# A HYBRID SYSTEM CONTROL APPROACH TO BIPED ROBOT CONTROL

Yingjie Yin, Shigeyuki Hosoe

**Abstract:**

*Human being can decide how to auto-adapt his motion to environmental changes. Comparing with this, the automation of the motion of biped robots is still very inefficient. In this paper, we review the state of the art of biped motion control in the field of hybrid system control. The main motivation is to fix the understanding of the research field and clarify strong and weak points of the available approaches. The presentation is illustrated by a typical example of biped machines.*

***Keywords:*** *biped robot, MLD model, periodic motion, hybrid system control.*

## 1. Introduction

The motion of a biped robot is naturally described as a hybrid dynamical system. Its behaviour is determined by interacting continuous and discrete dynamics. Just like living systems change posture (such as running, walking, balancing) variously according to the purpose of work, control also changes according to it. However, the problems of hybrid dynamical systems are inherently difficult because of their combinatorial nature.

In recent years, the control problems of biped motions have received considerable attention. The systems are tried to design from the hybrid system point of view. However, the understanding of hybrid systems is rather limited at present, and most of proposed approaches are schemes based on heuristic rules inferred from practical plant operation. New paradigm for the modelling of biped motions is required and systematic approaches for the synthesis of the biped system are expected.

An important control objective for biped locomotion systems is to generate reference motion trajectories that are consistent with underlying principles of hybrid movement, and then control the robot to track it. In this trend, a series of papers have been published (see [9], [11], [12], and [13]), where the periodic orbits of the bipeds are approximated by polynomials for numerical simplicity, and numerical algorithms are proposed to generate the trajectories. There are designed feedback controllers that enforce the biped robots to track the pre-planned trajectories.

The difficulties of this kind of approaches are that the movements are non-adaptable to environmental changes, and unexpected events are not pre-computable. Besides, the polynomial approximation of the joint trajectory may be relatively rough because of the dimension. In this meaning, despite the huge amount of works on the topic, walking/running control of biped robots is still inefficient especially in terms of stability, adaptability and robustness.

In contrast with this, human being can decide to auto-adapt his walking/running to environmental changes rather than predicting it a very long time ahead. A consideration inspired by human behaviour is to introduce the path changes on-line when stability and motion in environment are guaranteed. Towards this direction, in [2], the zero moment point (ZMP) was controlled for preserving the dynamic equilibrium. In [4], a trajectory, better adapted to the encountered situation, was chosen on-line amongst the sets of stored trajectories. In [10], a continuous set of parameterized trajectories was used as the candidate of choice. In [7], [8], [17], by optimizing the joint motion over a receding control horizon, biped robots are controlled without pre-computed reference trajectory such that the approaches are adaptable to the environment changes.

In this paper we summarize some of the currently hybrid control ideas for the motion control of biped robots. The main motivation is to fix understanding of the research field and clarify strong and weak points of the available approaches. In the consideration of motion planning then control, we propose that a hybrid external system can be used for producing a periodic locomotion pattern of running instead of the polynomial approximation. In the consideration of on-line motion control without predescribed trajectory, our main proposal is to express the biped motion as a unified modelling from the point of view of the hybrid system, which is called the mixed logic dynamical (MLD) model. Based on the MLD model, the motion planning problem of biped motion is formulated as an optimal control problem where the change of discrete configurations defines gait patterns. Finally, we conclude the proposal and promote some potential trends from the aspects of system control theory for the biped motion control.

## 2. Modelling of the biped motion

Consider a typical biped system shown in Fig.1. Motions are assumed to take place in the sagittal plane, consist of successive phases of single support, flight, and collision event without slip.

