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Perfect observers for standard linear systems

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Abstract

Perfect observers for linear continuous-time and discrete-time systems are considered. Necessary and sufficient conditions for existence of the perfect observers are established and the procedures for computation of the perfect observer matrices are derived. Numerical examples and some simulation results are also presented. Some interesting advantages of the perfect observers are presented and illustrated by numerical examples.

Keywords: perfect observers, singular systems, observers for unstable systems.

Obserwatory doskonałe standardowych układów liniowych

Streszczenie

W pracy rozważana jest teoria obserwatorów doskonałych liniowych układów ciągłych i dyskretnych. Zostały tu przedstawione warunki konieczne i wystarczające istnienia tych obserwatorów dla układów ciągłych oraz dyskretnych. Omówiony został sposób wyznaczania obserwatorów doskonałych wraz z procedurami pozwalającymi na wyznaczanie tych obserwatorów dla standardowych układów ciągłych oraz dyskretnych. Sposób wyznaczania obserwatorów doskonałych został pokazany poprzez przykłady numeryczne wraz z wynikami symulacji. Nie zapominając o oczywistej wadzie obserwatorów doskonałych polegającej na tym, iż sygnał wyjściowy może zależeć od pochodnej wymuszenia, pokazano kilka niewątpliwie ważnych zalet tych obserwatorów. Zamieszczone wnioski zostały poparte przykładami numerycznymi.

Słowa kluczowe: obserwatory doskonałe, układy singularne, obserwatory układów niestabilnych.

1. Introduction

The design of observers for linear systems has been considered in many papers and books [1, 2, 4-15]. Necessary and sufficient conditions for the existence of perfect observers for linear standard continuous-time and discrete-time systems have been established in [4-6, 8, 9]. A new concept of perfect observers for continuous-time linear systems has been proposed in [8, 9].

The subject of this paper is to present new method for designing perfect observers for continuous-time and discrete-time linear standard systems. Necessary and sufficient conditions will be established for the existence of the observers, and procedures for computation of the matrices of the perfect observer will be derived. The procedures will be illustrated by numerical examples and some simulation results. Some interesting advantages will be also presented.

2. Problem formulation

Let $R^{n\times m}$ be the set of real matrices of dimension $n\times m$ ($R^n:=R^{n\times 1}$) and the identity matrix will bee denoted by I^n.

Consider the discrete-time linear system

$$Ex_{k+1} = Ax_k + Bu_k \tag{1}$$

$$y_k = Cx_k + Du_k \tag{2}$$

where $x\in R^n$, $u\in R^m$ and $y\in R^p$ are the state input and output vectors, respectively, and $E,A\in R^{n\times n}$, $B\in R^{n\times m}$ and $C\in R^{p\times n}$, $D\in R^{p\times m}$ are known real matrices. Without loose of generality we assume that m=1, p=1 and D=0.

Definition 1. System (1) – (2) is called singular (descriptor) discrete-time system if and only if det$E$=0, rank$E = r < n$.

Definition 2. A singular discrete-time linear system

$$Q\hat{x}_{k+1} = F\hat{x}_k + Gu_k + Hy_k \tag{3}$$

where $Q,F\in R^{n\times n}$, $G\in R^{n\times m}$, $H\in R^{n\times p}$ and $\hat{x}_k$ is the estimate of the vector x_k, is called perfect observer of (1) – (2) if and only if

$$\hat{x}_k = x_k \text{ for } k>0 \tag{4}$$

Consider the continuous-time linear system

$$E\dot{x} = Ax + Bu \tag{5}$$

$$y = Cx + Du \tag{6}$$

where $\dot{x} = \frac{dx}{dt}$, $x\in R^n$, $u\in R^m$ and $y\in R^p$ are the state, input and output vectors, respectively, and E, $A\in R^{n\times n}$, $B\in R^{n\times m}$, and $C\in R^{p\times n}$, $D\in R^{p\times m}$ are known real matrices. Without loose of generality we assume that m=1, p=1 and D=0.

Definition 3. System (5) – (6) is called singular (descriptor) continuous-time system if and only if $\det E = 0$, $\mathrm{rank}E = r < n$.

Definition 4. A singular continuous-time linear system

$$Q\dot{\hat{x}} = F\hat{x} + Gu + Hy \tag{7}$$

where $Q,F\in R^{n\times n}$, $G\in R^{n\times m}$, $H\in R^{n\times p}$ and $\hat{x}$ is the estimate of the vector x, is called perfect observer of (5) – (6) if and only if

$$\hat{x}(t) = x(t) \text{ for } t>0 \tag{8}$$

The aim of this paper is to design the perfect observers for continuous-time and discrete-time linear systems.

3. Perfect observers for discrete-time linear systems

The standard discrete-time linear system (1) – (2) is observable if and only if the observability matrix O has full column rank [3, 10, 12].

