METHODS FOR CLASSIFICATION OF TACTILE PATTERNS IN GENERAL POSITION BY HELMHOLTZ'S EQUATION

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Abstract:

This paper describes new and original methods of tactile pattern recognition in general position applying the solution of Helmholtz's equation for a tactile transducer. Three groups of methods have been formed, based on: (a) calculation of the A matrix eigen value, with the matrix being formed either from the whole pattern, or from the limit points; (b) the scalar characteristic distribution of the components of the pattern's A matrix; (c) the geometrical properties of the A matrix of the pattern. The patterns have been classified into five groups.

Keywords: *pattern recognition, Helmholtz´s equation, tactile image, tactile information.*

1. Introduction

The ability to make right decision and the orientation of intelligent robot in given surroundings are connected directly to the efficiency of the sensor system and the ability to process the obtained information. Here the processing of the tactile information is of major importance.

Several processing methods may be used. This paper deals with the processing of two-dimensional tactile patterns by solving Helmholtz's equation to create the tactile image that is in general position.

New methods of processing the primary tactile information can be used for the tactile transducer treated. Because of their universality, these methods can also be applied to other types of tactile transducers. In [6] solution of pattern recognition by basic pattern position is described. Then in this paper by general position.

2. Solution of the Helmholtz's equation

If we consider the part of the tactile sensor that comes into contact with the object as a diaphragm deformed by pressure, where a definite function determines the pressure distribution on the diaphragm, the methods for computing elastic diaphragms may be analogously used for determining the tactile pattern. The use of Helmholtz's equation, a special type of general partial differential equation, seems to be especially advantageous. Let us identify the pattern limit by the border C and the internal pattern zone by R. There Helmoltz's equation may be written in the form:

$$\Delta\Phi+\lambda\Phi=0 \quad \text{in } R$$
$$\Phi=0 \quad \text{in } C$$

where

λ - eigen value of continuous pattern,

Φ - pressure function on transducer area that $\Phi\neq 0$.

If the pattern is discretized by the sensor or transducers, Helmholtz's equation has to be solved in differential form. The continuous Laplace operator is transformed into the five-point one

$$\Delta\Phi(I,J)=\frac{1}{h^2}\left[\Phi(I+1,J)+\Phi(I-1,J)+\Phi(I,J+1)+ +\Phi(I,J-1)-4\Phi(I,J)\right]$$

or the nine-point one

$$\Delta\Phi(I,J)=\frac{1}{6h^2}[\Phi(I+1,J+1)+\Phi(I+1,J-1)+ +\Phi(I-1,J+1)+\Phi(I-1,J-1)+ +4\Phi(I+1,J)+4\Phi(I-1,J)+4\Phi(I,J+1)+ +4\Phi(I,J-1)-20\Phi(I,J)]$$

where

h - the separation of the sensor centers,

I, J - the coordinates of the sensor centers.

The matrix A characterizes the tactile pattern and is formed from the appropriate differential Laplace operator. The Helmholtz equation changes into the form:

$$\left[-\frac{1}{h^2}\mathrm{A}+\lambda_h\mathrm{E}\right]\vec{\Phi}=0$$

which is solved in form:

$$det(\Lambda E\text{-}A)=0$$

where

h - distance between sensors centre.

λ_h - eigen value of discretized pattern.

Λ - minimum eigen value of matrix A.

$\underline{A}$ - pattern matrix type NxN.

$\vec{\Phi}$ - vector of dimension N representing value of function Φ.

Three groups of methods are formed, based upon:

1. the computation of the eigen value of the matrix A:
   a) the matrix A was formed for the whole pattern;
   b) the matrix A was produced from the ground points after the internal points are filtered by use of the five- or nine-point Laplace operator;

2. the scalar characteristic distribution of the components of matrix A of the tactile pattern;
3. the geometric properties of the matrix A.

For all cases five pattern groups were selected: circles, rectangles, squares, rectangle triangles and isosceles triangles.

3. Methods using the eigen value of the matrix A

The plane is considered to be the pattern space. Therefore two features are necessary to indicate the tactile pattern. The first is the minimum eigen value Λ of the matrix A; the other must be carefully selected.

If we take into consideration that the eigen values depend upon the pattern area, the latter may be that area which unit is h^2. The dependence of Λ_5 (the minimum eigen value for the matrix A made by the five-point Laplace operator) upon the pattern area is the same for all patterns. Another feature may be the number N of activated sensors 'inside' the discretized pattern (in the area R_{hr} of the discretized pattern, see Fig. 1). Feature Λ_5 or Λ_9 (the minimum eigen value for the matrix A formed by the nine-point Laplace operator) may be chosen. In both cases the basic pattern groups are distinguishable.

