

INVESTIGATION ON NUMERICAL SOLUTION FOR A ROBOT ARM PROBLEM

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Ramasamy Ponalagusamy, Sukumar Senthilkumar

Abstract:

The aim of this article is focused on providing numerical solutions for a Robot arm problem using the Runge-Kutta sixth-order algorithm. The parameters involved in problem of a Robot control have also been discussed through RK-sixth-order algorithm. The précised solution of the system of equations representing the arm model of a robot has been compared with the corresponding approximate solutions at different time intervals. Experimental results and comparison show the efficiency of the numerical integration algorithm based on the absolute error between the exact and approximate solutions. The stability polynomial for the test equation $\dot{y} = \lambda y$ (λ is a complex Number) using RK-Butcher algorithm obtained by Murugesan *et al.* [1] is not correct and the stability regions for RK-fourth order (RKAM) and RK-Butcher methods have been presented incorrectly. They have made a mistake in determining the range for real parts of λh (h is a step size) involved in the test equation for RKAM and RK-Butcher algorithms. In the present paper, a corrective measure has been taken to obtain the stability polynomial for the case of RK-Butcher algorithm, the ranges for the real part of λh and to present graphically the stability regions of the RKAM and the RK-Butcher methods. The stability polynomial and stability region of RK-Sixth order are also reported. Based on the numerical results it is observed that the error involved in the numerical solution obtained by RK-Sixth order is less in comparison with that obtained by the RK-Fifth order and RK-Fourth order respectively.

Keywords: Runge-Kutta (RK) method, RK-Arithmetic mean, RK- Fifth order algorithm, RK-Sixth order algorithm, Ordinary Differential Equations (ODE), robot arm problem.

1. Introduction

Extensive research work is still being carried out on variety of aspects in the field of robot control, especially about the dynamics of robotic motion and their governing equations. The dynamics of Robot arm problem was initially discussed by Taha [3]. Research in this area is still active and its applications are enormous. This is because of its nature of extending accuracy in the determination of approximate solutions and its flexibility. Many investigations [4-8] have analysed the various aspects of linear and non-linear systems.

Most of the Initial Value Problems (IVPs) are solved through Runge-Kutta (RK) methods which in turn being applied to compute numerical solutions for variety of problems that are modelled as and the differential equations are discussed by Alexander and Coyle [9], Evans

[10], Hung [11], Shampine and Watts [12], [20]. Shampine and Watts [12], [23], [26] have developed mathematical codes for the Runge-Kutta fourth order method. Nanayakkara *et al.* [25] proposed a method for the identification of complex non-linear dynamics of a multi-link robot manipulator using Runge-Kutta-Gill neural networks (RKGNNs) in the absence of input torque information. Runge-Kutta formula of fifth order has been developed by Butcher [13-15]. The applications of Non-linear Differential Algebraic Control Systems to Constrained Robot Systems have been discussed by Krishnan and Mcclamroch [7]. Also, Asymptotic observer design for Constrained Ro-bot Systems have been analysed by Huang and Tseng [22].

Said Oucheriah [4] discussed about the robust tracking and model following the uncertain dynamic delay systems by memoryless linear controllers. David Lirn and Homayoun Seraji [5] presented the configuration control of a mobile dexterous robot. Polycarpou and Loannou [6] discussed about a robust adaptive non-linear control design. Zluehua Qu [8] analysed robust control of a class of non-linear uncertain systems because of the-linear and coupled characteristics nature the design of a robot control system is made complex. Using fourth order RK method based on Heronian mean (RKHeM) an attempt has been made to study the parameters concerning the control of a robot arm model along with the Single Term-Walsh Series (STWS) technique [24]. Hung [27] discussed about the dissipativity of Runge-Kutta methods for dynamical systems with delays.

Yang *et al.* [29] addressed the placement of an open-loop robotic manipulator in a working environment is characterized by defining the position and orientation of the manipulator's base with respect to a fixed reference frame. Jong-Seok Rho *et al.* [30] presented a disk-type travelling wave B14 rotary ultrasonic motor (RUSM). Also, they proposed the analysis and design methodology of the B14 RUSM using a numerical method (3-D FEM) combined with an analytic method taking the contact mechanism into consideration in a linear operation. Mohamed Bakari *et al.* [31] designed a two-arm mobile delivery platform for application within nuclear decommissioning tasks. They examined the modelling and development of a real-time control method using Proportional-Integral-Derivative (PID) and Proportional-Integral-Plus (PIP) control algorithms in the host computer with National Instruments functions and tools to control the manipulators and obtain feedback through wireless communication.

