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Analysis of the error caused by possible displacements of the reference axis used for the assessment of geometrical deviations of cylindrical machine elements

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Abstract

A comprehensive analysis of one component of total error occurring in the determination of geometrical deviations of machine elements set in V-blocks is presented. Analyzed is the error component arising due to possible displacement of the reference axis adopted for the assessment of geometrical deviations, reference axis position in particular. Changes in the position of that axis may result form displacements of measured roundness profile centres that define the reference line. The analysis also allowed to determine the relative error values of the determined deviation of axis position resulting from changes in reference axis position.

Keywords: error analysis, reference measurements, geometric deviations.

Analiza błędu spowodowanego możliwością zmiany położenia osi odniesienia przyjętej do oceny odchyłek geometrycznych zespołu powierzchni cylindrycznych

Streszczenie

W artykule przedstawiona została analiza jednej ze składowych sumarycznego błędu wyznaczania odchyłek geometrycznych części maszyn bazowanych w pryzmach. Przedstawiona analiza jest częścią szeroko pojętej teoretycznej analizy błędów opracowanej metodyki pomiarów odchyłek geometrycznych zespołu powierzchni cylindrycznych elementów maszyn, które z uwagi na swoje duże gabaryty i masy bazowane są najczęściej w pryzmach. Analizowano składową błędu spowodowaną możliwością zmiany położenia osi odniesienia przyjętej do oceny odchyłek geometrycznych, w szczególności odchyłki położenia osi. Zmiany położenia tej osi wynikać mogą z przemieszczeń środków mierzonych zarysów okrągłości, w oparciu o które wyznaczana jest oś odniesienia. W wyniku przeprowadzonej analizy wyznaczone zostały wielkość pola i granice możliwych przemieszczeń punktu usytuowanego na zmieniającej swoje położenie osi odniesienia, w zależności od zmieniającego również swoje położenie punktu usytuowanego na osi rozpatrywanej powierzchni cylindrycznej. Wymienione punkty określają odległość odpowiadającą (zgodnie z przyjętym kryterium tworzenia elementów odniesienia i sposobem wyznaczania odchyłek geometrycznych) poszukiwanej odchyłce współosiowości. W oparciu o przeprowadzoną analizę określone zostały również wartości błędu względnego wyznaczanej odchyłki położenia osi, wynikającego ze zmian położenia osi odniesienia przyjętej do oceny odchyłek geometrycznych.

Słowa kluczowe: analiza błędów, pomiary odniesieniowe, odchyłki geometryczne.

1. Introduction

For measurements and machining, large cylindrical machine parts are commonly placed in two or more V-blocks. However, it should be borne in mind that elements thus supported are constantly displaced. This displacement is due to a constant contact between normally irregular profile points and the V-block generatrices [1, 2, 3, 4]. Measurement conditions, often complex geometry and substantial flexibility of such elements require that additionally the issue of their deformability should be taken into account. For this reason a strict definition of functional errors requires that they are separated as geometrical errors and errors of elastic deformation. The developed and implemented system of elastic support of the measured object supported in V-blocks allows to eliminate elastic deformation of the measured object by propoer selection of system parameters, i.e. the number, distribution and pressure values applied in lightening supports [5].

The research to date, aimed at the determination of total theoretical error of the developed measurement methodology, geometrical errors of cylindrical machine elements based in V-blocks, allowed to define the field of the profile centre displacements of an object set in a V-block that result from the interaction between the object being measured and constant support points. It has been proved that for a roller V-block and an assumed profile deviation *ΔR* this area has a shape as shown in Fig.1. The limits of that shape T_x and T_y can be described by these relationships [6]:

$$T_x = \frac{4(R+r)\Delta R}{b}, \tag{1}$$

$$T_y = \sqrt{[(R+r)+\Delta R]^2 - \left(\frac{b}{2}\right)^2} - \sqrt{[(R+r)-\Delta R]^2 - \left(\frac{b}{2}\right)^2}. \tag{2}$$

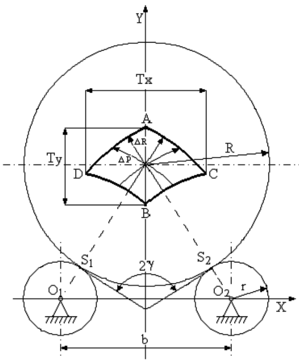

Fig. 1. Characteristic dimensions of a V-block and marked values of limit roundness profile centre displacements due to interaction between the measured object and constant support points

Rys. 1. Wymiary charakterystyczne pryzmy rolkowej wraz z oznaczeniami granicznych przemieszczeń środka zarysu okrągłości przedmiotu mierzonego, wynikającego ze współpracy mierzony element – stałe punkty podparcia

A similar shape has the field of roundness profile centre displacements obtained when we take into account the tolerance of dimensions of characteristic elements that make up the V-block (rolls axle distance, roll diameters, diameter of the measured object).

