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Minimum exposure path in an enclosure of randomly placed radioactive sources

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Abstract In this paper, two models to establish the minimum exposure path in a randomly placed radioactive sources enclosure are developed. The first model establishes the minimum exposure rate path and the second establishes the normal, to the inlet and outlet surfaces, path that gives a minimum exposure. The path that gives the least exposure is chosen. Although the point kernel technique is utilized and the enclosure is assumed rectangular with randomly placed radioactive point sources, the two models are independent techniques and can be easily incorporated with deterministic and statistical methods as well as other types of enclosures and source geometries.

Key words radiation protection • exposure minimization

Introduction

According to the International Commission of Radiation Protection (ICRP) recommendations, operations involving exposure to ionizing radiation should be designed so that any unnecessary exposure should be avoided and that all exposures should be kept as low as reasonably achievable (ALARA) with economical and social factors being taken into consideration. The ICRP recommends that this ALARA principle be implemented based on optimizing radiation protection efforts. Optimization is the balance attainment between the benefits of radiation protection obtained from the resources committed to radiation protection and the benefits obtained by committing these resources to other avenues [1–4].

It is sometimes mandatory to work in or pass through an area that has many randomly placed radioactive sources. Randomly placed sources are usually encountered in nuclear laboratories, radioactive waste sites, and areas of mountains that have high natural radioactivity levels in which roads are to be constructed through.

For such cases, stringent safety procedures have to be employed to minimize exposure of workers to radiation. One stringent safety condition is to provide a walkway (hereafter called the path) in the radiation enclosure that guarantees exposure rates when integrated over the path length give a minimum exposure value.

This condition, when implemented, guarantees that workers will receive minimized radiation exposures when passing through the radiation area.

In this paper, a model is developed to establish this path and to establish the location of the minimum exposure rate inside the radiation enclosure.

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In order to show the practicality of this paper, one example is given in which the enclosure is assumed rectangular. Although the enclosure is assumed rectangular with randomly placed point sources, the methodology presented here can easily be extended to model other types of enclosures and source geometries.

To extend the method to incorporate sources of other regular geometries, the point kernel method can still be used. However, when sources are of complicated geometries, the point kernel method may not warrant the desired accuracy. In such a case, any Monte Carlo code such as MCNP can be easily used with the developed algorithm.

It has to be mentioned that the presented model is a technique independent and other exposure calculation techniques whether deterministic or statistical can be used in conjunction with this model.

To clarify this, there is no dependency of the algorithms established in this paper on the exposure rate determination. The exposure rate can be found using any exposure finding technique and the models can then be applied. The exposure finding technique finds the exposure at grid points inside the enclosure and the minimum exposure rate path and the minimum normal path are found using the established algorithms.

Although, a homogeneous medium is assumed, the model is not restricted to homogeneous media. If the medium is of regular geometries, the point kernel method can be used, however, if the medium is complicated, having complex source geometries and/or backscattering is of importance, a Monte Carlo code such as MCNP can be used to find the exposure rates at all grid points.

The linearity of the differential equation in which the point kernel is derived from allows the summation (superposition) of exposure rates due to every point source calculated independently. This summation gives the total exposure rate at any chosen point inside the enclosure due to the contribution of all point sources.

Definition of variables

$\vec{r}_0$ – vector originating from a reference point to the point source (cm),
$\vec{r}$ – vector originating from a reference point to the points where exposure rates are to be determined (cm),
S – point source exposure rate strength (C cm^2)/(kg s) air,
μ – linear attenuation coefficient of the medium (cm^{-1}),
a – length of the enclosure (cm),
b – width of the enclosure (cm),
x – x Cartesian coordinate points inside the enclosure (cm),
y – y Cartesian coordinate points inside the enclosure (cm),
l – source index,
F_{tp} – total exposure rate (C/(kg s)) air,
dx – increment in the x-direction (cm),
dy – increment in the y-direction (cm),
ΔS – incremental length on the path (cm),
ν – average walking speed (cm/s),
F_s – superimposed exposure rate (C/(kg s)) air,
k – index for points on the minimum exposure path corresponding to i and j,
i – index for points on the x-direction where exposure is to be found,
j – index for points on the y-direction where exposure is to be found,
np – number of points on the minimum exposure rate path.