The dynamics of a robot can be described by a set of coupled non linear equations in the form of gravitational

torques, Coriolis and centrifugal forces. The significance of these forces is dependent in the physical parameters of the robot, the load it carries and the speed, which the robot operates with. If accuracy is required then compensation for these parameter variations and disturbances becomes much more serious. Therefore, the design of the control system becomes much more complex. The theory of variable structure system (VSS) [28] is developed and applied to solve wide variety of applications in the control process essentially; it is a system with discontinuous feedback control. Operating such a system in sliding mode makes it insensitive to parameter variations and disturbances.

The rest of the article is organized as follows. Section 2 provides the formulation of the robot arm problem and its sub-section gives an idea about the basics of robot arm model problem with variable structure control and controller design. Section 3 deals with the RK-sixth order algorithm in detail and section 4 discusses about the stability analysis of the RK-sixth order algorithm. Finally discussion and conclusion are given in section 5.

2. Formulation of the problem

2.1. Robot arm model and essential of variable structure

It is well known that non-linearity and coupled characteristics are involved in designing a robot control system and its dynamic behaviour. A set of coupled non-linear second order differential equations in the form of gravitational torques; Coriolis and Centrifugal forces represent the dynamics of the robot. The importance of the above three forces are dependent on the two physical parameters of the robot namely the load it carries and the speed at which the robot operates. The design of the control system becomes more complex when the end user needs more accuracy based on the variations of the parameters mentioned above. A detailed version of a robot's structure with proper explanation is given in [21]. Keeping the objective *solving the robot dynamic equations in real time computation* in view, an efficient numerical technique is required. Taha [3] discussed about the dynamics of robot arm problem and it can be represented in the following form.

$$T = A(Q)\ddot{Q} + B(Q, \dot{Q}) + C(Q) \quad (1)$$

where $A(Q)$ indicates the coupled inertia matrix, $B(Q, \dot{Q})$ represents the matrix of Coriolis and centrifugal forces. $C(Q)$ denotes the gravity matrix, T is the input torques applied at various joints.

For a robot with two degrees of freedom, by considering lumped equivalent massless links, i.e. it means point load or in this case the mass is concentrated at the end of the links, the dynamics are represented by

$$\begin{aligned} T_1 &= D_{11}\ddot{q}_1 + D_{12}\ddot{q}_2 + D_{122}(\ddot{q}_2)^2 + D_{112}(\dot{q}_1\dot{q}_2) + D_1 \\ T_2 &= D_{21}\ddot{q}_1 + D_{22}\ddot{q}_2 + D_{211}(\dot{q}_1)^2 + D_2 \end{aligned} \quad (2)$$

where

$$\begin{aligned} D_{11} &= (M_1 + M_2)d_2^2 + 2M_2d_1d_2 \cos(q_2) \\ D_{12} &= M_2d_2^2 + M_2d_1d_2 \cos(q_2) \\ D_{21} &= D_{12} \\ D_{22} &= M_2d_2^2 \\ D_{112} &= -2M_2d_1d_2 \sin(q_2) \\ D_{122} &= -M_2d_1d_2 \sin(q_2) \\ D_{211} &= D_{122} \\ D_1 &= [(M_1 + M_2)d_1 \sin(q_1) + M_2d_2 \sin(q_1 + q_2)]g \\ D_2 &= [M_2d_2 \sin(q_1 + q_2)]g \end{aligned}$$

The values of the robot parameters used are $M_1 = 2$ kg, $M_2 = 5$ kg, $d_1 = d_2 = 1$ m. In the case of problem of set point regulation the state vectors are represented as

$$X = (X_1, X_2, X_3, X_4)^T = (q_1 - q_{1d}, \dot{q}_1, q_2 - q_{2d}, \dot{q}_2)^T, \quad (3)$$

where:

q_1 and q_2 are the angles at joints 1 and 2 respectively, and q_{1d} and q_{2d} are constants. Hence, equation (2) may be written in state space representation as:

$$\begin{aligned} \dot{e}_1 &= x_2 \\ \dot{x}_2 &= \frac{D_{22}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \\ &\quad - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2) \\ \dot{e}_3 &= x_4 \\ \dot{x}_4 &= \frac{-D_{12}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \\ &\quad - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2) \end{aligned} \quad (4)$$

Here, the robot is simply a double inverted pendulum and the Lagrangian approach is used to develop the equations.