The setting of an object measured in two V-blocks (on condition its deflections are eliminated) causes the reference axis of the measured object (theoretically passing through the centres of roundness profiles contacting the V-block generatrices) possible changes of its position within the areas of the shape shown in Figure 1, placed from each other by distance *l*. The displacement field boundaries are described by relationships (1) and (2), while *l* is equivalent to the distance between two V-blocks.

2. Analysis of the error related with the displacement of the theoretical reference axis assumed for axis position deviations measurement

As noted before, the rotation axis of a measured object set in two V-blocks may change its position during measurements, which obviously affects the value of the determined axis position deviation. According to the adopted methodology of measurements and the interpretation of geometrical deviations [7, 8], the axis position deviation defined in relation to a previously defined reference axis, is a distance *d* of point F from that axis. The point *F* is located on axis O_o of the examined cylindrical surface half way between the extreme cross-sections selected for the assessment of the coaxiality (Fig. 2, Fig. 7). The area ranges through which the reference axis can pass are located at a distance *l* from each other of the corresponding length between V-blocks.
The centres of roundness profiles that contact the V-block generatrices can lie at any point within the areas of their possible displacement. The same refers to the position of reference axis that results from these changes. Consequently, the determined value of axis position deviation will be changing, too. The values of this deviation, understood as a distance of point F from the reference axis, depend on the position of point S on the reference axis as the deviation is determined in relation to this point. The performed analysis has shown that point S, due to a variable position of the reference axis, may change its position within a spatial figure. One particular example of this figure is presented in Fig. 2. The shape and boundaries of changes in the spatial figure dimensions (Fig. 2) may vary depending on: limit displacements T_x, T_y and T_z and the position of point F in the reference system (defined by coordinates *p*, *q*, and l_l) adopted for the assessment of axis position deviation.

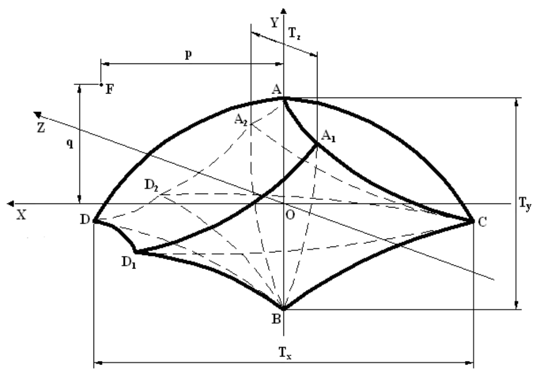

Fig. 2. The shape of the figure describing possible displacements of point S located on the position-changing reference axis adopted for the determination of axis position deviations with notations of limit displacements T_x, T_y and T_z

Rys. 2. Kształt figury opisującej możliwe przemieszczenia punktu S usytuowanego na zmieniającej swoje położenie osi odniesienia przyjętej do wyznaczania odchyłek położenia osi z oznaczeniami granicznych przemieszczeń T_x, T_y i T_z

An example of changes in shape and dimensions of the displacement area of point S (Figs. 2, 7) situated on the reference axis is shown in Fig. 3. The reference axis may change its position depending on position changes of point F assumed for the determination of axis position deviation *d*. In the case under consideration it is assumed that both point F and reference axis change their positions in one common plane Y-Z (Fig. 2). The difference between the measured axis position deviation d_1, interpreted as a distance between point F and any point S located within an area of its possible displacements relative to the nominal value of axis position deviation *d* is equivalent of an error of thus determined axis position deviation Δd. The limit displacement T_z of point S depends on the shape of the zone, i.e. on the limit displacements T_x and T_y and on the coordinates *p*, *q*, l_1 and *l* (l_i is a coordinate determining the longitudinal position of the profile, in which the coaxiality deviation is found, and *l* corresponds with the distance between the V-blocks).

For each plane in which the axis position deviation is to be determined, two characteristic shapes of displacement zones and corresponding limit displacement values T_z can be distinguished. Their examples are shown in Fig. 3.

