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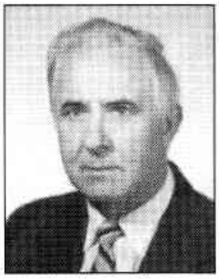
Full-order perfect observers for continuous-time linear systems



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Streszczenie

Podano nową koncepcję obserwatorów doskonałych pełnego rzędu dla ciągłych układów liniowych. Sformułowano warunki dostateczne istnienia obserwatorów doskonałych oraz podano procedurę wyznaczania tych obserwatorów.

Abstract

A new concept of the full – order perfect observer for continuous-time linear systems is presented. Conditions for the existence of the perfect observer are established and its design procedure is derived.

Key Words: full-order, perfect observer, continuous-time, linear system

1. Introduction

The observer problem for standard and singular (descriptor) continuous-time and discrete-time linear systems has been considered in many papers and books [1-6, 8-19]. Observers have many applications in state feedback control, system supervision and fault diagnosis. In the last decade great interest [1-3,5,10, 15-19] has been observed in studying observer problems of linear singular systems. Most investigations [3,4, 9-19] have been devoted to the Luenberger – type observers for singular systems while less attention has been devoted to singular observers [1-3,5]. Recently in [5] necessary and sufficient conditions have been established for the existence of a generalised observers for singular linear systems. Dai has shown [1,2] that it is possible to construct a singular observer which exactly reconstructs the state *x(k)* of the singular system $Ex(k+1)=Ax(k)+Bu(k)$, $y(k)=Cx(k)$ for all k=0,1,...

The main subject of this paper is to extend the Dai's concept of the perfect observer for standard continuous-time linear systems. Conditions will be established under which there exist the full – order perfect observer for standard continuous-time linear systems. A design procedure of the perfect observer will be derived and illustrated by a numerical example.

2. Perfect observer for singular systems

Consider the singular continuous-time linear system

$$E\dot{x}=Ax+Bu,\ \ x(0)=x_0 \tag{1a}$$

$$y=Cx \tag{1b}$$

where $\dot{x}=\frac{dx}{dt}$, $x(t)\in R^n$, $u=u(t)\in R^m$, $y=y(t)\in R^p$ are the state, input and output vectors respectively and $E,A\in R^{n\times n}, B\in R^{n\times m}, C\in R^{p\times n}$ and E=0.

Definition 1. The singular system

$$E\dot{\bar{x}}=A\bar{x}+Bu+K(C\bar{x}-y),\ \ \bar{x}(0)=\bar{x}_0 \tag{2}$$

is called full-order perfect observer of the system (1) if

$\bar{x}(t)=x(t)$ for $t>0$

and any initial conditions x_0 and $\bar{x}_0$ where $\bar{x}\in R^n$, u, y and *E, A, B, C* are the same as for (1) and $K\in R^{n\times p}$.

Lemma 1. [1,2,8] The matrix *K* can be chosen for the singular system (1) so that

$$\det[Es-(A+KC)]=\alpha\neq 0 \tag{3}$$

if and only if

$$rank\begin{bmatrix} E \\ C \end{bmatrix}=n \tag{4a}$$

and

$$rank\begin{bmatrix} Es-A \\ C \end{bmatrix}=n \quad \text{for all finite } s\in \mathbf{C} \text{ (the field of complex numbers)} \tag{4b}$$

where α is a constant independent of s.

Theorem 1. There exists a perfect observer (2) for the singular system (1) if the conditions (4) are satisfied

Proof. Let. $e = x - \hat{x}$ Then from (1) and (2) we obtain

$$E\dot{e} = E\dot{x} - E\dot{\hat{x}} = (A + KC)e \tag{5}$$

By Lemma 1 if the conditions (4) are satisfied then (3) holds and

$$[Es - (A + KC)]^{-1} = \sum_{i=-\mu}^{\infty} \Phi_i s^{-(i+1)} = \sum_{i=-\mu}^{-1} \Phi_i s^{-(i+1)}$$

and

$$\Phi_i = 0 \quad \text{for} \quad i \geq 0 \tag{6}$$

It is easy to show that (6) implies $e(t)=0$ for $t>0$. □

3. Perfect observer for standard systems

Consider the standard system

$$\dot{x} = Ax + Bu, \quad x(0) = x_0 \tag{7a}$$

$$y = Cx \tag{7b}$$

with the derivative output feedback

$$u = v - F\dot{y} = v - FC\dot{x} \tag{8}$$

where $F \in R^{m \times p}$ and v is the new input.
Substitution of (8) into (7a) yields the closed-loop system

$$E\dot{x} = Ax + Bv, \quad x(0) = x_0 \tag{9a}$$

$$y = Cx \tag{9b}$$

where

$$E := \mathbf{I}_n + BFC \tag{10}$$

Conditions will be established under which there exists a matrix F such that the matrix (10) is singular. Then applying the concept of perfect observer to the singular system (9) we may construct a full-order perfect observer for the standard systems (7).