It is observed that by selecting the suitable parameters, the non-linear equations (4) of the two-link robot-arm model may be reduced to the following system of linear equations [3]:

$$\begin{aligned} \dot{e}_1 &= x_2 \\ \dot{x}_2 &= B_{10}T_1 - A_{11}x_2 - A_{10}e_1 \\ \dot{e}_3 &= x_4 \\ \dot{x}_4 &= B_{20}T_2 - A_{21}x_4 - A_{20}e_3 \end{aligned} \quad (5)$$

where the values of the parameters concerning the joint-1 are given by:

$$A_{10} = 0.1730, A_{11} = -0.2140, B_{10} = 0.00265$$

and the values of parameters concerning the joint-2 are given by:

$$A_{20}= 0.0438, A_{21} = 0.3610, B_{20} = 0.0967$$

Choosing $T_1=\alpha$ (constant) and $T_2=\beta$ (constant), it is not possible to find the complementary functions of equation (5) because the nature of the roots of auxiliary equations (A. Es) of (5) is unpredictable. Owing to this reason and for simplicity, we take $T_1 = T_2 = 1$.

Considering, $q_1 = q_2 = 0, q_{1d} = q_{2d} = 1, \dot{q}_1 = \dot{q}_2 = 0$, the initial conditions are

$$[e_1(0) \ e_2(0) \ e_3(0) \ e_4(0)]^T = [-1 \ 0 \ -1 \ 0]^T.$$

The corresponding exact solution is given by:

$$e_1(t) = e^{0.107t} [-1.15317919 \cos(0.401934074 t) + 0.306991074 \sin(0.401934074 t)] + 0.15317919$$

$$e_2(t) = e^{0.107t} [0.463502009 \sin(0.401934074 t) + 0.123390173 \cos(0.401934074 t)] + e^{0.107t} [-1.15317919 \cos(0.401934074 t) + 0.306991074 \sin(0.401934074 t)]$$

$$e_3(t) = 1.029908976 e^{-0.113404416t} - 6.904124484 e^{-0.016916839t} + 4.874215508$$

$$e_4(t) = -0.116795962 e^{-0.113404416t} + 0.116795962 e^{-0.016916839t} \tag{6}$$

The exact solutions (Equation (6)) are similar to the solutions presented in [2].

3. Outline of RK-Sixth order Algorithm

The general s-stage RK method for solving an Initial Value Problem

$$y' = f(x, y) \tag{7}$$

with the initial condition $y(x_0) = y_0$ is defined by

$$y_{n+1} = y_n + h \sum_{i=1}^s b_i k_i$$

where $k_i = f(x_n + c_i h, y_n + h \sum_{j=1}^s a_{ij} k_j)$ and $c_i = \sum_{j=1}^s a_{ij}$,

$i = 1, 2, \dots, s$ with c and b are s-dimensional vectors and $A(a_{ij})$ be the $s \times s$ matrix. The RK- Sixth order array is of the form,

c_1	a_{11}			
c_2	a_{21}	a_{22}		
c_3	a_{31}	a_{32}	a_{33}	
\cdot	\cdot	\cdot	\cdot	
\cdot	\cdot	\cdot	\cdot	
\cdot	\cdot	\cdot	\cdot	
c_s	a_{s1}	$a_{s2} \dots a_{s-1}$	a_{ss}	
	b_1	$b_2 \dots b_{s-1}$	b_s	