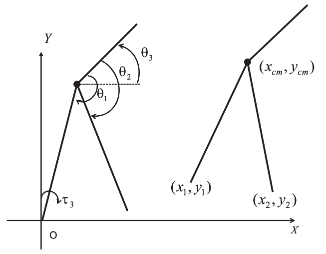


*Fig. 1. A typical biped robot with three links.*

### A. Dynamical equations of flight and stance

In the flight phase, the dynamical model is obtained by the method of Lagrange as

$$\begin{aligned} &M_f(\theta_b)\ddot{\theta} + C_f(\theta_b, \dot{\theta}_f)\dot{\theta} + G_f(\theta_f) = B_f \tau_f, \\ &m_o \ddot{x}_{cm} = 0, \\ &m_o \ddot{y}_{cm} + m_o g_0 = 0, \end{aligned} \tag{1}$$

where $\theta_f = [\theta^T x_{cm} y_{cm}]^T$ is the generalized coordinates in the flight phase, with $\theta = [\theta_1 \theta_2 \theta_3]^T$ the joint angles, and $(x_{cm}, y_{cm})$ the centre position of the mass. $\theta_b = [\theta_1 \theta_2]^T$ is the joint angles actuated in the flight phase, $\tau_f = [\tau_1 \tau_2]^T$ is the applied torque. $m_0$ is the mass of the biped. Note that $\tau_3$ is not applicable in flight, thus the joint $\theta_3$ is under-actuated.

In the stance phase, the end of the stance leg is fixed and the robot is fully actuated. The dynamical equation of stance is described as

$$M_s(\theta)\ddot{\theta} + C_s(\theta, \dot{\theta})\dot{\theta} + G_s(\theta) = B_s \tau_s. \tag{2}$$

where $\tau_s = [\tau_1 \ \tau_2 \ \tau_3]^T$.

### B. Motion transitions

An impact occurs when the advancing leg touches the ground, i.e. $y_2 = 0$. Under the assumption of inelastic impact, the post-impact velocities of angular joints are discontinous,

$$\dot{\theta}^{s+} = \bar{\Delta}(\theta^{f-})\dot{\theta}_f^-,$$

where $\dot{\theta}^{s+}$ is the initial value of $\dot{\theta}$ in the stance phase, $\theta^{f-}$ is the terminal value of $\theta$ in the flight phase, respectively. $\dot{\theta}_f^-$ is the terminal value of the generalized coordinates $\dot{\theta}_f$ in the flight phase.

The roles of the two legs are swapped after impact,

$$\theta^{s+} = R\theta^{f-},$$

where $\theta^{s+}$ is the initial value of $\theta$ in the stance phase. Therefore, the state transition from flight to stance is

$$\begin{bmatrix} \theta^{s+} \\ \dot{\theta}^{s+} \end{bmatrix} := \Delta_f^s(\theta_f^-, \dot{\theta}_f^-) = \begin{bmatrix} R\theta^{f-} \\ R\bar{\Delta}(\theta^{f-})\dot{\theta}_f^- \end{bmatrix}. \tag{3}$$

This transition takes place in an infinitesimal small length of time, hence there exists no double support phase.

On the other hand, the transition from stance to flight is initiated by accelerating the stance leg off the ground. Suppose the stance-to-flight transition occurs at a pre-determined state where $H_s^f(\theta, \dot{\theta}) = 0$. The transition is continuous in position and velocity,

$$\begin{bmatrix} \theta_f^+ \\ \dot{\theta}_f^+ \end{bmatrix} := \Delta_s^f(\theta^{s-}, \dot{\theta}^{s-}) = \begin{bmatrix} \theta^{s-} \\ f_{cm}(\theta^{s-}) \\ \dot{\theta}^{s-} \\ \frac{\partial}{\partial\theta} f_{cm}(\theta^{s-})\dot{\theta}^{s-} \end{bmatrix}, \tag{4}$$

where $f_{cm}(\theta) = (x_{cm}(\theta), y_{cm}(\theta))^T$.

