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Positive realizations for descriptor continuous-time linear systems

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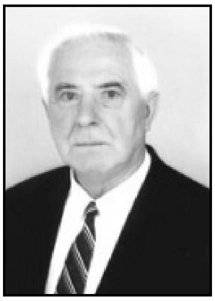

Abstract

Conditions for the existence of positive realizations for descriptor continuous-time linear systems are established. A procedure for computation of positive realizations for improper transfer matrices is proposed. The effectiveness of the method is demonstrated on numerical example.

Keywords: positive, realization, procedure, descriptor, linear system.

Dodatnie realizacje dla singularnych układów liniowych ciągłych

Streszczenie

W pracy podano warunki wystarczające na istnienie dodatnich realizacji dla singularnych układów liniowych ciągłych. Podano procedurę wyznaczania tych dodatnich realizacji na podstawie danych niewłaściwych macierzy transmitancji. Proponowaną metodę zilustrowano przykładem numerycznym.

Słowa kluczowe: dodatnie, realizacje, procedura, liniowy układ deskryptorowy.

1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive octant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [2, 9]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc. The positive fractional linear systems have been addressed in [6, 7, 18].

An overview on the positive realization problem is given in [1, 2, 9, 10]. The realization problem for positive continuous-time and discrete-time linear systems has been considered in [3, 4, 13, 16, 17] and the positive minimal realization problem for singular discrete-time systems with delays in [12]. The realization problem for fractional linear systems has been analyzed in [11, 14, 18] and for positive 2D hybrid systems in [15]. A method based on the similarity transformation of the standard realizations to the desired form has been proposed in [13].

Positive stable realizations problem for continuous-time standard and fractional linear systems has been addressed in [4, 11] and computation of realizations of discrete-time cone systems in [5].

In this paper a method for computation of positive realizations of descriptor continuous-time linear systems will be proposed.

The paper is organized as follows. In section 2 the positive realization problem for standard continuous-time linear systems is recalled. The positive realization problem for descriptor continuous-time linear systems is formulated and solved in section 3. The proposed procedure for computation of positive realizations of a given improper transfer matrix is illustrated by numerical example in section 4. Concluding remarks are given in section 5.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, $\Re^{p\times m}(s)$ - the set of $p\times m$ rational matrices in s with real coefficients, $\Re^{p\times m}[s]$ - the set of $p\times m$ polynomial matrices in s with real coefficients, M_n - the set of $n\times n$ Metzler matrices, I_n - the $n\times n$ identity matrix.

2. Preliminaries and positive realization problem for standard systems

Consider the standard continuous-time linear system

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{2.1a}$$

$$y(t) = Cx(t) + Du(t) \tag{2.1b}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $y(t)\in\Re^p$ are the state, input and output vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $D\in\Re^{p\times m}$.

Definition 2.1. The system (2.1) is called (internally) positive if $x(t)\in\Re_+^n$, $y(t)\in\Re_+^p$, $t\ge 0$ for any initial conditions $x(0)=x_0\in\Re_+^n$ and all inputs $u(t)\in\Re_+^m$, $t\ge 0$.

Theorem 2.1. [2, 9] The system (2.1) is positive if and only if

$$A\in M_n\,,\ B\in\Re_+^{n\times m}\,,\ C\in\Re_+^{p\times n}\,,\ D\in\Re_+^{p\times m}\,. \tag{2.2}$$

The transfer matrix of the system (2.1) is given by

$$T(s) = C[I_n s - A]^{-1}B + D \in \Re^{p\times m}(s)\,. \tag{2.3}$$

The transfer matrix $T(s)\in\Re^{p\times m}(s)$ is called proper if and only if

$$\lim_{s\to\infty} T(s) = K \in \Re^{p\times m} \tag{2.4}$$

and it is called strictly proper if K=0. Otherwise the transfer matrix is called improper.

Definition 2.2. Matrices (2.2) are called a positive realization of transfer matrix $T(s)$ if they satisfy the equality (2.3). The realization is called positive stable if the matrix A of (2.2) is an asymptotically stable Metzler matrix.

Theorem 2.2. [2,9] The positive system (2.1a) (or the Metzler matrix $A\in M_n$) is asymptotically stable if and only if the coefficients of the polynomial are positive, i.e. $a_k > 0$ for $k = 0,2,\ldots,n-1$.

Other asymptotic stability tests of positive linear systems are given in [9, 18].

Different methods for computation of a positive realization (2.2) if a given proper transfer matrix T(s) have been proposed in [4, 9, 11, 18].

3. Positive realization problem for descriptor systems

Consider the descriptor continuous -time linear system

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad (3.1a)$$

$$y(t) = Cx(t) \quad (3.1b)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, input and output vectors and $E, A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$.

It is assumed that det E = 0 and the pencil of (E, A) is regular, i.e.

$$\det[Es - A] \neq 0 \text{ for some } s \in \mathrm{C}. \quad (3.2)$$

where C is the field of complex numbers.