For Equation (7) the form of the the RK-sixth-order algorithm is stated as follows

$$\begin{aligned}
 k_1 &= hf(x_n, y_n), \\
 k_2 &= hf(x_n + \frac{1}{2}, y_n + \frac{k_1}{2}), \\
 k_3 &= hf(x_n + \frac{2}{3}, y_n + \frac{2k_1}{9} + \frac{4k_2}{9}), \\
 k_4 &= hf(x_n + \frac{1}{3}, y_n + \frac{7k_1}{36} + \frac{2k_2}{9} - \frac{k_3}{12}), \\
 k_5 &= hf(x_n + \frac{5}{6}, y_n - \frac{35k_1}{144} - \frac{55k_2}{36} + \frac{35k_3}{48} + \frac{15k_4}{8}), \\
 k_6 &= hf(x_n + \frac{1}{6}, y_n - \frac{k_1}{360} - \frac{11k_2}{36} - \frac{k_3}{8} + \frac{k_4}{2} + \frac{k_5}{10}), \\
 k_7 &= hf(x_n + h, y_n - \frac{41k_1}{260} + \frac{22k_2}{13} + \frac{43k_3}{156} - \frac{118k_4}{39} + \\
 &\quad + \frac{32k_5}{195} + \frac{80k_6}{39}), \tag{8}
 \end{aligned}$$

The corresponding RK-sixth-order array to represent Equation (8) takes form as follows:

$$\begin{aligned}
 y_{n+1} = y_n + [&\frac{13}{200} k_1 + \frac{11}{40} k_3 + \frac{11}{40} k_4 + \frac{4}{25} k_5 + \\
 &+ \frac{4}{25} k_6 + \frac{13}{200} k_7] \tag{9}
 \end{aligned}$$

0						
$\frac{1}{2}$	$\frac{1}{2}$					
$\frac{2}{3}$	$\frac{2}{9}$	$\frac{4}{9}$				
$\frac{1}{3}$	$\frac{7}{36}$	$\frac{2}{9}$	$\frac{-1}{12}$			
$\frac{5}{6}$	$\frac{-35}{144}$	$\frac{-55}{36}$	$\frac{35}{48}$	$\frac{15}{8}$		
$\frac{1}{6}$	$\frac{-1}{360}$	$\frac{-11}{36}$	$\frac{-1}{8}$	$\frac{1}{2}$	$\frac{1}{10}$	
1	$\frac{-41}{260}$	$\frac{22}{13}$	$\frac{43}{156}$	$\frac{-118}{39}$	$\frac{32}{195}$	$\frac{80}{39}$
	$\frac{13}{200}$	0	$\frac{11}{40}$	$\frac{11}{40}$	$\frac{4}{25}$	$\frac{4}{25}$
						$\frac{13}{200}$

4. Stability Analysis

It is of importance to mention that one has to determine the upper limit of the step-size (h) in order to have a stable numerical solution of the given ordinary differential equation with IVP. Keeping this in view, we consider the test equation $\dot{y} = \lambda y$ where λ is a complex constant and it has been used to determine the stability region of the RK-Sixth order method.

$$k_1 = f(y_n) = \lambda y_n,$$

$$k_2 = f(y_n + \frac{hk_1}{2}) = \lambda y_n (1 + \frac{h\lambda}{2}),$$

$$k_3 = f(x_n + \frac{2}{3}, y_n + \frac{2k_1}{9} + \frac{4k_2}{9}) = \lambda y_n [1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})],$$

$$k_4 = f(x_n + \frac{1}{3}, y_n + \frac{7k_1}{36} + \frac{2k_2}{9} - \frac{k_3}{12}) = \lambda y_n [1 + \frac{7h\lambda}{36} + \frac{2h\lambda}{9} (1 + \frac{h\lambda}{2}) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))],$$

$$k_5 = f(x_n + \frac{5}{6}, y_n - \frac{35k_1}{144} - \frac{55k_2}{36} + \frac{35k_3}{48} + \frac{15k_4}{8}) = \lambda y_n [1 - \frac{35h\lambda}{144} - \frac{55h\lambda}{36} (1 + \frac{h\lambda}{2}) + \frac{35h\lambda}{48} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) + (\frac{15h\lambda}{8} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - 1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))],$$