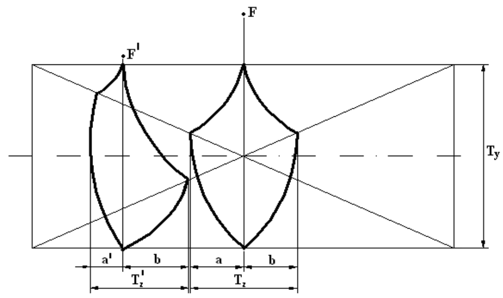

Fig. 3. Auxiliary diagram for defining limit values of displacements of point S located on the reference line for determining axis position deviations

Rys. 3. Schemat pomocniczy do wyznaczania wartości granicznych przemieszczeń punktu S usytuowanego na osi odniesienia przyjętej do wyznaczania odchyłek położenia osi

For the case presented in Fig. 3 (point F):

$$T_z = a + b = \frac{2 \cdot q \cdot T_y \cdot l}{T_y^{\,2} + l^2} \tag{3}$$

while:

$$a = \left[q + \frac{T_y}{2} - \frac{T_y \cdot l_1}{l} \right] \frac{T_y \cdot l}{T_y^{\,2} + l^2} \tag{4}$$

$$b = \left[q - \frac{T_y}{2} + \frac{T_y \cdot l_1}{l} \right] \frac{T_y \cdot l}{T_y^{\,2} + l^2} \tag{5}$$

For the case presented in Fig. 3 (point F'):

$$T_z' = a' + b = \frac{1}{2}\left[\sqrt{\left(\frac{T_y}{2} + q\right)^2 + l_1^2} - l_1 \right] + \left[q - \frac{T_y}{2} + \frac{T_y \cdot l_1}{l} \right] \frac{T_y \cdot l}{T_y^{\,2} + l^2} \tag{6}$$

while:

$$a' = \frac{1}{2}\left[\sqrt{\left(\frac{T_y}{2} + q\right)^2 + l_1^2} - l_1 \right] \tag{7}$$

$$b = \left[q - \frac{T_y}{2} + \frac{T_y \cdot l_1}{l} \right] \frac{T_y \cdot l}{T_y^{\,2} + l^2} \tag{8}$$

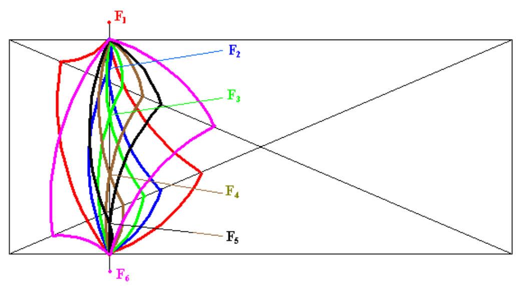

Fig. 4. A change of the shape of displacements zone for point S lying on the reference axis assumed for the determination of axis position deviations depending on point F placed on the axis of cylindrical surface

Rys. 4. Zmiana pola przemieszczeń punktu S usytuowanego na osi odniesienia przyjętej do wyznaczania odchyłek położenia osi w zależności od położenia punktu F usytuowanego na osi rozpatrywanej powierzchni cylindrycznej

The variations of displacement zones of the considered point S in the assumed plane of the axis position deviation depending on the position of point F (in each shape range) are depicted in Fig. 4.

The equations of curves limiting the displacement zone of point S in the assumed reference system (Fig. 5) for the measured profile defined by the longitudinal coordinate $l_{i,}$ can be written as follows:

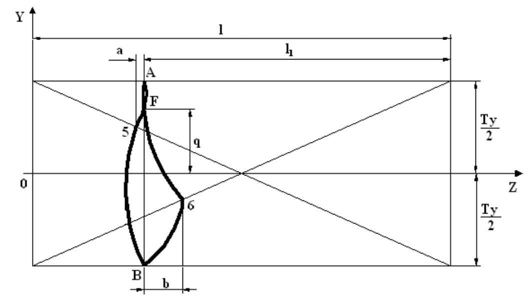

Fig. 5. Auxiliary diagram for calculations of the displacement zone for point S located on the reference axis assumed for defining axis position deviations