### C. State equation expression

Introducing the state vector

$$x_f := \begin{bmatrix} \theta_f \\ \dot{\theta}_f \end{bmatrix}, \quad x_s := \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix},$$

the models of (1) and (2) along the state transitions of (3) and (4) can be expressed as

$$\Sigma_f : \begin{cases} \dot{x}_f = f_f(x_f) + g_f(x_f)\tau_f, & x_f \in X_f, \\ x_f^+ = \Delta_s^f(x_s^-), & x_s^- \in S_s^f, \end{cases} \tag{5}$$

$$\Sigma_s : \begin{cases} \dot{x}_s = f_s(x_s) + g_s(x_s)\tau_s, & x_s \in X_s, \\ x_s^+ = \Delta_f^s(x_f^-), & x_f^- \in S_f^s, \end{cases} \tag{6}$$

where $S_s^f, S_f^s$ are the hyper-surfaces of motion switching, $S_s^f = \{x_s : H_s^f(x_s) = 0\}$, $S_f^s = \{x_f : y_2(x_f) = 0\}$.

(5) and (6) formulate a hybrid system whose discontinuous behaviour is caused by a motion transition.

### D. A mixed logic dynamical model

The hybrid system of biped robots can be represented as a mixed logic dynamical (MLD) model by introducing logic auxiliary variable $\delta_f$, $\delta_s$ to associate the events of motion transition[17],

$$\begin{aligned} &\{x_s \in S_s^f\} \leftrightarrow \{\delta_f = 1\}, \\ &\{x_f \in S_f^s\} \leftrightarrow \{\delta_s = 1\}. \end{aligned} \tag{7}$$

By (7), the system of (5), (6) can be equivalently transformed to the following relations by the similar way as in [17],

$$\begin{aligned} &\dot{x} = A(t)x + B_\tau(t)\tau + B_z(t)z + B_\delta(t)\delta + b(t), \\ &E_1(t)x + E_2(t)\tau + E_3(t)z + E_4(t)\delta \le E_5(t), \end{aligned} \tag{8}$$

where $x = [x_f^T, x_s^T]^T$, $\delta = [\delta_f, \delta_s]^T$. $z$ is the continuous auxiliary variable introduced for variable linearization. The details of the coefficient matrices are omitted here.

The MLD model of (8) represents both the continuous motions and the discrete events in a unified framework, allows the synthesis of the hybrid system in a systematic way.

## 3. Progressing constraint

A successful walking/running should be a stable and successive progress forward. For that, conditions of stable and successive walking/running have to be taken into account as constraints subject to the hybrid dynamical equations.

### A. Erected body

The movable range of joint 1 is limited,

$$-\theta_1^* \le \theta_1 \le \theta_1^*, \tag{9}$$

which results in the hip position of the robot above a positive level to avoid falling.

### B. Progression

For propel the body in the intended direction with a progression rhythm, horizontal velocity of the swing toe should be kept positive. For that, we set

$$\dot{x}_2 \ge \epsilon_v \tag{10}$$

where $\epsilon_v > 0$.

### C. Environment

For walking/running on non-smooth ground, the surface is supposed to be known, expressed by a set of mathematical inequalities,

$$y = \psi(x). \tag{11}$$

Then the impact occurrence condition becomes $y_2 = \psi(x_2)$. In addition, the constraints of $y_1 \geq \psi(x)$, $y_2 \geq \psi(x)$ have to be added for generating the biped motion.

(9)~(11) can be included into the constraint inequality of (8). The resulted MLD model becomes a unified description for both the physical phenomena and the control constraints.

## 4. Motion control of biped robots by trajectory tracking

The motion control of biped robots is frequently performed by tracking a pre-defined periodic orbit. For that, the problem is generally separated into two steps: $(i)$ the synthesis of motion patterns[5], [9], [6] and $(ii)$ the control of the robot to track the prescribed trajectory [12], [11].

The problems of these approaches are that the movements are non-adaptable to environment changes, and unexpected events are not pre-computable.

### A. The periodic orbit of running

Denoted by $\mathcal{O}$, the periodic orbit of running consists of trajectories of the stance motion and the flight motion with impulse effect. The desired path of the stance motion is $(\theta_o^s, \dot{\theta}_o^s)$, which initiates from $(\theta_o^{s+}, \dot{\theta}_o^{s+})$ and terminates at $(\theta_o^{s-}, \dot{\theta}_o^{s-})$.