Lemma 2. For the standard system (7)

$$rank \begin{bmatrix} I_n s - A \\ C \end{bmatrix} = n \quad \text{for all} \quad s \in \mathbf{C} \tag{11}$$

if and only if for singular system (9)

$$rank \begin{bmatrix} Es - A \\ C \end{bmatrix} = n \quad \text{for all finite} \quad s \in \mathbf{C} \tag{12}$$

Proof. Using (10) we may write

$$rank \begin{bmatrix} Es - A \\ C \end{bmatrix} = rank \begin{bmatrix} \mathbf{I}_n s - A + BFCs \\ C \end{bmatrix} = rank \left(\begin{bmatrix} \mathbf{I}_n & BFs \\ 0 & \mathbf{I}_p \end{bmatrix} \begin{bmatrix} \mathbf{I}_n s - A \\ C \end{bmatrix} \right) = rank \begin{bmatrix} \mathbf{I}_n s - A \\ C \end{bmatrix}$$

for all $s \in \mathbf{C}$

Lemma 3. For singular system (9)

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = n \quad \text{for any } F \tag{13}$$

Proof. Using (10) we may write

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = rank \begin{bmatrix} \mathbf{I}_n + BFC \\ C \end{bmatrix} = rank \left(\begin{bmatrix} \mathbf{I}_n & BF \\ 0 & \mathbf{I}_p \end{bmatrix} \begin{bmatrix} \mathbf{I}_n \\ C \end{bmatrix} \right) = rank \begin{bmatrix} \mathbf{I}_n \\ C \end{bmatrix} = n \quad \square$$

It is well – known [6] that if (11) holds then there exist a nonsingular matrix $P \in R^{n \times n}$ such that

$$\overline{A} = P^{-1}AP = \begin{bmatrix} \overline{A}_{11} & \dots & \overline{A}_{1p} \\ \overline{A}_{p1} & \dots & \overline{A}_{pp} \end{bmatrix}, \quad \overline{B} = P^{-1}B, \quad \overline{C} = CP = [C_1 \;\; C_2 \;\; \dots \;\; C_p] \tag{14a}$$

where

$$\overline{A}_{ii} = \begin{bmatrix} 0 & \vdots & \\ I_{d_i - 1} & \vdots & a_i \end{bmatrix} \in R^{d_i \times d_i}, \quad \overline{A}_{ij} = [0 \;\vdots\; a_{ij}] \in R^{d_i \times d_j} \quad (i \neq j, \; i, j = 1, \dots) \tag{14b}$$

$$C_i = [0 \;\vdots\; c_i] \in R^{p \times d_i}, \quad c_i = [0 \;\; \dots \;\; 0 \;\; 1 \;\; c_{1,i+1} \;\; \dots \;\; c_{1p}]^T, \quad n = \sum_{i=1}^{p} d_i$$

(T denotes the transpose)

Let us define

$$\hat{C} := block\ diag[\hat{c}_1 \dots \hat{c}_p], \quad \hat{c}_i := [0 \;\; \dots \;\; 0 \;\; 1] \in R^{1 \times d_i}$$

It is easy to check that

$$\overline{C} = \tilde{C}\hat{C} \tag{15}$$

where

$$\tilde{C} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ c_{21} & 1 & \dots & 0 \\ c_{p1} & c_{p2} & \dots & 1 \end{bmatrix} \tag{16}$$

Note that

$$\overline{C}\overline{B} = CPP^{-1}B = CB \tag{17}$$

and

$$\overline{E} := (I_n + \overline{B}F\overline{C}) = P^{-1}(I_n + BFC)P = P^{-1}EP \tag{18}$$

Using (18) and (15) we obtain

$$\overline{E} = I_n + \overline{B}\overline{F}\hat{C} \tag{19}$$

where

$$\overline{F} := F\tilde{C} \tag{20}$$

Theorem 2. Let the condition (11) be satisfied and the matrices $\overline{A}$, $\overline{C}$ have the form (14). There exists a matrix F such that

$$\overline{E} = \begin{bmatrix} I_{t_1} & \overline{e}_1 & 0 \\ 0 & 0 & 0 \\ 0 & \overline{e}_2 & I_{t_2} \end{bmatrix}, \quad t_1 + t_2 = n - 1, \quad \overline{e}_1 \in R^{t_1}, \quad \overline{e}_2 \in R^{t_2} \tag{21}$$

if and only if

$$CB \neq 0 \tag{22}$$

Proof. Necessity

$$\det\begin{bmatrix} I_n & B \\ -FC & I_m \end{bmatrix} = \det\begin{bmatrix} I_n + BFC & B \\ 0 & I_m \end{bmatrix} = \det[I_n + BFC]$$

but also

$$\det\begin{bmatrix} I_n & B \\ -FC & I_m \end{bmatrix} = \det\begin{bmatrix} I_n & B \\ 0 & I_m + FCB \end{bmatrix} = \det[I_m + FCB]$$