Rys. 5. Schemat pomocniczy do wyliczania wielkości pola przemieszczeń punktu S usytuowanego na osi odniesienia przyjętej do wyznaczania odchyłek położenia osi

$$Z_1 = \pm\sqrt{\left(\frac{w_1}{2}\right)^2 - \left[y - \left(\frac{T_y}{4} + \frac{q}{2}\right)\right]^2} + \left(l - \frac{l_1}{2}\right) \tag{9}$$

$$Z_2 = \pm\sqrt{\left(\frac{w_2}{2}\right)^2 - \left[y + \left(\frac{T_y}{4} + \frac{q}{2}\right)\right]^2} + \left(l - \frac{l_1}{2}\right) \tag{10}$$

$$Z_3 = \pm\sqrt{\left(\frac{w_3}{2}\right)^2 - \left[y + \left(\frac{T_y}{4} - \frac{q}{2}\right)\right]^2} + \left(\frac{l - l_1}{2}\right) \tag{11}$$

$$Z_4 = \pm\sqrt{\left(\frac{w_4}{2}\right)^2 - \left[y - \left(\frac{T_y}{4} + \frac{q}{2}\right)\right]^2} + \left(\frac{l - l_1}{2}\right) \tag{12}$$

with:

$$w_1 = \sqrt{\left(\frac{T_y}{2} - q\right)^2 + l_1^2}$$

$$w_2 = \sqrt{\left(\frac{T_y}{2} + q\right)^2 + l_1^2}$$

$$w_3 = \sqrt{\left(\frac{T_y}{2} + q\right)^2 + (l - l_1)^2}$$

$$w_4 = \sqrt{\left(\frac{T_y}{2} - q\right)^2 + (l - l_1)^2}$$

The size of displacement zone of the examined point S can be determined, in turn, by the integration of curve equations described by formulas (9-12) bounding the varying shape of the zone in each range of its variation.

The derived mathematical relationships written in a generalized form, allowed to carry out simulation computations of the size of displacement zone of the considered point S on the reference axis O. The changes in this zone directly affect the value of the determined axis position deviation. The calculations were made using an author's computational program enabling the entering of necessary input data, such as measuring system parameters, i.e. quantities l_1 and l. With these parameters, the program makes it possible to calculate the size of displacement zone of point S for one selected plane, for which the axis position deviation is determined. It was assumed that the plane to be considered was the plane Y-Z (Fig. 2), comprising a component T_y and a coordinate q. Thus, the program enabled the computing of the magnitude of displacement zone of the considered point S for assumed value ranges of the T_y and the coordinate q. The results, shown on 3D charts, are presented in Fig. 6.

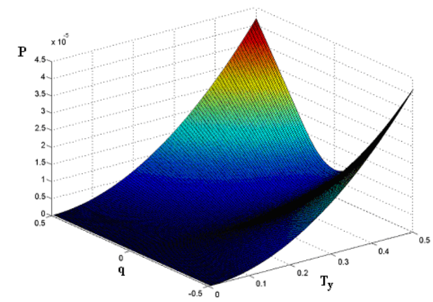

Fig. 6. A change in values of the displacement zone of point S in a selected plane of possible displacements depending on changes in the value of T_y and the coordinate q defining the position of point F located on the axis of the examined cylindrical surface

Rys. 6. Zmiana wielkości pola przemieszczeń punktu S w jednej z wybranych płaszczyzn możliwych jego przemieszczeń w zależności od zmian wartości T_y i wartości współrzędnej q określającej w przyjętym układzie odniesienia położenie punktu F usytuowanego na osi rozpatrywanej powierzchni cylindrycznej

The investigation, supported by a series of simulated computations, leads to a statement that the size of displacement zone of the considered point increases along with the zone width defined by value T_y and an increase of the coordinate q value (defining the position of point F in the adopted coordinate system) in relation to the width T_y.

An analysis of the relationship between changes in the position of the reference axis and the value of the axis deviation d relative to the reference axis was based on an auxiliary diagram shown in Fig. 7.

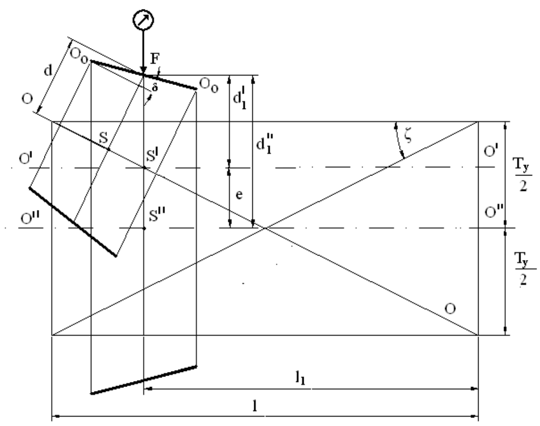

Fig. 7. An auxiliary diagram for the determination of the relative error value of the measured deviation of axis position resulting from displacements of the reference axis adopted for the determination of geometrical deviations of cylindrical machine elements based in two V-blocks