Kharkevich [4] has proved that the edges of the pattern bring more information than its area. This has also been proved in [5]. With this knowledge, the number of activated sensors P_0 (black points) at the border C_{hr5} of the area R_{hr} (see Fig. 2) is chosen as the second feature. This border is computed from the matrix A formed by the five-point Laplace operator.

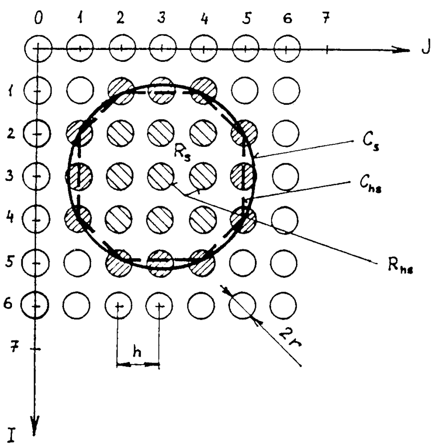

Fig. 1. The discrete pattern of a circle.

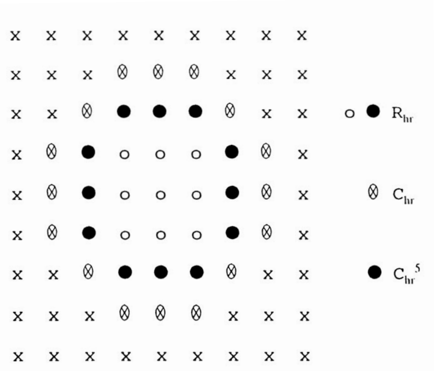

Fig. 2. Border P05 (black points) of the tactile pattern of a circle.

The matrix A can be formed by all points of the tactile pattern. Further improvement is reached if the inside points of the pattern are filtered off and the matrix A is formed from the five- or nine-point Laplace operator for border points only. These are obtained by applying both the operators mentioned. Different combinations of appropriate numbers and sensor numbers at the border may be formed.

4. Method using the scalar characteristic distribution of the components of matrix A

If we start from the knowledge that the vertices and edges are the points containing the most information and that the inside pattern points bring less information, another method can be formulated.

For the term "information brought by the pattern", the following relation may be presented by formula:

$$\widetilde{I} = -\sum_{i=1}^{N}\sum_{j=1}^{N} a(i,j)$$

where

$a(i,j)$ - elements of matrix A,
N - pattern sensor numbers (order of matrix A),
$\widetilde{I}$ - coefficient of pattern ability.

This relation satisfies the condition presented in the introduction of Section 3.

If we want to accent vertices and edges we use formula:

$$\widetilde{I} = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} a(i,j)\right)^2$$

The information measure in the pattern is expressed by

$$\widetilde{I}_k = \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} a(i,j)\right)^2$$

where

k - the Laplace operator used for forming the matrix A,
$\widetilde{I}_k$ - the norm coefficient of pattern ability.

5. Methods using the geometric properties of the matrix A

The operator being the representation that in our case transforms the matrix of activated sensors to the matrix A, the geometric properties of this matrix may be investigated.

The matrix A is always square and in order to get comparable results for different matrices, let us introduce a normalization, after which the matrix area will have unity dimension. If we suppose the elements of matrix A are the points in a plane, then after normalization the element with the coordinates (i,j) is transformed to (ii,jj). The normalization is given by the relations.

$$ii = \frac{i-1}{N-1}\ ; \qquad jj = \frac{j-1}{N-1}$$

Considering that the matrix A is symmetrical, which is a disadvantage in this case, because its centrum of gravity will always lie at the main diagonal, its lower triangular part will be used. The centrum of gravity coordinates may be expressed in two ways: in rectangular Cartesian coordinates $T(x,y)$ or in polar coordinates $T(V,\alpha)$, where V is the distance from the origin of coordinates (ii, jj) and a is the angle between the axis jj and the connecting line between the origin and the centrum of gravity T (see Fig. 3).

The elements of matrix A act as weighting coefficients during the computation. The centrum of gravity coordinates in the Cartesian system are determined according to the relations:

$$x = \frac{1}{N_o}\sum_{i=1}^{N}\sum_{j=1}^{i-1}\frac{i-1}{N_o-1}a(i,j)$$

$$y = \frac{1}{N_o}\sum_{i=1}^{N}\sum_{j=1}^{i-1}\frac{j-1}{N_o-1}a(i,j)$$

$$N_o = \sum_{i=1}^{N}\sum_{j=1}^{i-1}a(i,j)$$

where

N - the activated sensor number in the pattern area R_{hr} $(dim A = N)$,

N_o - elements number of down triangle normalized matrix A.

