NUKLEONIKA 2003;48(Supplement 2):S21–S24

Space charge dominated beam transport in the K130 cyclotron injection line

Pauli Heikkinen

Abstract High intensity H^- beams are injected into the K130 cyclotron [1] for isotope production and for proton induced fission studies. Earlier, when protons were accelerated as positive ions, the beam intensity was limited by beam losses in the extraction system from the cyclotron. Stripping extraction of negative ions removed this limitation. However, now space charge effects in the injection beam line limit the beam intensity. At present, the maximum practical H^- beam intensity at the inflector is about 0.25 mA, which gives 40–50 μA of extracted proton beam. Calculations predict that the injection beam line from the ion source to the matching quadrupoles below the cyclotron can transfer about 1 mA of 6 keV H^- beam, which also was measured. The quadrupole section has a smaller transmission. Also a significant portion of the beam is lost during the last 2 m in the axial hole. General rules for maximum beam intensity as a function of beam line parameters such as beam tube aperture, distance of focusing elements, beam charge, mass and energy are given for different kinds of focusing systems (solenoids, FODO and FOFDOD quadrupole structures). As a conclusion, some suggestions to improve the transmission of the injection line are given.

Key words cyclotron • injection • space charge

P. Heikkinen
Department of Physics,
University of Jyväskylä,
P.O. Box 35, FIN-40014 University of Jyväskylä, Finland,
Tel.: +358 14/ 2602 414, Fax: +358 14/ 2602 401,
e-mail: pauli.heikkinen@phys.jyu.fi

Received: 14 October 2002, Accepted: 24 January 2003

Introduction

Coulomb repulsion limits the beam intensity in low energy beam lines such as injection lines. Ion sources give easily beam currents of several mA to several tens of mA. The easiest way of reducing space charge effects is to use high beam energy. However, the injection of the K130 cyclotron was designed mainly for low and medium intensity heavy ions, and space charge was not a problem. Later, high proton intensities were needed and space charge became the major limiting factor in beam intensity. The geometry of the cyclotron central region fixes the injection energy for each beam: for 30 MeV protons it is about 6 keV.

Methods

Theoretical background

In order to see the functional dependence of space charge force acting on a charged particle on the surface of the beam we derive the transport matrix for δs long drift with space charge force. The matrix transports a phase space point (x, x') as

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\delta s} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 .$$

For a very short distance δs we can clearly write

$$x_s = 1 \cdot x_0 + \delta s \cdot x_0'$$
$$x_s' = \frac{\partial x'}{\partial x} \cdot x_0 + 1 \cdot x_0' \; .$$

It only remains to find out the element C'. For simplicity, we consider a round beam with constant charge density. Assuming a small beam divergence, we have only radial force, which is due to electric field, generated by the beam itself. The beam current can be written as

$$I = \rho v A = \rho \pi r_0^2 v,$$

where ρ is the charge density, r_0 the beam radius and v the velocity. For non-relativistic particles we can write

$$I = \rho \pi r_0^2 \sqrt{\frac{2qU}{m}} \quad \text{or} \quad \rho = \frac{I}{\pi r_0^2}\sqrt{\frac{m}{2qU}},$$

where m and q are the particle mass and charge, respectively, and U is the accelerating voltage. Now we can write the electric field acting upon a charged particle at radius $r \le r_0$ using Gauss' Law (for a ΔL long beam element; the length cancels for the electric field)

$$\int \vec{E} \cdot \delta\vec{s} = E \cdot 2\pi r \cdot \Delta L = \frac{Q}{\varepsilon_0} = \frac{\pi r^2 \Delta L \rho}{\varepsilon_0}$$
$$\Rightarrow \quad E = \frac{r\rho}{2\varepsilon_0} = \frac{rI}{2\varepsilon_0 \pi r_0^2}\sqrt{\frac{m}{2qU}}.$$