Rys. 7. Schemat pomocniczy do wyznaczania wartości błędu względnego mierzonej odchyłki położenia osi wynikającego z przemieszczeń osi odniesienia przyjętej do wyznaczania odchyłek geometrycznych cylindrycznych elementów maszyn bazowanych w dwóch pryzmach

Consideration was given to two cases of the varying position of the reference axis assumed for the assessment of axis position deviation d. One case corresponded to a situation when the reference axis is displaced within the maximum limits resulting from the zone with a width T_y, given that this position is constant during rotation of the measured object (the axis of the examined cylindrical pin rotates only relative to axis O). The measured value of axis position deviation is d_1' in Fig. 7, while the relative error of the measured deviation of axis position $w_{\Delta d1'}$ (expressed relative to d) equals:

$$w_{\Delta d_1'} = \frac{d_1' - d}{d} \cdot 100\% \tag{13}$$

because :

$$d_1' = \frac{d\sqrt{l^2 + T_y^2}}{l} \tag{14}$$

so finally:

$$w_{\Delta d_1'} = \frac{\sqrt{l^2 + T_y^2} - l}{l} \tag{15}$$

When the pin axis O_o is parallel to the reference axis O, the error has a constant value dependent on angle ζ. If the pin axis is not parallel to the reference axis O, the determined error value $w\Delta d_1'$ will additionally depend on the inclination angle δ of pin axis O_o.

The other case referred to a situation when the reference axis is displaced within the limits of maximum displacement resulting from the zone with a width T_y, with an additional assumption that the position of this axis may vary during rotation of the measured object within the limits of T_y. In this case the measured pin axis rotates around the axis O and makes a relative rotational motion around the axis O''.

The measured value of axis position deviation is equivalent to a section d_1'' in Fig. 7, while the relative error of the axis position deviation $w_{\Delta d1''}$ equals:

$$w_{d_1''} = \frac{d_1'' - d}{d} \cdot 100\% \tag{16}$$

The value of this error changes depending on the position of point F relative to axis O_1'' (this position is defined by coordinate l_1 in the adopted reference system).

The change of the measured axis position deviation d_1'' can be written in this form (Fig. 7):

$$d_1'' = d_1' + e = d_1' + T_y\left(\frac{2l_1 - l}{2l}\right) \tag{17}$$

The relationship (17) allows to determine the range of changes in the deviation d_1'' values contained within:

$$d_{1\min}'' = d_1' = \frac{d\sqrt{l^2 + T_y^2}}{l} \quad \text{when: } l_1 = \frac{l}{2} \tag{18}$$

$$d_{1\max}'' = d_1' + \frac{T_y}{2} = \frac{d\sqrt{l^2 + T_y^2}}{l} + \frac{T_y}{2} \quad \text{when: } l_1 = l \tag{19}$$

and finally:

$$w_{\Delta d_{1\min}''} = \frac{\sqrt{l^2 + T_y^2} - l}{l} \tag{20}$$

$$w_{\Delta d_{1\max}''} = \frac{\sqrt{l^2 + T_y^2} - l}{l} + \frac{T_y}{2d} \tag{21}$$

For the latter case of reference axis position under consideration, the relative error $w_{\Delta d1''}$ changes its value from $w_{\Delta d1'}$ to the values increased by a component $\frac{T_y}{2d}$. This component substantially affects the error value $w_{\Delta d1''}$. As a result, this error may assume significant values primarily dependent on parameters T_y and d.

Example computations of the error value $w_{\Delta d1''}$ for preset values T_y=0.5 mm, d=0.5 mm and l=3000 mm have shown error $w_{\Delta d1''}$ variation up to 50%. The effect of error $w_{\Delta d1'}$ (formula 15), being a component of error $w_{\Delta d1''}$, is slight and practically can be neglected in the error analysis.

3. Conclusions

The error analysis herein described has unequivocally shown that in reference measurements of geometrical errors of cylindrical machine components supported by two V-blocks, a change in the position of reference axis (created by the object V-blocks system) has a major influence on the value of the determined axis position deviation. This refers mainly to a situation when the reference axis during the measured object rotation will be displaced within the maximum limits of the possible displacements zone. As a result, measurements of axis position deviations will contain a component of 'eccentric' displacement of point S' located on the reference axis O which thus may change its position. The adoption of a reference axis should be preceded by a incisive analysis of possible displacements of an object rotated during measurements that are due to its irregular profile. Such analysis can be performed using harmonic or wavelet methods for examining roundness profiles [10, 11, 12, 13, 14], as these methods take account of the influence of irregularities of object profile measured to determine its geometrical deviations.

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