For polar coordinates we get

$$V = \sqrt{(x^2 + y^2}$$

$$\alpha = arctg\,\frac{x}{y}$$

By analogy with the foregoing cases, the matrix A may also be formed from the five- or nine-point Laplace operator, for either all the activated sensors or for the reduced matrix. The individual Cartesian coordinates of the polar centrum of gravity T in combination with a number P_{05} of activated sensors may be selected for marks.

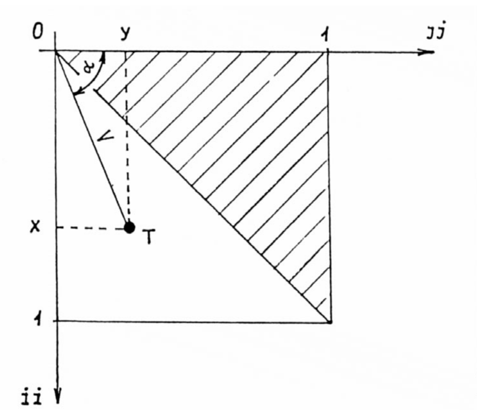

Fig. 3. The centrum of gravity of the normalized matrix A.

6. Way of making rotation patterns

Experimentally has been chosen pattern rotation about 4° in interval 180°. Self pattern rotation is executed by next rules:

1. *Determine pattern centrum of gravity:*

$$T_x = \frac{\sum_{i=1}^{N} L_{xi}}{N} \quad ; \qquad T_y = \frac{\sum_{i=1}^{N} L_{yi}}{N}$$

where

L_{xi} - distance i - point from axis y thorough coordinate basic origin,

L_{yi} - distance i - point from axis x thorough coordinate basic origin,

N - activation sensors number.

2. *Pattern completion by fictive points*

By computing and the pattern discretization, the various abnormality and teeth rise on patterns border. Then we insert other fictive points between original points that present active tactile sensors. By this way we increase texture and increase expressively rotate pattern quality. On Fig. 4 triplet of adjacent points (they are big and represent active tactile sensors) and created rectangle isosceles triangle we complete by next 18 points.

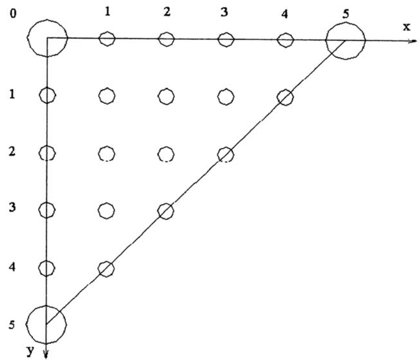

Fig. 4. Pattern completing by fictive points.

3. *Sequential rotation of all pattern points*

In last step cycle is executed, in which all pattern points are rotated (real and fictive) around its centrum of gravity step by step about 0, 4, 8, ..., 180°. We calculate features in each from these angles. E. q. for each feature we obtain (180°/4°) + 1 = 46 values. From these values we used only minimal, maximal or difference between maximal and minimal feature value.

After rotation calculation we decrease texture for next processing (calculation features). For next calculation we keep only points, which fulfill condition:

$$x\,mod\,5 = 0 \quad \text{together} \quad y\,mod\,5 = 0$$

where

x, y - are points coordinates (real and fictive) in are tactile sensor,

mod - residue after division (e.g. $6\,mod\,5 = 1$).

From pattern with decrease texture we calculate all features and then we make next rotation about 4°.

Analogously as irregularities and teeth rise on the pattern border by the pattern rotation from computer calculation, than holes rise inside of pattern from the same reason. Than we have to make three rules for correction turning pattern.

1. rule - if in axis y of tactile sensor area rises situation sensor - space - sensor then we insert sensor instead the space;
2. rule - if in axis x of tactile sensor area rises situation sensor - space - sensor then we insert sensor instead the space;
3. rule - if in diagonal of tactile sensor area rises situation sensor - space - sensor then we insert sensor instead the space.

Using of these rules shows Fig. 5

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Rule 1            Rule 2

X      X
-  =>  X          X - X => X X X
X      X

Rule 3

X          X             X          X
  -    =>  X      or       -    =>  X
    X          X       X          X
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Fig. 5. Rules for holes correction in pattern.

7. Classification and the used features

For classification we used net features. All were defined up.

α_9 - polar coordinate of centrum of gravity of normalize matrix A,
N - number of activate sensors in area R_{hr},
I_{t5} - scalar characteristic distribution of the components of matrix A,
P_{05} - number of active sensors on border R_{hr}, border is created by 5 point Laplace´s operator.

In front of these features we will write symbols min, max, dif. Min or max present minimal or maximal features value through its turn. Dif presents difference between maximal and minimal value.