Since the takeoff of robots is continuous,

$$(\theta_o^{f+}, \dot{\theta}_o^{f+}) = (\theta_o^{s-}, \dot{\theta}_o^{s-}) \tag{12}$$

where $(\theta_o^{f+}, \dot{\theta}_o^{f+})$ is the initial of the flight phase. The position of the centre of mass at the takeoff point can be calculated by the terminal state of stance,

$$\begin{aligned} (x_{cm,o}^{s-}, y_{cm,o}^{s-}) &= f_{cm}(\theta_o^{s-}), \\ (\dot{x}_{cm,o}^{s-}, \dot{y}_{cm,o}^{s-}) &= \frac{\partial f_{cm}}{\partial \theta_o}(\theta_o^{s-})\dot{\theta}_o^{s-}. \end{aligned}$$

The desired path of the flight phase is $(\theta_o^f, \dot{\theta}_o^f)$ which initiates from $(\theta_o^{f+}, \dot{\theta}_o^{f+})$ and terminates at $(\theta_o^{f-}, \dot{\theta}_o^{f-})$. The ending of flight triggers the (next) initial of stance by impact event,

$$\begin{aligned} \theta_o^{s+} &= R\theta_o^{f-}, \\ \dot{\theta}_o^{s+} &= \Delta_{\dot{\theta}}(\theta_o^{f-})(\dot{\theta}_o^{f-}, \dot{x}_{cm,o}^{f-}, \dot{y}_{cm,o}^{f-}), \end{aligned} \tag{13}$$

where $\dot{x}_{cm,o}^{f-}$, $\dot{y}_{cm,o}^{f-}$ can be obtained by the time derivation of the following collision condition.

$$(x_{cm,o}^{f-}, y_{cm,o}^{f-}) = f_2(\theta_o^{f-}).$$

Therefore, the periodic orbit $\mathcal{O}$ is a closed-route consisting of two curves of $\theta_o^s$ and $\theta_o^f$. A pre-defined terminates of the stance phase, $(\theta_o^{s+}, \dot{\theta}_o^{s+})$ and $(\theta_o^{s-}, \dot{\theta}_o^{s-})$, determine the terminates of the flight phase,

$(\theta_o^{f+}, x_{cm,o}^{f+}, y_{cm,o}^{f+}, \dot{\theta}_o^{f+}, \dot{x}_{cm,o}^{f+}, \dot{y}_{cm,o}^{f+})$ and

$(\theta_o^{f-}, x_{cm,o}^{f-}, y_{cm,o}^{f-}, \dot{\theta}_o^{f-}, \dot{x}_{cm,o}^{f-}, \dot{y}_{cm,o}^{f-})$.

Consequently, the duration of the flight phase and the progressing distance of one step in running is also specified.

The curves of $\theta_o^s$ and $\theta_o^f$ which connect at the previous terminates can be chosen optimally by ensuring the control constraints of (9)~(11) while satisfying the boundary conditions of (12) and (13). In [9], [11], [12], [13], the $\theta_o^s$ and $\theta_o^f$ are approximated by a set of polynomials for numerical simplicity.

Here we remark that the paths of $\theta_o^s$ and $\theta_o^f$ can also be considered as the trajectories generated by a hybrid external dynamical system by the technique of [1], [16]. The trajectories can be produced according to some theoretical instruction, and they can be much smooth including rich information. The results on this topic will be reported in our coming publication.

