# SLIDING MODE SPEED CONTROL FOR MULTI-MOTORS SYSTEM

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# Abstract:

Continuous processes in the plastics, textile paper and other industries, require several drives working in synchronism. The aim of this paper is to control speed of the multi motors system, and to maintain a constant mechanical tension between the rollers of the system. Several controllers are considerer, including Proportional-Integral (PI) and sliding-mode control (SMC). Since the PI control method can be applied easily and is widely known, it has an important place in control applications. But this method is insensitive to parameter changes. The advantage of an SMC is its robustness and ability to handle the non-linear behaviour of the system, and is indicated in comparison with traditional proportional-integral (P1) control scheme. Theoretical analysis and simulation results are provided to evaluate the consistency and performances of this control technique (SMC).

*Keywords:* Multi-Motors systems, sliding mode control, proportional-integral (P1) control.

## 1. Introduction

Paper industries require several drives for paper processing. In the past, a single mechanical line shaft was used for all drives. Nowadays independent drives are used. Electronic synchronization has to be ensured for quality of produced paper rolls. Several control strategies have been suggested based on robust control or electronic emulation of mechanical line shaft [1]. Since the decentralized PI control method can be applied easily and is widely known, it has an important place in control applications, where many industrial web transport systems have used this type of controllers [2]. But this method is insensitive to parameter changes. A nonlinear decoupled control is designed for multi-motors multi motors system. At the first, an ideal feedback linearization control system is adopted in order to decouple the tensions and velocity of the web winding system is presented in [3]centralized and decentralized fixed order  $H\infty$ controller results with model based feed-forward for multi motors systems which provide improved the tension and velocity regulation is presented in [4]. In this work the design of sliding-mode (SMC) to control a multi motors system are proposed in order to improve the performances of the control system, which are coupled mechanically, and Synthesis of the robust control and their application to synchronize the five sequences and to maintain a constant mechanical tension between the rollers of the system [6]. The advantage of an SMC is its robustness and ability to handle the non-linear behaviour of the

system.

The model of the multi-motors system and in particular the model of the mechanical coupling are developed and presented in Section II. Section III shows the development of sliding mode controller's design for winding system. The proposed structure of the studied propulsion system is given in the section IV. Simulation results using MATLAB SIMULINK of different studied cases. Finally, the conclusions are drawn in Section V.

#### 2. System Model's

In the mechanical part, the motor M1 carries out unreeling, M3 drives the fabric by friction and M5 is used to carry out winding, each one of the motors M2 and M4 drives two rollers *via* gears "to grip" the band (Fig.1). Each one of M2 and M4 could be replaced by two motors, which each one would drive a roller of the stages of pinching off. The elements of control of pressure between the rollers are not represented and not even considered in the study. The stage of pinching off can make it possible to isolate two zones and to create a buffer zone. [6,7].

$$\left(\frac{di_{ds}}{dt} = \frac{1}{\sigma \cdot L_s} \left( -\left(R_s + \left(\frac{L_m}{L_r}\right)^2 \cdot R_r\right) \cdot i_{ds} + \sigma L_s \omega_e i_{qs} + \cdots \right) \right. \\ \left. \frac{L_m \cdot R_r}{L_r^2} \cdot \phi_{dr} + \frac{L_m}{L_r} \cdot \phi_{qr} \cdot \omega_r + V_{ds} \right) \\ \left. \frac{di_{qs}}{dt} = \frac{1}{\sigma \cdot L_s} \left( -\sigma L_s \omega_e i_{ds} - \left(R_s + \left(\frac{L_m}{L_r}\right)^2 \cdot R_r\right) \cdot i_{qs} - \cdots \right) \right. \\ \left. \frac{L_m}{L_r} \cdot \phi_{dr} \cdot \omega_r + \frac{L_m \cdot R_r}{L_r^2} \cdot \phi_{qr} + V_{qs} \right) \\ \left. \frac{d\phi_{dr}}{dt} = \frac{L_m \cdot R_r}{L_r} \cdot i_{ds} - \frac{R_r}{L_r} \cdot \phi_{dr} + \left(\omega_e - \omega_r\right) \cdot \phi_{dr} \right. \\ \left. \frac{d\phi_{qr}}{dt} = \frac{L_m \cdot R_r}{L_r} \cdot i_{qs} - \left(\omega_e - \omega_r\right) \cdot \phi_{dr} - \frac{R_r}{L_r} \cdot \phi_{qr} \\ \left. \frac{d\omega_r}{dt} = \frac{P^2 \cdot L_m}{L_r \cdot J} \cdot \left(i_{qs} \phi_{dr} - i_{ds} \phi_{qr}\right) - \frac{f_c}{J} \cdot \omega_r - \frac{P}{J} \cdot T_l \right.$$

$$(1)$$

Where  $\boldsymbol{\sigma}$  is the coefficient of dispersion and is given by:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \tag{2}$$

50



Fig. 1. Five motors web transport system.