In first step, by classification was used, features dependence $dif\,\alpha_9 = f(N)$. This dependence shows Fig. 6.

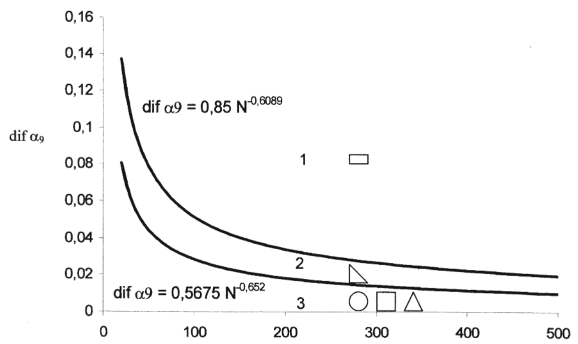

Fig. 6. Features dependence $dif\,\alpha_9 = f(N)$.

By two curves we select the graph to three parts. To area 1 the rectangle pattern class belongs, to area 2 rectangle triangles pattern class belongs and to area 3 remaining patterns classes (circles, squares, equilateral triangles).

For selecting of equilateral triangles from circles and squares the dependence min $I_{t5} = f(min\ P_{05})$. This dependence shows Fig. 7. The pattern classes of circles and squares are in area under curve and the pattern classes of triangles are up curve.

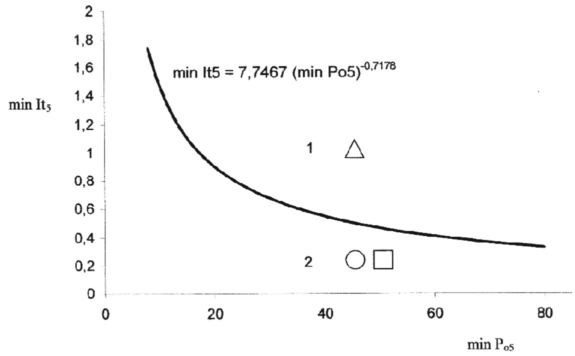

Fig. 7. Features dependence $min\, I_{t5} = f(min\, P_{05})$.

By combination of up procedures we can select four pattern classes: rectangle triangles, equilateral triangles, rectangles and last group create squares and circles. For differentiation of circles from squares we used dependence $dif\, I_{t5} = f(difP_{05})$, see Fig. 8.

By three features dependences (Fig. 6 to Fig. 8) we can right classify five pattern classes by probability dependent from inside sensors N. For inside sensors number $N \geq 35$ is right pattern recognition more as 90 %, for $N \geq 100$ is right pattern recognition about 99,9 %. Minimal size of pattern is $N=20$. In this case is right pattern recognition about 70%.

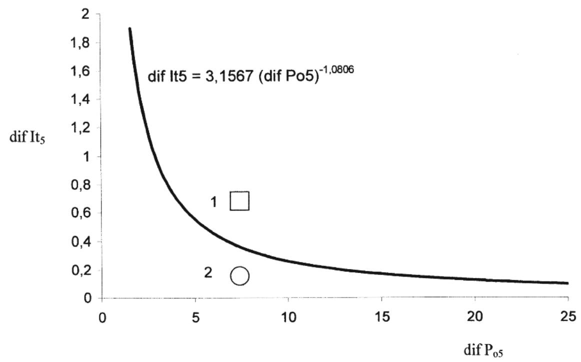

Fig. 8. Features dependence $dif\, I_{t5} = f(min\, P_{05})$.

8. Conclusion

This paper has dealt with the new and original methods for processing tactile information and distinguishing tactile patterns. These methods are based on the solution of Helmholtz's differential equation in discrete form.

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References

[1] Dyga R., Chart P., *Raspoznavanije obrazov i analyz scen*, Mir: Moscow 1976.

[2] Forsythe G.E., Wasow W.R., *Finite Difference Methods for Partial Differences Equations*, Wiley: New York, 1967.

[3] Pugh A., Robot Sensors, vol. 2, *Tactile and Nonvision Sensors*, Springer, Berlin, 1986.

[4] Kharkevich A.A., "O cennosti informacii", *Problemy Kibernetiky*, no.4, 1960, pp. 53-57.

[5] Kanajev E.M. and Shnejder A.J., *Principy postrojenija os-jazatelnych racpoznajuscich ustrojstv*, Mechanika Masin, Nauka, Moscow, 1974, pp. 29-32.

[6] Volf J., "Methods of Processing Tactile Information based on the Solution of Helmholtz's Equation", *Sensors and Actuators A. Physical*, vol 41, 1994, no. 1-3, ELSEVIER-SEQUOIA S.A.: Lausanne, Switzerland, SSDI 0924-4247(93)00648-N, pp. 174-179.