### B. Trajectory tracking

For tracking the pre-planned periodic orbit, the controller design is accomplished by asymptotically regulating the tracking errors $e_s$, $e_f$ to zero,

$$\begin{aligned} e_s &= h_s(\theta) = \theta - \theta_o^s, \\ e_f &= h_f(\theta_b) = \theta_b - \theta_{o,b}^f, \end{aligned} \tag{14}$$

where $\theta_{o,b}^f$ is the first two elements of $\theta_o^f$ for the 3-link biped. The twice time derivation of $e_s$ along (6) is

$$\ddot{e}_s = L_{f_s}^2 h_s + (L_{g_s} L_{f_s} h_s)\tau_s. \tag{15}$$

The controller of

$$\begin{aligned} \tau_s &= \tau_{s,o} + v_s, \\ \tau_{s,o} &= -(L_{g_s} L_{f_s} h_s)^{-1} L_{f_s}^2 h_s \end{aligned} \tag{16}$$

results (15) to be

$$\ddot{e}_s = v_s.$$

Then a finite-time control law of $v_s$ renders $e_s$ to zeros with finite time before takeoff.

On the other hand, biped systems are underactuated in the flight phase. For the convenience of synthesis, a coordinate transformation

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} h_f(x_f) \\ L_{f_f} h_f(x_f) \\ \eta_1(x_f) \\ \eta_2(x_f) \end{bmatrix} := \Phi(x_f)$$

is used to transform the equation (5) to a normal form,

$$\dot{\xi}_1 = \xi_2, \tag{17}$$

$$\begin{aligned} \dot{\xi}_2 &= L_{f_f}^2 h_f(x_f) \mid_{x_f=\Phi^{-1}(\xi,\eta)} + \\ &+ L_{g_f} L_{f_f} h_f(x_f) \mid_{x_f=\Phi^{-1}(\xi,\eta)} \tau_f \\ &:= \beta(\xi,\eta) + \alpha(\xi,\eta)\tau_f, \end{aligned} \tag{18}$$

$$\dot{\eta}_1 = L_{f_f}\eta_1 + (L_{g_f}\eta_1)\tau_f, \tag{19}$$

$$\dot{\eta}_2 = L_{f_f}\eta_2 + (L_{g_f}\eta_2)\tau_f. \tag{20}$$

For the 3-link robot shown in Fig1, the joint $\theta_3$ is unactuated in the flight phase. Thus $\eta_1$, $\eta_2$ can be chosen to be

$$\eta_1(x_f) = \begin{bmatrix} \theta_3 \\ x_{cm} \\ y_{cm} \end{bmatrix}, \quad \eta_2(x_f) = \begin{bmatrix} \sigma_{cm} \\ \dot{x}_{cm} \\ \dot{y}_{cm} \end{bmatrix}, \tag{21}$$

where $\sigma_{cm}$ is the angular momentum of the biped about its centre of mass,

$$\sigma_{cm} = M_{f,3}(\theta_b)\dot{\theta}, \tag{22}$$

$M_{f,3}(\theta_b)$ is the 3rd row of the inertia matrix $M_f(\theta_b)$. It can be confirmed by (1) that

$$\dot{\sigma}_{cm} = 0. \tag{23}$$

(22) also implies that

$$\dot{\theta}_3 := \lambda(\xi_1, \xi_2, \sigma_{cm}) = = \frac{\sigma_{cm}}{M_{f,33}(\theta_b)} - \sum_{i=1}^{2} \frac{M_{f,3i}(\theta_b)\theta_i}{M_{f,33}(\theta_b)}. \tag{24}$$

By (21), (23), (24), and (1), the equations of (19), (20) become

$$\dot{\eta}_1 = \begin{bmatrix} \lambda(\xi_1, \xi_2, \sigma_{cm}) \\ \dot{x}_{cm} \\ \dot{y}_{cm} \end{bmatrix}, \quad \dot{\eta}_2 = \begin{bmatrix} 0 \\ 0 \\ -g_0 \end{bmatrix}. \tag{25}$$

Note that (25) is the internal dynamics of the biped system in the flight phase, which can not be directly regulated by $\tau_f$.