Definition 3.1. The descriptor system (3.1) is called (internally) positive if $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$, $t \geq 0$ for any initial conditions $x(0) = x_0 \in \Re_+^n$ and all inputs $u^{(k)}(t) = \frac{d^k u(t)}{dt^k} \in \Re_+^m$, $t \geq 0$, $k = 0,1,\ldots,q$.

The transfer matrix of the system (3.1) is given by

$$T(s) = C[Es - A]^{-1} B \in \Re^{p\times m}(s). \quad (3.3)$$

If the nilpotency index μ of the matrix E is greater or equal to 1 [8] then the transfer matrix (3.3) is improper and can be always written as the sum of the strictly proper part $T_{sp}(s)$ and the polynomial part $P(z)$, i.e.

$$T(s) = T_{sp}(s) + P(s) \quad (3.4)$$

where

$$P(s) = D_0 + D_1 s + \ldots + D_q s^q \in \Re^{p\times m}[s], \quad q \in N = \{1,2,\ldots\} \quad (3.5)$$

and $q = \mu - 1$.

Theorem 3.1. Let the matrices

$$A \in M_n, \ B \in \Re_+^{n\times m}, \ C \in \Re_+^{p\times n} \quad (3.6)$$

be a positive and asymptotically stable realization of the strictly proper transfer matrix $T_{sp}(s)$. Then there exists a positive asymptotically stable realization of $T(s) \in \Re^{p\times m}(s)$ of the form

$$\overline{E} = \begin{bmatrix} I_n & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & I_m & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & I_m & 0 \end{bmatrix} \in \Re_+^{\overline{n}\times\overline{n}},$$

$$\overline{A} = \begin{bmatrix} A & B & 0 & \ldots & 0 & 0 \\ 0 & I_m & 0 & \ldots & 0 & 0 \\ 0 & 0 & I_m & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & I_m \end{bmatrix} \in M_n, \ \overline{B} = -\begin{bmatrix} 0 \\ I_m \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \Re_+^{\overline{n}\times m}, \quad (3.7)$$

$$\overline{C} = [C \ \ D_0 \ \ D_1 \ \ \ldots \ \ D_q] \in \Re_+^{p\times\overline{n}}, \ \ \overline{n} = n + (q+1)m$$

if and only if

$$D_k \in \Re_+^{p\times m} \text{ for } k = 0,1,\ldots,q. \quad (3.8)$$

Proof. If the matrices (3.6) are a positive realization of $T_{sp}(z)$ then the standard system

$$x(t) = Ax(t) + Bu(t) \quad (3.9a)$$

$$y(t) = Cx(t) \quad (3.9b)$$

is positive and asymptotically stable and $x(t) \in \Re_+^n$, $t \geq 0$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \geq 0$. Defining the new state vector

$$\overline{x}(t) = \begin{bmatrix} x(t) \\ u(t) \\ \dot{u}(t) \\ \vdots \\ u^{(q)}(t) \end{bmatrix} \in \Re^{\overline{n}} \quad (3.10)$$

and using (3.7) we obtain

$$\overline{E}\dot{\overline{x}}(t) = \overline{A}\overline{x}(t) + \overline{B}\overline{u}(t) \quad (3.11a)$$

$$\overline{y}(t) = \overline{C}\overline{x}(t) \quad (3.11b)$$

From (3.11) if follows that $\overline{x}(t) \in \Re_+^{\overline{n}}$ and $\overline{y}(t) \in \Re_+^p$ for $t \geq 0$ if and only if the condition (3.8) is met since $x(t) \in \Re_+^n$, $t \geq 0$ and by assumption $u^{(k)} \in \Re_+^m$ for $t \geq 0$. Using (3.7), (3.3) and (3.4) it is easy to verify that

$$\overline{C}[\overline{E}s - \overline{A}]^{-1}\overline{B} = [C \ \ D_0 \ \ D_1 \ \ \ldots \ \ D_q]$$

$$\times \begin{bmatrix} I_n s - A & -B & 0 & \ldots & 0 & 0 \\ 0 & -I_m & 0 & \ldots & 0 & 0 \\ 0 & I_m s & -I_m & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & I_m s & -I_m \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -I_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= [C \ \ D_0 \ \ D_1 \ \ \ldots \ \ D_q] \begin{bmatrix} [I_n s - A]^{-1} B \\ I_m \\ I_m s \\ \vdots \\ I_m s^q \end{bmatrix} \quad (3.11)$$

$$= C[Es - A]^{-1} B + D_0 + D_1 s + \ldots + D_q s^q$$

$$= T_{sp}(s) + P(s) = T(s).$$

□

The positive asymptotically stable realization problem for the descriptor system can be stated as follows. Given an improper asymptotically stable rational matrix $T(s) \in \Re^{p\times m}(s)$, find its positive asymptotically stable realization (3.7).

If the conditions of Theorem 3.1 are satisfied then the desired positive realization (3.7) of $T(s)$ can be computed by the use of the following procedure.

Procedure 3.1.

Step 1. Decompose the given matrix $T(s)$ into the strictly proper part $T_{sp}(s)$ and the polynomial part $P(s)$ satisfying (3.4).

Step 2. Using one of the well-known methods [4, 9, 11] find the positive realization (3.6) of $T_{sp}(s)$.