The tension model in web transport systems is based on Hooke's law, Coulomb's law, [8], [9] mass conservation law and the laws of motion for each rotating roll.

# A. Hooke's law

The tension T of an elastic web is function of the web strain  $\boldsymbol{\epsilon}$ 

$$T = ES\varepsilon = ES\frac{L - L_0}{L_0}$$
(3)

Where E is the Young modulus, S is the web section, L is the web length under stress and  $L_0$  is the nominal web length (when no stress is applied).

#### B. Coulomb's law

The study of a web tension on a roll can be considered as a problem of friction between solids, see [8] and [9]. On The roll, the web tension is constant on a sticking zone of arc length a and varies on a sliding zone of arc length g (cf. Fig. 2, where  $V_k(t)$  is the linear velocity of the roll k). The web tension between the first contact point of a roll and the first contact point of the following roll is given by:

$$\begin{split} & \varepsilon(x,t) = \varepsilon_1(t) & \text{if } x \le a \\ & \varepsilon(x,t) = \varepsilon_1(t)e^{\mu(x-a)} & \text{if } a \le x \le a+g \\ & \varepsilon(x,t) = \varepsilon_2(t) & \text{if } a+g \le x \le L_t \end{split} \ (1.51)$$

Where  $\mu$  is the friction coefficient, and  $L_t = a + g + L$ . The tension change occurs on the sliding zone. The web velocity is equal to the roll velocity on the sticking zone. (1.54)



Fig. 2. Web tension on the roll.

#### C. Mass conservation law

Consider an element of web of length  $L = L_0(1+\varepsilon)$ . With a weight density  $\rho$ , under an unidirectional stress. The cross section is supposed to be constant. According to the mass conservation law, the mass of the web remains constant between the state without stress and the state with stress

$$\rho SL = \rho_0 SL_0 \implies \frac{\rho}{\rho_0} = \frac{1}{1+\varepsilon}$$
(4)

#### D. Tension model between two consecutive rolls.

The equation of continuity, cf. [8], applied to the web gives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0$$
(5)

By integrating on the variable x from 0 to  $L_t$  (cf. Fig. 2), taking into account (4), and using the fact that a + g << L, we obtain

$$\frac{d}{dt}\left(\frac{L}{1+\varepsilon_2}\right) = \frac{V_1}{1+\varepsilon_1} - \frac{V_2}{1+\varepsilon_2}.$$
  
Therefore:

$$-L\frac{d\varepsilon_2}{dt} = V_1 \frac{(1+\varepsilon_2)^2}{1+\varepsilon_1} - V_2 (1+\varepsilon_2).$$
(6)

This equation can be simplified by using the approximation

$$\varepsilon_{1} << 1 \quad \text{and} \quad \varepsilon_{2} << 1$$

$$\frac{(1+\varepsilon_{2})^{2}}{1+\varepsilon_{1}} \approx (1-\varepsilon_{1})(1+2\varepsilon_{2})$$
(7)

And using Hook's law, we get:

$$L_{k-1} \frac{dT_k}{dt} \cong ES(V_k - V_{k-1}) + T_{k-1}V_{k-1} - T_k(2V_{k-1} - V_k).$$
(8)

k = 2, 3, 4, 5.

where  $L_{k-1}$  is the web length between roll k-1 and roll k,  $T_k$  is the tension on the web between roll k-1 and roll k,  $V_k$  is the linear velocity of the web on roll k,  $\Omega k$  is the rotational speed of roll k,  $R_k$  is the radius of roll k, E is the Young modulus and S is the web section.

#### E. Roll velocity calculation

The law of motion can be obtained with a torque balance:

$$\frac{d(J_k\Omega_k)}{dt} = R_k(T_{k+1} - T_k) + Cem_k + C_f$$
(9)

Where  $\Omega_k = V_k/R_k$ , is the rotational speed of roll  $k \ Cem_k$ is the motor torque (if the roll is driven) and  $C_f$  is the friction torque.