$$k_6 = f(x_n + \frac{1}{6}, y_n - \frac{k_1}{360} - \frac{11k_2}{36} - \frac{k_3}{8} + \frac{k_4}{2} + \frac{k_5}{10}) = \lambda y_n [1 + \frac{h\lambda}{360} - \frac{11h\lambda}{36} (1 + \frac{h\lambda}{2}) + \frac{h\lambda}{48} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) + \frac{h\lambda}{2} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))], \frac{1}{10} h\lambda (1 - \frac{35h\lambda}{144} - \frac{55h\lambda}{36} (1 + \frac{h\lambda}{2}) + \frac{35h\lambda}{48} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) + (\frac{15h\lambda}{8} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})))],$$

$$k_7 = f(x_n + h, y_n - \frac{41k_1}{260} + \frac{22k_2}{13} + \frac{43k_3}{156} - \frac{118k_4}{39} + \frac{32k_5}{195} + \frac{80k_6}{39}) = \lambda y_n [1 + \frac{41h\lambda}{260} - \frac{22h\lambda}{13} (1 + \frac{h\lambda}{2}) + \frac{43h\lambda}{156} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) - \frac{118h\lambda}{39} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))] \frac{32}{195} h\lambda (1 - \frac{35h\lambda}{144} - \frac{55h\lambda}{36} (1 + \frac{h\lambda}{2}) + \frac{35h\lambda}{48} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))) + (\frac{15h\lambda}{8} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))) \frac{80}{39} h\lambda (1 + \frac{h\lambda}{360} - \frac{11h\lambda}{36} (1 + \frac{h\lambda}{2}) - \frac{h\lambda}{8} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})) + \frac{h\lambda}{2} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))) \frac{1}{10} h\lambda (1 - \frac{35h\lambda}{144} - \frac{55h\lambda}{36} (1 + \frac{h\lambda}{2}) + \frac{35h\lambda}{48} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2}))) + (\frac{15h\lambda}{8} (1 + \frac{7h\lambda}{36} + \frac{2}{9} h\lambda (1 + \frac{h\lambda}{2})) - \frac{h\lambda}{12} (1 + \frac{2h\lambda}{9} + \frac{4h\lambda}{9} (1 + \frac{h\lambda}{2})))].$$

Taking $z = h\lambda$, we have

$$k_1 = f(y_n) = \lambda y_n, \quad k_2 = f(y_n + \frac{hk_1}{2}) = \lambda y_n (1 + \frac{z}{2}),$$

$$k_3 = f(x_n + \frac{2}{3}, y_n + \frac{2k_1}{9} + \frac{4k_2}{9}) = \lambda y_n [1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})],$$

$$k_4 = f(x_n + \frac{1}{3}, y_n + \frac{7k_1}{36} + \frac{2k_2}{9} - \frac{k_3}{12}) = \lambda y_n [1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2}) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))],$$

$$k_5 = f(x_n + \frac{5}{6}, y_n - \frac{35k_1}{144} - \frac{55k_2}{36} + \frac{35k_3}{48} + \frac{15k_4}{8}) = \lambda y_n [1 - \frac{35z}{144} - \frac{55z}{36} (1 + \frac{z}{2}) + \frac{35z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})) + (\frac{15z}{8} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2})) - 1 + \frac{2h\lambda}{9} + \frac{4z}{9} (1 + \frac{z}{2}))],$$

$$k_6 = f(x_n + \frac{1}{6}, y_n - \frac{k_1}{360} - \frac{11k_2}{36} - \frac{k_3}{8} + \frac{k_4}{2} + \frac{k_5}{10}) = \lambda y_n [1 + \frac{z}{360} - \frac{11z}{36} (1 + \frac{z}{2}) + \frac{z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})) + \frac{z}{2} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2})) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))] \frac{1}{10} z (1 - \frac{35z}{144} - \frac{55z}{36} (1 + \frac{z}{2}) + \frac{35z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) + (\frac{15z}{8} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2})) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})))].$$