Step 3. Knowing the realization (3.6) and the matrices $D_k \in \Re_+^{p\times m}$, $k = 0,1,\ldots,q$ of (3.5) find the desired realization (3.7).

4. Example

Find a positive asymptotically stable realization (3.7) of the transfer matrix

$$T(s) = \begin{bmatrix} \dfrac{2s^3+5s^2+5s+2}{s+1} & \dfrac{3s^2+7s+2}{s+2} \\ \dfrac{s^3+2s^2+2s+4}{s+2} & \dfrac{z^3+5s^2+7s+3}{s+3} \end{bmatrix}. \qquad (4.1)$$

Using Procedure 3.1 we obtain the following.

Step 1. The transfer matrix (4.1) has the strictly proper part

$$T_{sp}(s) = \begin{bmatrix} \dfrac{2}{s+1} & \dfrac{1}{s+2} \\ \dfrac{1}{s+2} & \dfrac{1}{s+3} \end{bmatrix} \qquad (4.2)$$

and the polynomial part

$$P(s) = \begin{bmatrix} 2s^2+3s+2 & 3s+1 \\ s^2+2 & s^2+2s+1 \end{bmatrix} = D_0 + D_1 s + D_2 s^2, \quad (q=2) \qquad (4.3a)$$

where

$$D_0 = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 3 & 3 \\ 0 & 2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}. \qquad (4.3b)$$

Step 2. The strictly proper transfer matrix (4.2) can be rewritten in the form

$$T_{sp}(s) = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} 2(s+2)(s+3) & (s+1)(s+3) \\ (s+1)(s+3) & (s+2)(s+1) \end{bmatrix} \qquad (4.4)$$

and the well-known Gilbert method can be applied to find its positive asymptotically stable realization [4, 9, 19] since the poles of (4.4) are distinct and negative ($s_1 = -1,\ s_2 = -2,\ s_3 = -3$). Following Gilbert method we compute the matrices

$$T_1 = \frac{1}{(s+2)(s+3)} \begin{bmatrix} 2(s+2)(s+3) & (s+1)(s+3) \\ (s+1)(s+3) & (s+2)(s+1) \end{bmatrix}_{s=-1} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix},$$
$$r_1 = \operatorname{rank} T_1 = 1, \quad T_1 = C_1 B_1, \quad C_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad B_1 = [1 \quad 0], \qquad (4.5a)$$

$$T_2 = \frac{1}{(s+1)(s+3)} \begin{bmatrix} 2(s+2)(s+3) & (s+1)(s+3) \\ (s+1)(s+3) & (s+2)(s+1) \end{bmatrix}_{s=-2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$
$$r_2 = \operatorname{rank} T_2 = 2, \quad T_2 = C_2 B_2, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad (4.5b)$$

$$T_3 = \frac{1}{(s+1)(s+2)} \begin{bmatrix} 2(s+2)(s+3) & (s+1)(s+3) \\ (s+1)(s+3) & (s+2)(s+1) \end{bmatrix}_{s=-3} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$r_3 = \operatorname{rank} T_3 = 1, \quad T_3 = C_3 B_3, \quad C_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_3 = [0 \quad 1]. \qquad (4.5c)$$

Thus, the positive asymptotically stable realization of (4.2) has the form

$$A = \text{blockdiag}[I_{r_1} s_1, I_{r_2} s_2, I_{r_3} s_3] = \text{diag}[-1,-2,-2,-3],$$
$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = [C_1 \quad C_2 \quad C_3] = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \qquad (4.6)$$

Step 3. Using (3.7), (4.3) and (4.5) we obtain the desired positive asymptotically stable realization of (4.1) in the form

$$\overline{E} = \begin{bmatrix} I_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & I_2 & 0 \end{bmatrix} \in \Re_+^{\overline{n}\times\overline{n}}, \quad \overline{A} = \begin{bmatrix} A & B & 0 & 0 \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & I_2 \end{bmatrix} \in M_n,$$
$$\overline{B} = \begin{bmatrix} 0 \\ -I_2 \\ 0 \\ 0 \end{bmatrix} \in \Re_+^{\overline{n}\times 2},$$
$$\overline{C} = [C \quad D_0 \quad D_1 \quad D_2] \in \Re_+^{2\times\overline{n}},$$
$$\overline{n} = n + (q+1)m = 5 + 3*2 = 11 \qquad (4.7)$$

and the matrices A, B, C, D_0, D_1, D_2 are given by (4.6) and (4.3b).

5. Conclusions

A method for computation of positive realizations for descriptor continuous-time linear systems has been proposed. Conditions for the existence of positive realizations for given improper transfer matrices have been established. A procedure for computation of positive realizations has been proposed and illustrated by a numerical example. The proposed method can be easily extended to asymptotically stable descriptor discrete-time linear systems. An open problem is an extension of this method for fractional descriptor linear systems.

This work was supported by National Science Centre in Poland under work no. N N514 6389 40.

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otrzymano / received: 25.04.2012
przyjęto do druku / accepted: 02.08.2012 *artykuł recenzowany / revised paper*

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