#### F. State space representation

The nonlinear state-space model is composed of (10) for the different web spans and of (11) for the different rolls. Under the assumption that  $J_k R_k$  (k = 2, 3, 4, 5) is varying only slowly, which is the case for thin webs,  $V_k$  can be chosen as a state variable in (11), leading to the

following linear model:

$$E_m X = A(t)X + BU$$
  

$$Y = C(t)X$$
(10)

where

$$X^{T} = \begin{bmatrix} T_{2} & T_{3} & T_{4} & T_{5} & J_{1} & T(t) \Omega_{1} & J_{2} \Omega_{2} & J_{3} \Omega_{3} & J_{4} \Omega_{4} & J_{5} & T(t) \Omega_{5} \end{bmatrix}$$

$$Y^{T} = [T_{2} \ T_{3} \ T_{4} \ T_{5} \ V_{1}], \tag{11}$$

$$U = [u_1 \, u_2 \, u_3 \, u_4 \, u_5] \tag{12}$$

# 3. Design of sliding mode speed and current controllers

A Sliding Mode Controller (SMC) is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that can map plant state to a control surface, whereas switching among different functions is determined by plant state represented by a switching function [8], [9].

To control the speed of the induction machine, the sliding surface is defined as follows:

$$S(w_m) = w_m^* - w_m \tag{13}$$

The derivative of the sliding surface can be given as:

$$\dot{S}(w_m) = \dot{w}_m^* + \dot{w}_m \text{ ok?}$$
(14)

Taking into account the mechanical equation of the induction motor defined in the system of equations (1), the derivative of sliding surface becomes

$$\dot{S}(w_m) = \dot{w}_m^* - \left(\frac{P_{\cdot}L_m \phi_{dr}^*}{JL_r} \cdot i_{qs} - \frac{f_c}{J} w_m - \frac{P_{\cdot}}{J} C_r\right)$$
(15)

The current control is given by:

$$i_{qs}^* = i_{qseq} + i_{qs} \tag{16}$$

To avoid the chattering phenomenon produced by the *Sign* function we use the Saturation function *Sat* in the discontinuous control defined as follow:

$$sat\left(\frac{S}{\phi}\right) = \begin{cases} \frac{S}{\phi} ; & if \quad \left|\frac{S}{\phi}\right| < 1\\ Sign\left(\frac{S}{\phi}\right) ; & if \quad \left|\frac{S}{\phi}\right| > 1 \end{cases}$$
(17)

Where  $\phi$  is the boundary layer thickness.

The discontinuous control action can be given as:

$$i_{qs}^{n} = k_{iqs} \cdot sat(s(\omega)/\phi_{\omega})$$
(18)

 $k_{ias}$ : Positive constant.

$$i_{qseq} = \frac{JL_{r}}{P^{2}L_{m}\phi_{dr}^{*}} \left( \dot{w}_{m}^{*} + \frac{f_{c}}{J} w_{m} + \frac{P}{J} C_{r} \right)$$
(19)

The Fig. 3 shows the SMC control strategy scheme for each induction motor



Fig. 3. Block diagram for each motor with SMC control.

For the indirect field-oriented control (IFOC) tuning parameters we need two surfaces  $S_1$  and  $S_2$  the first for the  $i_{ds}$  regulator and the second for  $i_{qs}$  regulator respectively where: [10,11]

$$S_1 = i_{ds}^* - i_{ds}$$
 (20)

$$S_2 = i_{qs}^* - i_{qs}$$
(21)

The derivate of  $S_1$  can be given as:

$$\dot{S}_1 = \dot{i}_{ds}^* - \dot{i}_{ds}$$

From equation (1) and (27) we can obtain:

$$\dot{S}_{1} = \dot{i}_{ds}^{*} - \left(-\frac{1}{\sigma . L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}}\right)^{2}\right) \dot{i}_{ds} + w_{s} . \dot{i}_{qs} + \frac{L_{m} . R_{r}}{\sigma . L_{s} L_{r}^{2}} \phi_{r}^{*} + \frac{1}{\sigma . L_{s}} V_{ds}\right)$$

The virtual voltage controller  $V_{ds}$  is given by:

$$V_{ds}^{*} = V_{dseq} + V_{dsn}.$$
 (22)

The voltage discontinuous control  $V_{asn}$  is defined as:

$$V_{dsn} = k_1 \cdot sat(s_1/\phi_1)$$
<sup>(23)</sup>

According to Lyapunov stability criteria [10], our speed loop system's stable if:  $S_1 S_1 < 0$  by means that  $K_1$  is positive constant.