The radial force is

$$F_r = qE_r = \frac{dp_r}{dt} = \frac{dp_r}{\delta s}\frac{\delta s}{dt} = \frac{dp_r}{\delta s} v = \frac{Ir}{2\pi\varepsilon_0 r_0^2} \cdot \sqrt{\frac{mq}{2U}}$$

and hence the increment of radial momentum is

$$\delta p_r = \frac{1}{v}\frac{Ir}{2\pi\varepsilon_0 r_0^2}\sqrt{\frac{mq}{2U}} \cdot \delta s = \frac{m}{U}\frac{Ir}{4\pi\varepsilon_0 r_0^2} \cdot \delta s.$$

Now we can write (with $p_s^2/2m = qU$) for the increment of divergence

$$\delta r' = \frac{\delta p_r}{p_s} = \frac{m}{U\sqrt{2mqU}}\frac{Ir}{4\pi\varepsilon_0 r_0^2} \cdot \delta s$$
$$= \sqrt{\frac{m}{2q}}\frac{Ir}{U^{3/2} 4\pi\varepsilon_0 r_0^2} \cdot \delta s$$

and hence the matrix element C' is

$$\frac{\partial x'}{\partial x} = \frac{\partial r'}{\partial r} = \sqrt{\frac{m}{2q}}\frac{I}{U^{3/2} 4\pi\varepsilon_0 r_0^2} \cdot \delta s.$$

Finally, we can write the transport matrix for a δs long drift as

$$\begin{pmatrix} r \\ r' \end{pmatrix}_{\delta s} = \begin{pmatrix} 1 & \delta s \\ \sqrt{\frac{m}{2q}}\frac{I}{U^{3/2} 4\pi\varepsilon_0 r_0^2} \cdot \delta s & 1 \end{pmatrix} \begin{pmatrix} r \\ r' \end{pmatrix}_0 .$$

We note that the determinant is not unity but smaller. This is due to the fact that the off-axis particles gain energy due to the radial electric field. There is no axial force and hence the total energy is increased, which decreases the emittance. The defocusing due to space charge has to be compensated by focusing in the beam line. The derived matrix suggests that the maximum current, which can be transported through a given beam line is proportional to $\sqrt{\frac{q}{m}}U^{3/2}R^2$, where R is the beam tube radius. This will be shown in the next chapter.

Above, constant charge density was assumed. In practice this is not quite true. The charge density usually is close to a (truncated) Poisson distribution or it is parabolic. In these cases, the quadrupole component of defocusing decreases with radius. This leads to an S-shape phase space area, which has been measured at several laboratories. A hollow beam gives a similar result, but the direction of distortion is the opposite. In this report, we do not handle other than constant charge density.

Numerical methods

Space charge defocusing was studied numerically with TRACE 3-D, which is a part of PBO-Lab package [2]. It assumes constant charge distribution. The calculations were made for long beam lines, which consist of solenoids or quadrupoles. FODO and FOFDOD quadrupole structures were studied. Maximum beam currents were found for different q/m-ratios, accelerating voltages and R/L-ratios, where R is the beam tube radius and L the distance between focusing elements (center-to-center). Increasing the beam current from zero requires stronger focusing elements to get the same beam envelope. In these calculations periodic solution for beam parameters were used as to mimic very long transfer lines. At a certain value for beam current it is not anymore possible to keep the beam inside the beam tube. This is the maximum current due to space charge.

Results

Solenoids as thin lenses

The simplest focusing is a thin lens that has the same strength in all directions. Such an element is a solenoid. Let us assume a beam line with short lenses at a distance L from each other, which is shown in Fig. 1. The emittance in this calculation is negligible. It turns out that the maximum current corresponds to focal length $L/4$ as can be seen

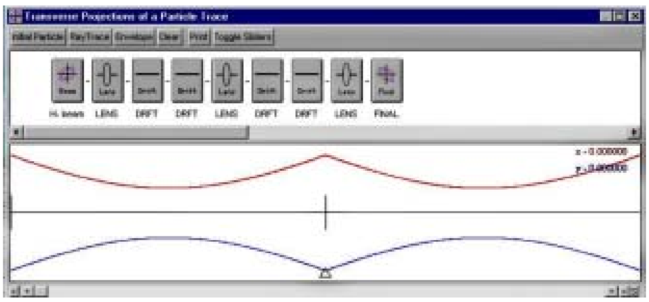

Fig. 1. Space charge limited beam focused by short solenoids (thin lenses).