Methodology

The point kernel technique gives the un-collided exposure rate at any point in space due to a point source; this can be represented mathematically as:

$$F = \frac{S}{4\pi\left(\vec{r}-\vec{r}_0\right)^2}\exp\left(-\mu\left|\vec{r}-\vec{r}_0\right|\right) . \tag{1}$$

The total (superimposed) un-collided exposure rate (will be called hereafter exposure rate) at any point inside the enclosure due to n point sources is given by the following equation:

$$F_{s_{ij}} = \sum_{l=1}^{l=n}\frac{S_l}{4\pi\left(\vec{r}-\vec{r}_{0l}\right)^2}\exp\left(-\mu\left|\vec{r}-\vec{r}_{0l}\right|\right) \tag{2}$$

where S_l is the exposure rate strength of the point source l normalized to the highest exposure rate source.

In order to determine the minimum exposure rate location we need to:
minimize:

$$F_{s_{ij}} = \sum_{l=1}^{l=n}\frac{S_l}{4\pi\left(\vec{r}-\vec{r}_{0l}\right)^2}\exp\left(-\mu\left|\vec{r}-\vec{r}_{0l}\right|\right) \tag{3}$$

subject to: $$\begin{matrix} 0 \le x \le a \\ 0 \le y \le b \end{matrix} .$$

To establish the minimum exposure rate path inside the enclosure the following algorithm is implemented.

1. Choose the surface of the enclosure you want to have the inlet on. Here we assume the left hand-side surface of Fig. 1 to be our inlet surface.
2. Determine the minimum exposure rate location on this surface by minimizing equation (2) subject to the constraints: $0 \le y \le b$ and $x = 0$ i.e. ($i = 1$). This point will be the inlet of the enclosure.
3. March on the x direction with an increment of dx and minimize equation (2) similar to step 2 but now with $x = dx$, i.e. this will give the minimum exposure rate location on the marching line at $x = dx$.
4. Repeat step 3 by incrementing x every time by dx until the right hand-side surface is reached. The location of the minimum exposure rate on the right surface will be the outlet.
5. Record the minimum exposure rate and its corresponding (x, y) points for every marching line. These points, when connected form the path of minimum exposure rate.
6. To calculate the total exposure, we need to integrate the exposure rate over this established path and divide the integration by the average walking speed. The following equation fulfills this purpose:

$$F_{tp} = \frac{1}{\upsilon}\sum_{k=1}^{k=np} F_k \Delta S_k \tag{4}$$

where: ΔS_k is the path increment length for point k on the minimum exposure rate path and can be calculated using:

$$\Delta S_k = ((y_{i+1,j+1} - y_{i+1,j})^2 + (x_{i+1,j} - x_{i,j})^2)^{1/2}. \tag{5}$$

It has to be noted that this generated minimum exposure rate path is due to marching on the x-direction. Marching on the y-direction will generate a different minimum exposure rate path.

It is worth mentioning that there is infinite number of marching directions and consequently each marching direction gives a different minimum exposure rate path. The choice of the marching direction, however, is dictated by the nature of the radiation enclosure geometry and the existence of potential constraints. In the example of this paper, x and y marching directions were chosen.

The above algorithm guarantees that the determined path is the minimum exposure rate path in the chosen marching direction; however, it does not guarantee that it gives a minimum exposure when exposure rate is integrated over that path from inlet to outlet.

This is due to the fact that the minimum exposure rate path is usually of a curvature nature and requires more walking time than a path normal to the inlet and outlet surfaces.

Due to this reason, one has to investigate whether a normal path gives a total exposure less than that of the minimum exposure rate path.

The following algorithm shows how to establish the normal path that gives a minimum exposure. This path will be called from now on the minimum normal path:

1. Assume we start from the lower surface of Fig. 1. The lower surface is our current normal path.
2. Calculate the exposure rates on the lower surface (at $y = 0$) of the enclosure at increments of dx's in the x-direction.
3. Calculate the exposure by integrating over the lower surface and record this value.
4. Increment y by dy and calculate the exposure rate similarly at points of dx increments on the x-direction. Integrate exposure rate on this line.
5. Repeat step 4 until the upper surface is reached.
6. Among all exposures calculated, determine which normal path gives minimum exposure. This line will be the minimum normal path.

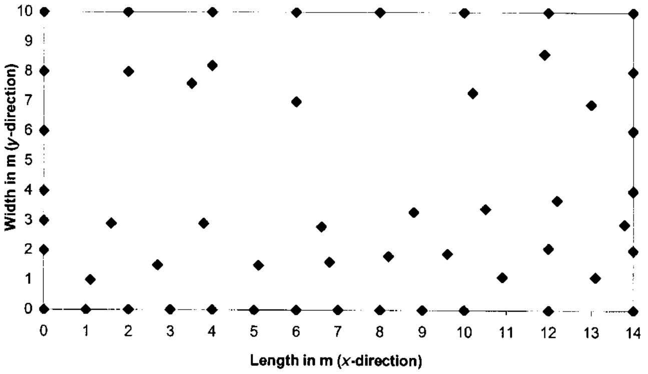

Fig. 1. Distribution of sources inside the enclosure.