$$\begin{aligned}
 k_7 = f(x_n + h, y_n - \frac{41k_1}{260} + \frac{22k_2}{13} + \frac{43k_3}{156} - \frac{118k_4}{39} + \frac{32k_5}{195} + \frac{80k_6}{39}) = \lambda y_n [& 1 - \frac{41z}{260} + \frac{22z}{13} (1 + \frac{z}{2}) + \frac{43z}{156} (1 + \frac{2z}{9} + \frac{4z}{9} \\
 (1 + \frac{z}{2})) - \frac{118z}{39} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2}) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) & \frac{32z}{195} (1 - \frac{35z}{144} - \frac{55z}{36} (1 + \frac{z}{2}) + \frac{35z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) \\
 + (\frac{15z}{8} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2}) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})))) & \frac{80z}{39} (1 + \frac{z}{360} - \frac{11z}{36} (1 + \frac{z}{2}) - \frac{z}{8} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) \\
 + \frac{z}{2} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2}) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) & \frac{z}{10} (1 - \frac{35z}{144} - \frac{55z}{36} (1 + \frac{z}{2}) + \frac{35z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2}))) \\
 + (\frac{15z}{8} (1 + \frac{7z}{36} + \frac{2z}{9} (1 + \frac{z}{2}) - \frac{z}{12} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})))) &] \lambda y_n [1 + \frac{z}{360} - \frac{11z}{36} (1 + \frac{z}{2}) + \frac{z}{48} (1 + \frac{2z}{9} + \frac{4z}{9} (1 + \frac{z}{2})))] \quad (10)
 \end{aligned}$$

Substituting the expressions of $k_1, k_2, k_3, k_4, k_5, k_6$ and k_7 into equation (9) one can obtain as,

$$\begin{aligned}
 y_{n+1} = y_n + \frac{h\lambda y_n}{2160} [& 2160 + 2160z + 1080z^2 + 360z^3 + \\
 + 90z^4 + 18z^5 + 3z^6 - z^7] \quad (11)
 \end{aligned}$$

From equation (11), the stability of the polynomial $Q(z) = y_{n+1} / y_n$ becomes,

$$Q(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{120} + \frac{z^6}{720} - \frac{z^7}{2160} \quad (12)$$

In a similar manner, the stability polynomial for the test equation $\dot{y} = \lambda y$ (λ is a complex constant) using the RK-Butcher method has been obtained as

$$Q_1(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{120} + \frac{z^6}{640} \quad (13)$$

At this juncture, it is pertinent to point out that Murugesan *et al.* [1], Murugesan and Murugesan [32], Park *et al.* [33], Park *et al.* [34] and Sekar *et al.* [2] have obtained in-correct stability polynomial for the same test equation by adapting the RK-Butcher method and it is given by

$$Q_2(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{360} - \frac{853z^5}{19008} + O(z^6) \quad (14)$$

The stability polynomial for the same test equation using the RKAM method is given by

$$Q_3(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} \quad (15)$$

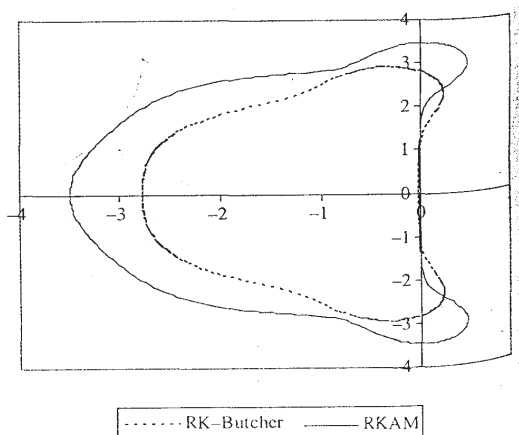


Fig. 1. Uncorrected stability region for RKAM and RK-Butcher Algorithm drawn by Murugesan *et al.* [1]; horizontal and vertical Axes represent Real Part of λh and Imaginary Part of λh .

Murugesan *et al.* [1], Murugesan and Murugesan [32], Park *et al.* [33], Park *et al.* [34] and Sekar *et al.* [2] have made a wrong comparative study of the stability regions of the RKAM and the RK-Butcher methods using uncorrected version of the stability polynomial (Equation (14)) obtained by them. Further, the stability region of the RKAM is also wrongly illustrated. For more detail, see Figure. 1 presented by Murugesan *et al.* [1], Murugesan and Murugesan [32], Park *et al.* [33], Park *et al.* [34] and Sekar *et al.* [2]. Also, they have made a critical mistake to determine the range for the real part of λh in the cases of RKAM and the RK-Butcher methods. The wrong range for the real part of λh in RK-fourth order (RKAM) is $-3.463 < \text{Re}(z) < 0.0$ and $-2.780 < \text{Re}(z) < 0.0$ in the RK-Butcher method. Similar types of severe mistakes have been detected in the paper authored by (Sekar *et al.* [2]). In view of this, we have presented the corrected version of the stability region of the RKAM and the RK-Butcher methods which are shown in Figures 2 and 3.