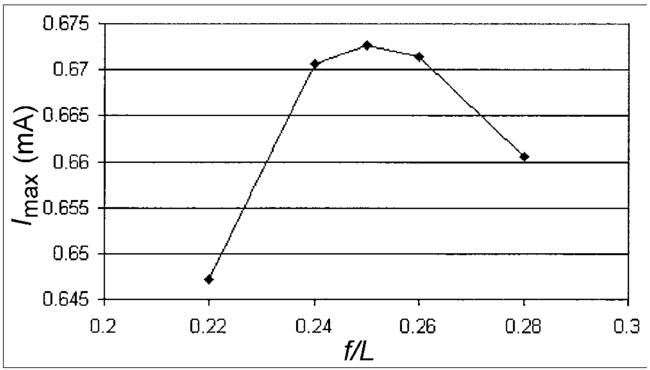

Fig. 2. Maximum beam current as a function of focal length-to-lens distance f/L for 8 keV protons in a beam line consisting of thin solenoid lenses (L = 2500 mm).

from Fig. 2. Using this focal length and changing parameters (charge state Q, mass number A, accelerating voltage U and radius to focal element distance ratio R/L) so that we get a periodic solution for the beam parameters, we finally get a law for the maximum beam current as

$$I_{\max} = 114.4\sqrt{\frac{\mathrm{Q}}{\mathrm{A}}}\,U^{3/2}\left(\frac{R}{L}\right)^2 \mathrm{mA}$$

for a solenoid system (thin lenses). Here the voltage U is given in kV. Beyond this current (proportionality factor 114.4) it is not possible to keep the beam within the given radius R with any focusing strength. This gives a maximum current of about 1 mA for 6 keV H^- beam (A = 1.008) when the beam tube has a radius of 50 mm and the solenoid distance (center-to-center) is 2000 mm. Clearly, if the thin lenses are replaced by real solenoids (length l) the purely defocusing section (drift) gets shorter and the maximum current increases.

Real solenoids

The main part of the injection line from ion sources to the K130 cyclotron consists of 400 mm long solenoids (2000 mm center-to-center). When we replace the thin lenses by real solenoids and use an emittance of 100π mm·mrad we get about 1.3 mA for the 6 keV H^- beam, which is roughly what we have measured. This gives then

$$I_{\max} = 140\sqrt{\frac{\mathrm{Q}}{\mathrm{A}}}\,U^{3/2}\left(\frac{R}{L}\right)^2 \mathrm{mA}$$

for a realistic 100π mm·mrad beam with 400 mm long solenoids.

Figure 3 shows the desired solenoid field for a 5.75 keV H^- beam in a solenoid system when a periodic solution has been found. As it can be seen, the space charge effects have to be taken into account also below the space charge limit. This also means that changes in the ion source current have an effect on beam phase space. This may cause problems if the extracted beam from the cyclotron is sensitive to the phase space of the beam in the central region of the cyclotron. The beam may, for example, hit undesired positions in the beam line when the ion source intensity is oscillating.