The equivalent control  $V_{dseq}$  is given as:

$$V_{dseq} = \sigma . L_s \left( i_{ds}^* + \frac{1}{\sigma . L_s} \left( R_s + R_r \cdot \left( \frac{L_m}{L_r} \right)^2 \right) i_{ds} - w_s . i_{qs} - \frac{L_m \cdot R_r}{\sigma . L_s L_r^2} \phi_r^* \right)$$
(24)

The derivate of can be given as:

$$\dot{S}_2 = \dot{i}_{qs}^* - \dot{i}_{qs}$$

From equation (1) and (34) we can obtain:

52

$$\dot{S}_{2}(i_{qs}) = \dot{i}_{qs}^{*} - \left(-w_{s} i_{ds} - \frac{1}{\sigma L_{s}}\right)$$

$$\left(R_{s} + R_{r} \cdot \left(\frac{L_{m}}{L_{r}}\right)^{2}\right) \dot{i}_{qs} - \frac{L_{m}}{\sigma L_{s} L_{r}} \cdot \phi_{r}^{*} \cdot w_{m} + \frac{1}{\sigma L_{s}} V_{qs}$$

The voltage controller  $V_{qs}$  is given by:

$$V_{qs}^* = V_{qseq} + V_{qsn} \tag{25}$$

The  $V_{asea}$  equivalent control actions defined as:

$$V_{qseq} = \sigma . L_s . (\dot{i}^*_{qslim} + w_s . \dot{i}_{ds} + \frac{1}{\sigma . L_s} \left( R_s + R_r \left( \frac{L_m}{L_r} \right)^2 \right) \dot{i}_{qs} + \frac{L_m}{\sigma L_s L_r} \phi_r^* w_m)$$
(26)

The voltage discontinuous control  $V_{dsn}$  is defined as:

$$V_{qsn} = k_2 \cdot sat(s_2/\phi_2)$$
<sup>(27)</sup>

For the same reason condition of  $K_1$ ,  $K_2$  are positives constant.

### 4. Simulation results

The winding system we modelled is simulated using MATLAB/SIMULINK software and the simulation is carried out on 10s.

To evaluate system performance we carried out numerical simulations under the following conditions: Start with the linear velocity of the web of 5m / s.

The motor M1 has the role of Unwinder a roll radius R1 (R1 = 2.25 m).

The motors M2, M3, M4 are the role is to pinch the tape. The motor M5 has the role of winding a roll of radius R5.

From the Fig (4-6), we can say that: the effect of the disturbance is neglected in the case of the SMC controller. It appears clearly that the classical control with PI controller is easy to apply. However the control with sliding mode controllers offers better performances in both of the overshoot control and the tracking error.



*Fig. 4. The linear speed of unwinder M1.* 

As shown in Figs (4-6). An improvement of the linear speed is registered, and has follows the reference speed for both PI controller and SMC control, but in case of PI controller, the overshoot in linear speed of Unwinder is 25%. Figs (4-6) show that with the SMC controller the

system follows the reference speed after 1 sec, in all motors, however, in the PI controller the system follows after 2 sec. Fig (9) shows the phase plot of the sliding surface(s) of SMC control.



Fig. 5. The linear speed of motors M2, M3 and M4.



Fig. 6. The linear speed of winder M5.

As shown in Fig (4-6). An improvement of the linear speed is registered, and has follows the reference speed for both PI controller and SMC control, but in case of PI controller, the overshoot in linear speed of Unwinder is 25%. Figs (4-6) show that with the SMC controller the system follows the reference speed after 1 sec, in all motors, however, in the PI controller the system follows after 2 sec. Fig (9) shows the phase plot of the sliding surface(s) of SMC control.



*Fig. 7. The phase plot of the sliding surface(s).* 

53

# 5. Conclusion

The objective of this paper consists in developing a model of a winding system constituted of five motors that is coupled mechanically by a strap whose tension is adjustable and to develop the methods of analysis and synthesis of the commands robust and their application to synchronize the five sequences and to maintain a constant mechanical tension between the rollers of the system.

The simulations results show the efficiency of the SMC controller technique, however the strategy of SMC Controller brings good performances, and she is more efficient than the classical PI controller.

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