By the coordinate transformation, the tracking error $e_f$ becomes $e_f = \xi_1$, and it's twice time derivation is (18), i.e.,

$$\ddot{e}_f = \beta(\xi, \eta) + \alpha(\xi, \eta)\tau_f.$$

For the tracking control of the flight motion, the controller of

$$\tau_f = \tau_{f,o} + v_f, \quad \tau_{f,o} = -\alpha^{-1}(\xi, \eta)\beta(\xi, \eta) \tag{26}$$

results in $\ddot{e}_f = v_f$. A PD control of $v_f$ will stabilize $e_f$ exponentially at zeros,

$$\ddot{e}_f + K_D\dot{e}_f + K_P e_f = 0.$$

Note that the value of $\eta$ is required for the generation of $\tau_f$ in (26). $\eta$ is the internal variable vector evolving along the internal dynamics of (25), and depends on the initial state of the flight phase.

Concluding the previous discussion, we have the following results.

***Theorem:*** Suppose the biped robot is fully actuated in the stance phase, and a periodic orbit is pre-defined for the biped running. If the matrix $L_{g_s}L_{f_s}h_s$ and $L_{g_f}L_{f_f}h_f$ are invertible upon the periodic orbit, then the stance motion controller (16) and the flight motion controller (26) result in asymptotical tracking of the biped motion to the pre-defined periodic orbit, if and only if the settle time of the finite-time stabilizing feedback controller $v_s$ is shorter than the duration of the stance phase.

The zero-error manifold of the hybrid system is

$$Z := \{(x_s, x_f) \mid h_s(x_s) = 0,\ L_{f_s}h_s(x_s) = = 0,\ h_f(x_f) = 0,\ L_{f_f}h_f(x_f) = 0.\} \tag{27}$$

## 5. Motion control of biped robots without pre-defined trajectory

Human being can decide his walking/running to auto-adapt to environment changes rather than predicting it a very long time ahead. Inspired by this, an important control objective for biped locomotion systems is to determine the motion trajectory online for current state and changed environment, while guarantee the stable motion in environment. The advantage of motion control without the using of predicted trajectory is the adaptability of the motion to environment.

The control problem of on-line walking adaptation has been studied by some researchers. In [2], [3], the zero moment point (ZMP) was controlled for preserving the dynamic equilibrium. However, this adaptation can only involve small changes of the original trajectory. In [4], a trajectory better adapted to the encountered situation was chosen on-line amongst the sets of stored trajectories. However, the switching from one trajectory to another may lead to unexpected effects in control. To cope with this problem, in [10], a continuous set of parameterized trajectories was used as the candidate of choice. The switches were suppressed, but the set of trajectories has to be defined beforehand.

A consideration different with the previous approaches is to adapt the model predictive control (MPC) to the on-line walking adaptation. MPC is a control method for the problem of multivariable systems that are constrained in the state and/or control variables. In the past years, MPC is well applicated in refining or chemical industry, but few works are dedicated to process with short time response such as robotic system. In fact, for the problem of walking pattern synthesis of biped robot subject to unilateral constraints or disturbances due to an unstructured environment, MPC method is suitable and some experience has been obtained in [7], [8], and [17].

### A. The model predictive control of biped walking

In [7], the criterion used for minimization in the model predictive control (MPC) can be

$$J = \sum_{i=0}^{N-1} (\|x(k+i+1|k) - x_r\|_{Q_1} + \|\tau(k+i)\|_{Q_2}) \tag{28}$$

where $x_r$ is the reference state where impact occurs. $x(k+i+1|k)$ is the sampled state which can be predicted by the dynamics of (6) from the current state $x(k)$ and applied control. $k$ denotes the $k-th$ sampling time.

In the MPC approach, the future control inputs $\tau(k+i)$, $i = 1, \ldots, N-1$, are calculated which minimize the criterion of (28). Then, only $\tau(k)$ is applied to the biped robot for the next sampling time which results in the updated state $x(k+1)$. The time is shifted from $k$ to $k+1$, the length of the optimization horizon is shortened to $N = N - 1$ in [7], and the process of optimization is repeated.

