.5916	0.0334	2.09
	4	4	10.1563	10.1563	9.5496	0.6067	6.35
sin(5t)	50	51	0.8125	0.7966	0.7958	0.0167	2.09
sin(40t)	6	6	6.7708	6.7708	6.3664	0.4044	6.35

In Table 2 the results of frequency identification and relative errors calculated using *r.m.q.* are presented. The real frequency of analyzed signal is presented in column 6. The scale number for which the centre frequency is the nearest to real frequency is shown in column 3. In column 8 the relative, acceptable errors of the employed continuous wavelet transform are shown. To recognize the scale on the diagram (figure 4b) the algorithm based on designation of a single number value calculated as *r.m.q.* (equation2) on the bases of the runs of $CWT_f(a,b)$ coefficient that resulted from wavelet transform for each a scales is calculated. Afterwards these values are normalized according to equation

$$Z(a) = \frac{1}{\sqrt{a}} \cdot \mathrm{X}(a), \tag{10}$$

where:
a – scale,
X(a) – *r.m.q.* value wavelet coefficient for a scale.

Dimension of the obtained matrix that result from above algorithm is 1 to a_{max} where a_{max} is scale number. The exemplary result of the employment of the algorithm described above is shown in Figure 5.

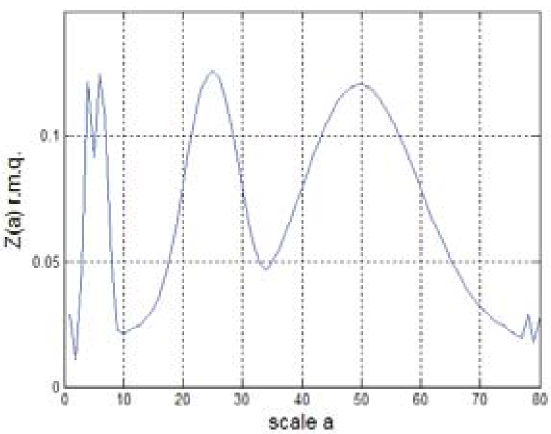

Fig. 5. Normalized values of r.m.q.as a scale number function

Four local maximums which correspond to signal frequency can be noted in Figure 5. When the number of scale is known for particular maximum the centre frequency can be calculated f_c (Tab. 2).

5. Wavelet analysis of acceleration signal

The signals of lateral accelerations recorded on the passenger seat pan of tram NGT6 are the subject of analysis. Relative long tram line allowed us to make a lot of measurements of acceleration for straight railway section, transaction curve and as well as for curve for various tram velocity The research was made with no passengers present. Signal was recorded with 50 [Hz] frequency.