Hence

$$\det E = \det[I_n + BFC] = \det[I_m + FCB]$$

If $CB=0$ then $E=1$ for any F.

Sufficiency.

If $CB = \overline{C}\overline{B} \neq 0$ then also $\hat{C}\overline{B} \neq 0$ since $\det\widetilde{C} \neq 0$. Hence for at least one k we have $\hat{c}_k\overline{b}_k = \overline{b}_{kk} \neq 0$ where $\overline{b}_k$ is the k-th row of $\overline{B}$ and $\hat{c}_k$ is k-th column of $\hat{C} = [\hat{c}_{ij}]$. Choosing the entries of $\overline{F}$ as follows

$$\bar{f}_{ij} = \begin{cases} -\dfrac{1}{\overline{b}_{kk}} & for\ i = j = k \\ 0 & otherwise \end{cases} \tag{23}$$

we obtain

$$\overline{E} = I_n + \overline{B}\overline{F}\hat{C} = I_n + \bar{f}_{kk}\overline{b}_k\hat{c}_k = \begin{bmatrix} I_{t_1} & \overline{e}_1 & 0 \\ 0 & 0 & 0 \\ 0 & \overline{e}_2 & I_{t_2} \end{bmatrix}$$

where $\overline{e}_1 = -\dfrac{1}{\overline{b}_{kk}}[\overline{b}_{k1}\,\overline{b}_{k2}\ldots\overline{b}_{k,k-1}]^T$, $\overline{e}_2 = -\dfrac{1}{\overline{b}_{kk}}[\overline{b}_{k,k+1}\,\overline{b}_{k,k+2}\ldots\overline{b}_{kn}]^T$. □

Theorem 3. There exist a gain matrices K satisfying (3) if and only if the conditions (11) and (22) are satisfied.

Proof. Sufficiency. If (11) and (22) hold then using (3), (14), (15) and (18) we may write

$$\det[Es - (A + KC)] = \det[P^{-1}(Es - (A + KC))P] =$$

$$= \det[\overline{E}s - (\overline{A} + P^{-1}K\overline{C})] = \det[\overline{E}s - (\overline{A} + \overline{K}\hat{C})] \tag{24}$$

where

$$\overline{K} = P^{-1}K\widetilde{C} \tag{25}$$

To simplify the notation without loss of generality we may assume that

$$\overline{E} = \begin{bmatrix} I_{n-1} & \overline{e} \\ 0 & 0 \end{bmatrix}, \quad \overline{e} = [\overline{e}_1\ \overline{e}_2\ \ldots\ \overline{e}_{n-1}]^T \tag{26}$$

Let

$$\overline{K} = [-\overline{A}_{n_1} + e_{n_1+1}, -\overline{A}_{n_2} + e_{n_2+1}, \Lambda, -\overline{A}_{n_{p-1}} + e_{n_{p-1}+1}, -\overline{A}_{n_p} - k], \quad \left(n_i := \sum_{j=1}^{i} d_j\right) \tag{27}$$

where $\overline{A}_i$ is the i–th column of $\overline{A}$, e_i is the i–th column of I_n and $k = [k_1\ k_2\ \ldots\ k_n]^T \in R^n$.

Using (14) and (27) it is easy to verify that

$$\overline{A} + \overline{K}\hat{C} = \begin{bmatrix} 0 & \\ I_{n-1} & -k \end{bmatrix} \tag{28}$$

Taking into account (24), (26) and (28) it is easy to check that

$$\det[Es - (A + BK)] = \det[\overline{E}s - (\overline{A} + \overline{K}\hat{C})] = \begin{vmatrix} s & 0 & \Lambda & 0 & \overline{e}_1 s + k_1 \\ -1 & s & \Lambda & 0 & \overline{e}_2 s + k_2 \\ 0 & 0 & \Lambda & s & \overline{e}_{n-1}s + k_{n-1} \\ 0 & 0 & \Lambda & -1 & k_n \end{vmatrix} =$$

$$= \overline{e}_1 s + k_1 + (\overline{e}_2 s + k_2)s + \ldots + (\overline{e}_{n-1}s + k_{n-1})s^{n-2} + k_n s^{n-1} = \tag{29}$$

$$= k_1 + (\overline{e}_1 + k_2)s + \ldots + (\overline{e}_{n-2} + k_{n-1})s^{n-2} + (\overline{e}_{n-1} + k_n)s^{n-1}$$