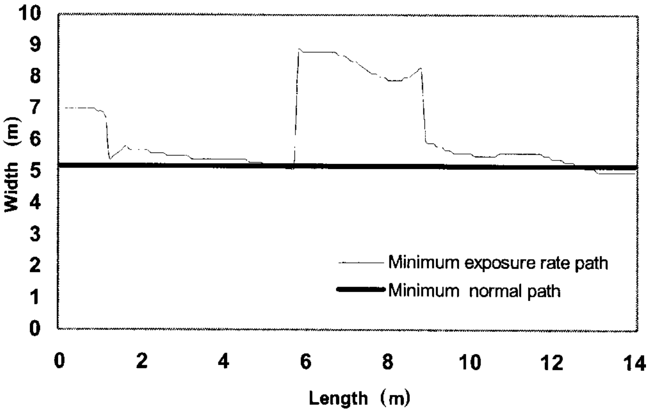

Fig. 2. Minimum exposure rate path and minimum normal path when marching in x-direction.

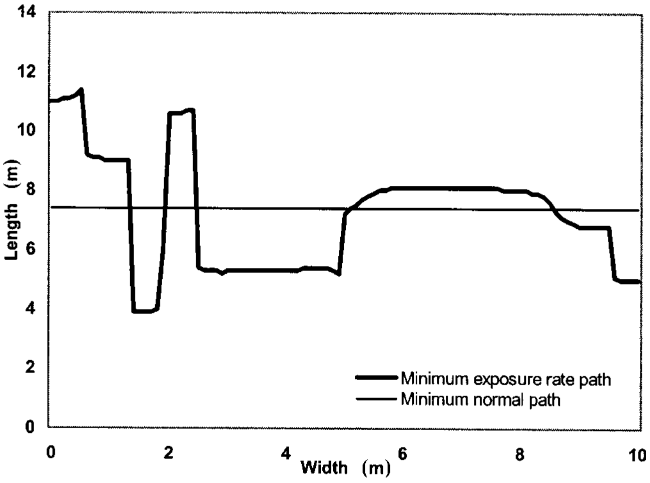

Fig. 3. Minimum exposure rate path and minimum normal path when marching in y-direction.

7. This minimum exposure is compared with the minimum exposure obtained from the first algorithm and the least is taken as the minimum exposure path.

Although the developed algorithms were applied on a two-dimensional enclosure, the same strategy can be followed when having a third dimension involved. To elaborate on this, let us assume that we have randomly placed sources inside a three-dimensional enclosure. The first step is to divide the enclosure into grid points in all dimensions and to find the exposure rate at every grid point due to all sources. It is known that when a human walks, he walks perpendicular to the ground (parallel to the z-direction) as if he is part of a line connecting the ground and the roof of the enclosure. If we assume a standard man height, the total exposure can now be found by integrating the exposure rate over each line from the ground to the standard man height. This can be accomplished by assuming that the standard man is standing on every single (x, y) grid points of the floor. The z-exposure dimension will collapse by integrating the exposure rate over the standard man. This will make the exposure known at every grid point on the xy plane. The minimum exposure rate path as well as the minimum normal path can now be found exactly similar to the two-dimensional case using the suggested algorithms.

Table 1. Normalized exposure rate strengths at coordinates (x, y)'s. x and y are in meters while S is normalized to the maximum exposure rate strength.

x	y	S	x	y	S	x	y	S	x	y	S	x	y	S
0	0	0.80	0	0	0.32	8.0	10.0	0.68	1.1	1	0.44	9	0	0.4
0	2	0.92	2	0	0.61	10.0	10.0	0.54	10.9	1.1	0.77	5	0	0.6
0	4	0.78	4	0	0.84	12.0	10.0	0.43	10.5	3.4	0.65	0	3	0.9
0	6	0.94	6	0	0.71	14.0	10.0	0.88	1.6	2.9	0.74	10.2	7.3	0.8
0	8	0.88	8	0	0.97	12.0	2.08	0.82	13.1	1.1	0.7	2	8	0.5
0	8	0.68	10	0	0.93	2.7	1.5	0.98	6.6	2.8	0.89	3.5	7.6	0.8
14	0	0.96	12	0	0.82	9.6	1.9	0.44	3.0	0	0.7	4	8.2	0.76
14	2	0.44	14	0	0.78	6.8	1.6	0.22	3.8	2.9	1	6	7	0.88
14	4	0.73	0	10	0.47	12.2	3.7	0.41	7.0	0	0.5	11.9	8.6	0.7
14	6	0.81	2	10	0.51	5.1	1.5	0.92	1.0	0	0.8			
14	8	0.83	4	10	25	8.2	1.8	0.55	8.8	3.3	0.9			
14	10	0.91	6	10	0.31	13.8	2.88	0.80	13.0	6.9	0.8			

Results and analysis

To show the practicality of the developed model, an example in which the two aforementioned algorithms are incorporated.