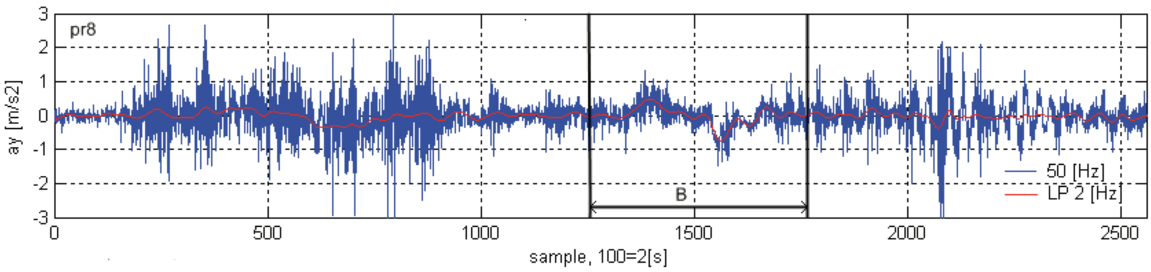

Fig. 6. Exemplary signal of lateral acceleration double curve

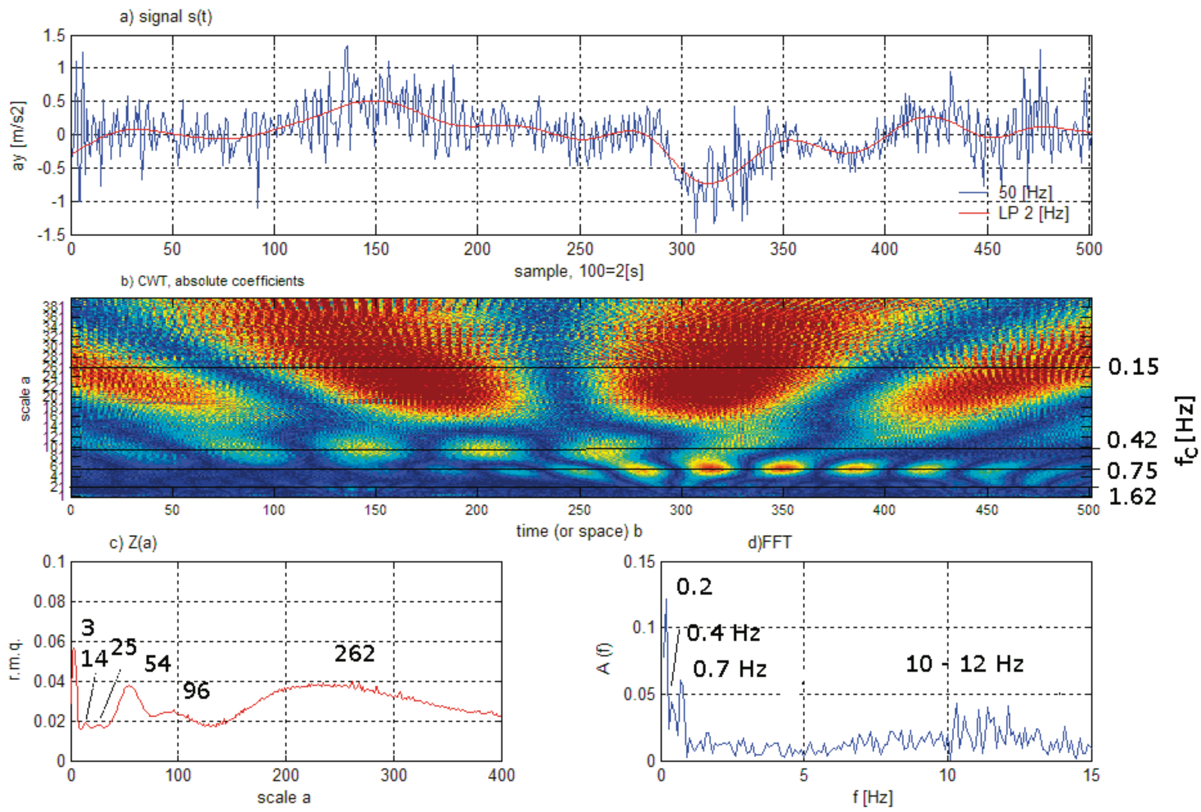

Fig. 7. Analysis of a signal

In Figure 6 the recording of lateral acceleration lasted 51.2 [s] is shown. The section from this record comprising two curves with opposite directions as situation with temporary comfort disturbances was selected. In Figure 7 the analysis of this signal is presented. The analyzed signal lasted 10s. The shape of the run of this signal is similar to sinusoid with 0.5 [m/s] amplitude and 0.15 [Hz] frequency. In Figure 7b the diagram of continuous wavelet transform for 400 scales is presented. In Figure 7c the values of *r.m.q.* used to calculate centre frequency is shown. The relative error between centre frequency and frequency from Fourier transform was calculated and is presented in Table 3.

Tab. 3. Scale and centre frequency of signal presented in Fig. 7

Scale: a	Central Frequency: fc [Hz]	FFT [Hz]	Error [%]
262	0.16	0.20	28.95
96	0.42	0.40	5.48
54	0.75	0.70	6.95
25	1.62	-	-
14	2.90	-	-
3	13.54	10-12	26-11

In Figure 7d the FFT of signal with frequencies of local maximum are presented. If the run of signal and its wavelet transform is compared, it can be noticed that time coincidence of a vibration character with maximum values of wavelet $CWT_f(a,b)$ coefficients appears (the darker the hue of red the higher values of wavelet coefficient). Owing to that, high conformity of a signal part and analyzing wavelet can be obtained. The analysis of transform diagram presents that after passing the first curve the changes from 0.42 [Hz] to 0.75 [Hz] of centre frequency can be noticed. These changes can be also noticed on a filtered run (red line Figure 7a) Moreover, the centre frequency of 1.62 [Hz] for sample from 140 to 150 and from 330 to 340 occurred. A 13.54 [Hz] centre frequency occurring all along the run of signal is invisible on the diagram. In Figure 7d Fourier Transform is presented and some frequencies are the same as centre frequencies of the analyzed wavelet.

6. Conclusion

An attempt of an employment of a continuous wavelet transform for evaluation of temporary comfort disturbances was performed. The results of this analysis confirmed the possibilities of wavelet transform employment for evaluation of temporary comfort disturbances. The values of main frequency obtained from FFT and wavelet transform differ slightly. Taking into account the advantages of wavelet transform in comparison with FFT especially for non- stationary signals this approach seems much better then conventional methods.

7. References

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