The approach [7] avoided to take into account the impact as an interior point of optimization horizon. Therefore, the effect of impact cannot be treated positively. Furthermore, the walking adaptability to environment is poor because it shortens the prediction horizon each iteration until to $N = 1$. In [8], the optimization horizon $N$ is kept constant. However, how to predict the occurrence of impact then compensate positively for the effect of impact by the MPC approach is not stated clearly.

### B. The MPC of biped walking based on the MLD model

Remind that the MLD model (8) for the biped motion encapsulates phases of continuous motion, switching between types of motion, occurrence of impacts, and represents all of the features in a unified model. Therefore, based on the MLD model, the synthesis of a biped motion can be carried out in a systematic way; meanwhile the effect of impact can be taken into well account.

According to this consideration, the criterion for minimization is as the following in [17].

$$L = \sum_{i=0}^{N-1} (\|x(k+i+1|k) - x_r\|_{Q_1} + \|\tau(k+i)\|_{Q_2} + \|z(k+i) - z_r\|_{Q_3} + \|\delta(k+i) - \delta_r\|_{Q_4}) \tag{29}$$

which is subject to the MLD model of (8). $N$ is the horizon for optimization and is kept to be constant. $Q_1, \ldots, Q_4$ are weighting matrices. $x(k+i+1|k)$ is the future state of $x(k)$ predicted by the MLD model. $x_r$ is set as a desired state of pre-impact, which can be time-varying or time invariant. $z_r$ and $\delta_r$ are the desired auxiliary continuous and logical states corresponding to $x_r$.

In the case that an impact occurs within the horizon, the minimization of (29) implies that the state before the impact point is regulated to approach the pre-impact state, and the state after the impact point is controlled towards the impact of the next step. The optimal control is generally effective for impact occurred either within or outside the optimization horizon $N$. The second term at the right side of (29) implies the minimal input control.

The optimization of (29) is carried out over a receded horizon $N$. The biped robot is controlled with neither precomputed reference trajectory nor switches at the algorithm level. Thus the resulted walking can easily autoadapts to environment changes. The MPC of the MLD system can be solved using powerful mixed integer quadric programming (MIQP) algorithm. Its solution corresponds to the objective-oriented optimization of the gaits. Simulation results are reported in [17], which show that the MLD model based MPC approach leaded to faster walking with smaller torque.

Theoretically, the MLD model based MPC approach is also effective for the synthesis of the biped running. Numerical test is required to confirm its validation.

## 6. Conclusions

A review of the state of the art in the field of hybrid system control of biped robot has been undertaken in this paper. Despite the huge amount of works on the motion control of biped robot, the problem is still inefficient especially in terms of stability, adaptability and robustness. On the other hand, hybrid systems represent a highly challenging research area which has a lot of application in robotics. However, straight-forward application of available frameworks faces the limit of computational complexity and lucks the theoretical prediction of system properties. In this connection, approaches based on application of MLD look more attractive.

Also, a human uses his predictive function based on an internal model together with his feedback function for motion, which is considered as a motor control model of a cerebellum [15]. Stimulated by this, a general theoretical studies for motion control of hybrid systems are reported in [16], [14] which are based on the MLD model or the piece-wise linear system model. Further developing this kind of theory will be useful for the realization of complex motion of bio-mimetic robots. Finally, we mention that biologically-inspired solution, as those based on the reinforcement learning paradigm, can be of high potential if the appropriate minimal simulation models can be constructed and validated.

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**AUTHORS**

**Yingjie Yin** - Measurement & Instrumentation Engineering Div.5, Toyota Technical Development Corporation, 1-21 Imae, Hanamoto-cho, Toyota, Aichi, 470-0334 Japan. E-mail: yingjie.yin@mail.toyota-td.jp.

**Shigeyuki Hosoe** - RIKEN-TRI Collaboration Center for Human-Interactive Robot Research, the Institute of Physical and Chemical Research (RIKEN), Moriyama-ku, Nagoya, 463-0003 Japan. E-mail: hosoe@bmc.riken.jp.

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