Example

An enclosure of 14 m length and 10 m width containing 57 sources distributed randomly with strengths and locations shown in Table 1. The source exposure rate strengths are normalized to 1. The attenuation coefficient of the medium is assumed to be 1×10^{-3} m^{-1} and the average walking speed, v, is assumed to be 1 m/s. It can be seen from Table 1 that there are several sources with the same coordinates but with different source strengths. This was done to check the functionality of the program and to make sure that it can sweep many sources at the same location. This attempt was made to model the possibility of having two or more sources having different strengths and energies.

Figure 1 shows the distribution of the point sources inside the enclosure.

After implementing the two algorithms discussed above, Figs. 2, 3, 4, and 5 were generated.

Figure 2 shows the minimum exposure rate path (first model) and the minimum normal path (second model) when marching on the x-direction. The first model gives an exposure of 3.1529, while the second model gives an exposure of 2.0861.

Similarly, Fig. 3 shows the minimum exposure rate path (first model) and the minimum normal path (second model) when marching on the y-direction. The first model gives an exposure of 8.2201 and the second model gives an exposure of 2.1068.

Although the first model gives a minimum exposure rate at every point on the path, the second model gives a minimum total exposure. This is, however, due to the large differences in lengths between the minimum exposure rate path and the minimum normal path. This large difference is very noticeable when marching in the y-direction.

Figure 4 shows the minimum exposure rate profiles and the minimum normal path when marching on the x-direction. Similarly, Fig. 5 shows the minimum exposure rate and the minimum normal path profiles when marching on the y-direction. The minimum exposure rate was found to be 0.1105 at x = 8 m and y = 7.9 m.

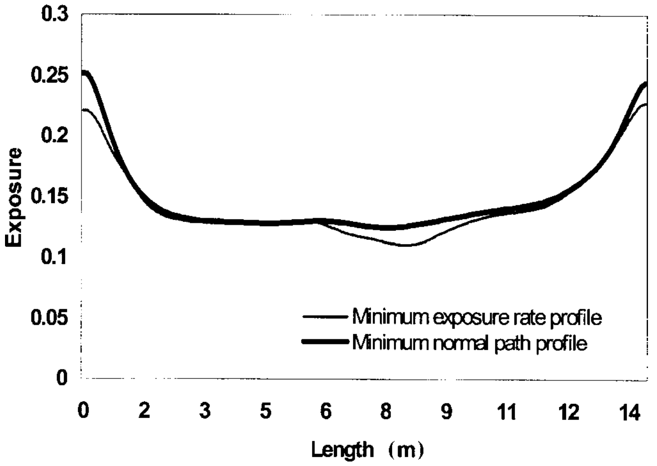

Fig. 4. Minimum x-direction exposure profiles.

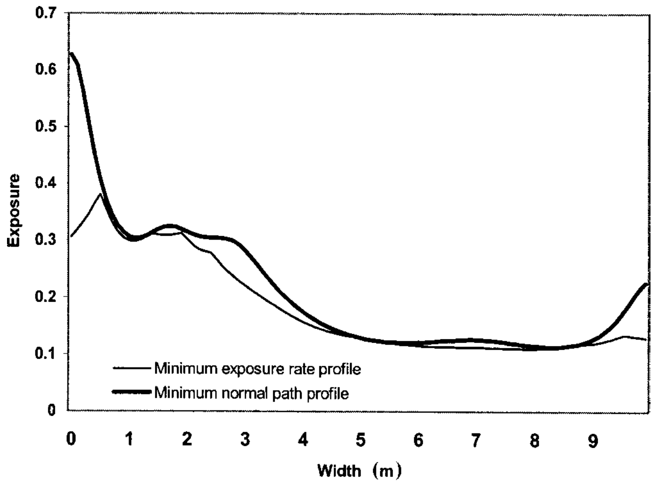

Fig. 5. Minimum y-direction exposure profiles.

Conclusion

Two models are incorporated in this paper; the first establishes the minimum exposure rate path. This minimum exposure rate path may not give a minimum total exposure due to its zigzagging nature, which may make it very long compared to paths that are normal to the inlet and outlet surface. The second model establishes the path, normal to the inlet and outlet surfaces, that gives a minimum exposure among all normal paths. The exposures obtained from the two models are then compared and the one that gives less exposure is chosen.

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