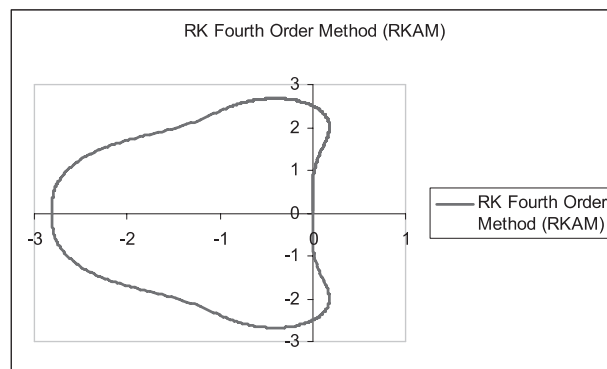


Fig. 2. Corrected stability region for RK Fourth Order Method (RKAM): horizontal and vertical axes represent Real Part of λh and Imaginary Part of λh .

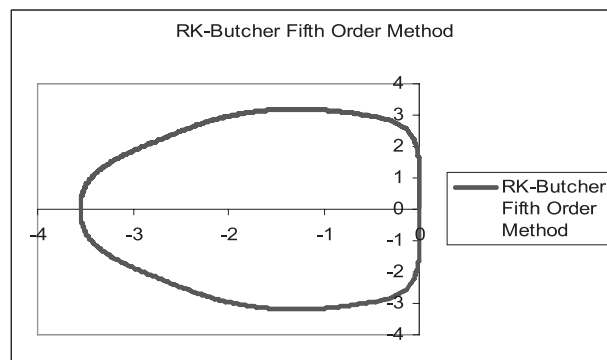


Fig. 3. Corrected stability region for RK-Butcher Fifth Order Method: horizontal and vertical axes represent Real Part of λh and Imaginary Part of λh .

In these stability regions, the range for the real part of λh in RKAM is $-2.780 < \text{Re}(z) < 0.0$ and $-3.463 < \text{Re}(z) < 0.0$ in the RK-Butcher algorithm.

5. Discussion and Conclusion

Using equations (5) and (9), the discrete and exact solutions of the robot arm model problem have been computed for different time intervals are depicted in Tables 1-4. The values of $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ are calculated for time t arranging from 0.0 to 1. The absolute error between the exact and discrete solutions for the RK methods based on RK-Fourth-order, RK-Fifth-order and RK-Sixth-order are calculated. For time $t = 0.0, 0.25, 0.05, 0.75$ and 1.0 the values are tabulated in Tables 1-4, respectively. It is pertinent to point out here that the obtained discrete solutions for the Robot Arm model problem using the RK-Sixth-order algorithm guarantees more accurate values compared to the classical RKAM method and RK-Fifth-order method. The numerical solutions computed by the RK-Sixth-order algorithm are in very close to the exact solutions of the robot arm model problem whereas the RKAM method gives rise to a considerable error. Hence this RK-Sixth-order algorithm is found to be more suitable for studying the Robot Arm model problem. It is of interest to mention that effort has been made to obtain the true stability polynomial for the test equation (considered in the present paper) using RK-Butcher method and the correct range for the real part of λh in cases of RK-Butcher algorithms and RKAM algorithms. Further, stability polynomial for the test equation by adapting RK-Sixth order formula has been obtained.

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URL: <http://www.nitt.edu>.

AUTHORS

Ramasamy Ponalagusamy, Sukumar Senthilkumar* - Department of Mathematics, National Institute of Technology, Tiruchi-rappalli-620 015, Tamilnadu, India.
E-mail: rpalagu@nitt.edu,
ssenthilkumar1974@yahoo.co.in.

* Corresponding author

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