Comparison of the right sides of (3) and (29) yields

$$k = [\alpha, -\overline{e}_1, \ldots, -\overline{e}_{n-1}]^T \tag{30}$$

The necessity can be shown using the same arguments as for standard systems. □

Theorem 4. There exists a full-order perfect observer of the form

$$E\dot{\overline{x}} = A\overline{x} + Bu + K(C\overline{x} - y) \tag{31}$$

for the standard system (7) if the conditions (11) and (22) are satisfied.

Proof. If the assumption (22) is satisfied then F can be chosen so that the closed-loop system (9) is singular. By Theorem 3 if (11) and (22) are satisfied then there exists K satisfying (3) and by Theorem 1 there exists a perfect observer (31) for the standard system (7). □

If the conditions (11) and (22) are satisfied then a full-order perfect observer (31) can be computed by the use of the following procedure.

Procedure

Step 1. Compute the matrix P satisfying (14).

Step 2. Using (23) find $\overline{F}$ and next compute

$$F = \overline{F}\widetilde{C}^{-1} \tag{32}$$

and $E = I_n + BFC$

Step 3. From (30), (27) and (25) compute k, $\overline{K}$ and

$$K = P\overline{K}\widetilde{C}^{-1} \tag{33}$$

Step 4. Compute the desired observer (31) in the form

$$E\dot{\overline{x}} = (A + KC)\overline{x} + Bu - Ky \tag{34}$$

Example

For the standard system (7) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{35}$$

compute the full-order perfect observer (34) for α=1.

It is easy to check that the system (7) with (35) satisfies the conditions (11) and (22) since

$$rank\begin{bmatrix} Is-A \\ C \end{bmatrix} = rank\begin{bmatrix} s & -1 & 0 & -2 \\ -1 & s-2 & 0 & 1 \\ 0 & -1 & s & 0 \\ 0 & -3 & -1 & s-1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = 4 \quad \text{for all } s \in \mathbf{C}$$

and $CB = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Using the procedure we obtain

Step 1. The matrices (35) have already the desired form (14) $\overline{A} = A$, $\overline{B} = B$, $\overline{C} = C$ and $P = I_4$.

Step 2. Using (23) and (32) we obtain

$$\overline{F} = [0 \ \ -1] \ , \ F = \overline{F}\widetilde{C}^{-1} = [0 - 1]\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = [1 \ \ -1]$$

and $E = I_n + BFC = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Step 3.
Using (30) and (27) and taking onto account that $\overline{e}_1 = 1$, $\overline{e}_2 = 0$ $\overline{e}_3 = 1$ and $\overline{A}_2 = [1 \ \ 2 \ \ 1 \ \ 3]^T$, $\overline{A}_4 = [2 \ \ -1 \ \ 0 \ \ 1]^T$ we obtain

$$k = [\alpha, -\overline{e}_1, -\overline{e}_2, -\overline{e}_3]^T = [1 \ \ 1 \ \ 0 \ \ -1]^T$$

$$\overline{K} = [-\overline{A}_2 + e_3, -\overline{A}_4 - k] = \begin{bmatrix} -1 & -3 \\ -2 & 0 \\ 0 & 0 \\ -3 & 0 \end{bmatrix}$$

and

$$K = P\overline{K}\widetilde{C}^{-1} = \begin{bmatrix} -1 & -3 \\ -2 & 0 \\ 0 & 0 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -3 \\ -2 & 0 \\ 0 & 0 \\ -3 & 0 \end{bmatrix}$$

Step 4. The desired observer has the form

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\dot{\hat{x}} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\hat{x} + \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}u - \begin{bmatrix} 2 & -3 \\ -2 & 0 \\ 0 & 0 \\ -3 & 0 \end{bmatrix}y$$

4. Concluding remarks

A new concept of the full-order perfect observer for standard continuous-time linear systems has been presented. Conditions have been established for the existence of full-order perfect observer for standard continuous-time linear system (7). A design procedure of the perfect observer have been derived and illustrated by a numerical example. With slight modifications the considerations can be extended for standard discrete-time linear systems. Applying the approach presented in [8] the considerations can be extended for reduced-order perfect observers for the standard continuous-time linear systems. The considerations can be also extended for two-dimensional singular and standard linear systems [6,7].

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