Quadrupole transfer lines

Sometimes an injection line consists of quadrupoles instead of solenoids. For long beam lines three main structures are used, FODO, FOFDOD or doublets. Here only FODO and FOFDOD structures are studied. In both systems, the beam envelope in one direction gets small at the defocusing quadrupoles (FODO) or between the defocusing quadrupoles (FOFDOD). The small beam size increases strongly defocusing ($1/r^2$ dependence in the matrix element). If a FOFDOD structure is described by thin lenses we get a maximum current as

$$I_{\max} = 17.2\sqrt{\frac{\mathrm{Q}}{\mathrm{A}}}\,U^{3/2}\left(\frac{R}{L}\right)^2 \mathrm{mA},$$

which is much smaller than for a solenoid system. If we then have a FODO structure with 200 mm long quadrupoles and 100π mm·mrad beam the maximum current becomes

$$I_{\max} = 29.5\sqrt{\frac{\mathrm{Q}}{\mathrm{A}}}\,U^{3/2}\left(\frac{R}{L}\right)^2 \mathrm{mA}.$$

The K130 cyclotron injection line

The K130 cyclotron injection line consists of solenoids, quadrupoles and dipoles. The beam intensity is decreased mainly at quadrupole sections. One matching quadrupole doublet is located in front of the first 32-degree dipole and another set of four quadrupoles is located under the cyclotron in front of the 90-degree dipole. Simulations show that with the present set-up it is not possible to increase the beam current from which has been reached (about 0.25 mA at the inflector). The maximum measured 6 keV H^- current at the end of the solenoid section corresponds to the simulated 1 mA including the quadrupole doublet and the dipole before the solenoids. The last 90-degree turn with matching quadrupoles cuts the beam intensity to about 0.5 mA.

Discussion

Numerical studies show that a solenoid transport system is more effective than a quadrupole system. The space charge

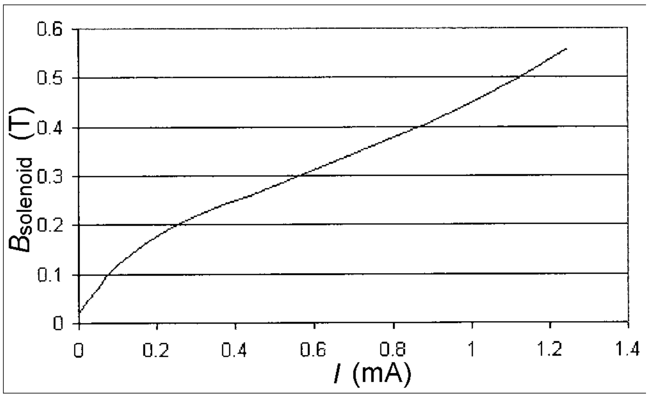

Fig. 3. Solenoid field as a function of 5.75 keV H^- beam current for the K130 cyclotron injection line: R = 50 mm, L_{drift} = 1600 mm, $L_{solenoid}$ = 400 mm, I_{max} = 1.25 mA.

limit can be increased by higher beam energy. Usually the cyclotron central region dictates the injection energy ($U^{3/2}$). Higher beam energy in the injection line calls for a decelerating system prior to the central region. If an existing beam line is to be upgraded for higher beam currents perhaps the most practical way of doing the upgrade is to replace as many quadrupoles with solenoids as possible or to increase the number of solenoids and hence shorten the focal distances. If, however, a new beam line is designed, attention should be paid to the tube diameter (R^2). There is also a trade-off between the number of matching quadrupoles and solenoids. Principally, four quadrupoles are needed to match four parameters (x, x', y, y'). In practice, this may lead to smaller beam intensity in spite of better phase space matching. Perhaps a good compromise would be to place a short solenoid in the middle of two quadrupoles and use only the solenoid with high beam current and the quadrupoles with low current when it is more crucial to match the beam emittance into the cyclotron acceptance.

References

1. Heikkinen P, Liukkonen E (2001) Cyclotron development program at Jyväskylä. In: Proc of the 16th Int Conf on Cyclotrons and their Applications held at NSCL/MSU, 13–17 May 2001, East Lansing, USA. AIP 600:89–93
2. Hill BW, Gillespie GH, Moore JM (2000) Using TRACE 3-D in the Particle Beam Optics Laboratory (PBO Lab). In: Proc of the 20th Int Linear Accelerator Conf, 21–25 August 2000, Monterey, USA, pp 824–826