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{9}$$

If system (1) – (2) is observable then there exists matrix T such that transforms system to its canonical observable form

$$\bar{x}_{k+1} = \begin{bmatrix} 0 & I_{n-1} \\ & \bar{q} & \end{bmatrix}\bar{x}_k + \begin{bmatrix} \bar{B}_1 \\ \bar{b}_2 \end{bmatrix}u_k \tag{10}$$

$$y_k = \begin{bmatrix} \bar{c}_1 & 0 \end{bmatrix}\bar{x}_k \tag{11}$$

where $\bar{x}_k = Tx_k$.

From (11) and middle n–1 (n-p) equations of (10) we obtain

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}\bar{x}_{k+1} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}\bar{x}_k + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}u_k + \begin{bmatrix} 0 \\ -1 \end{bmatrix}y_k \tag{12}$$

Using $\bar{x}_k = Tx_k$ and (12) we can create perfect observer (3).

Dynamic of the error equation $e_k = \hat{x}_k - x_k$ for this observer is equal

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}e_{k+1} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}e_k \tag{13}$$

so condition (4) is satisfied.

Therefore the perfect observer for standard discrete-time system exists if and only if system is observable. Perfect observer for observable discrete-time linear system can be obtained by the use of the following procedure.

Procedure 1.

Step 1. Find matrix T such that transforms system (1) – (2) to its canonical form (10) – (11) and count matrices

$$\bar{A} = TAT^{-1}, \bar{B} = TB, \bar{C} = CT^{-1} \tag{14}$$

Step 2. Using canonical form (10) – (11) construct (12)

Step 3. Using $\bar{x}_k = Tx_k$ create perfect observer for discrete-time linear system

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}T\hat{x}_{k+1} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}T\hat{x}_k + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}u_k + \begin{bmatrix} 0 \\ -1 \end{bmatrix}y_k \tag{15}$$

where $\hat{x}_k$ satisfies (4).

4. Perfect observers for continuous-time linear systems

The standard continuous-time linear system (5) – (6) is observable if and only if the observability matrix O has full column rank [3, 10, 12]

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{16}$$

If system (5) – (6) is observable then there exists matrix T such that transforms system to its canonical observable form

$$\dot{\bar{x}} = \begin{bmatrix} 0 & I_{n-1} \\ & \bar{q} & \end{bmatrix}\bar{x} + \begin{bmatrix} \bar{B}_1 \\ \bar{b}_2 \end{bmatrix}u \tag{17}$$

$$y = \begin{bmatrix} \bar{c}_1 & 0 \end{bmatrix}\bar{x} \tag{18}$$

where $\bar{x}(t) = Tx(t)$.

From (18) and middle n–1 (n-p) equations of (17) we obtain [7-9, 13]

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}\dot{\bar{x}} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}\bar{x} + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}u + \begin{bmatrix} 0 \\ -1 \end{bmatrix}y \tag{19}$$

Using $\bar{x}(t) = Tx(t)$ and (19) we can create perfect observer (7).

Dynamic of the error equation $e(t) = \hat{x}(t) - x(t)$ for this observer is equal

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}\dot{e} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}e \tag{20}$$

so condition (8) is satisfied.

Therefore the perfect observer for standard continuous-time linear system exists if and only if system is observable [9, 10]. Perfect observer for observable continuous-time linear system can be obtained by the use of the following procedure

Procedure 2.

Step 1. Find matrix T such that transforms system (5) – (6) to its canonical form (17) – (18) and count matrices

$$\bar{A} = TAT^{-1}, \bar{B} = TB, \bar{C} = CT^{-1} \tag{21}$$

Step 2. Using canonical form (17) – (18) construct (19)

Step 3. Using $\bar{x} = Tx$ create perfect observer for continuous-time linear system

$$\begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}T\dot{\hat{x}} = \begin{bmatrix} 0 & I_{n-1} \\ \bar{c}_1 & 0 \end{bmatrix}T\hat{x} + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}u + \begin{bmatrix} 0 \\ -1 \end{bmatrix}y \tag{22}$$

where $\hat{x}(t)$ satisfies (8).

5. Examples and simulations results

Example 1.

Find perfect observer for observable discrete-time linear system with matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{23}$$

Step 1. System (23) is in the canonical form (10) – (11) so matrix $T = I_n$.

Step 2. Using n-p equations from the state and output equations we obtain

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\hat{x}_{k+1}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\hat{x}_k+\begin{bmatrix}1\\2\\0\end{bmatrix}u_k+\begin{bmatrix}0\\0\\-1\end{bmatrix}y_k \quad (24)$$

Step 3. Perfect observer is equal (24), because of $T=I_n$.

The output signal of the perfect observer (24) depends on the future values of the observer input signals. Future values of the input signals may not be attainable so they can be obtained using previous ones.

$$x_{k+1}=x_k+\Delta x_k\approx x_k+(x_k-x_{k-1})=2x_k-x_{k-1} \quad (25)$$

Using (25) we obtain

$$\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\hat{x}_{k+1}=\begin{bmatrix}-2&1&0&1&0\\0&-2&1&0&1\\1&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\end{bmatrix}\hat{x}_k+\begin{bmatrix}1\\2\\0\\0\\0\end{bmatrix}u_k+\begin{bmatrix}0\\0\\-1\\0\\0\end{bmatrix}y_k \quad (26)$$

Simulation results are presented below.

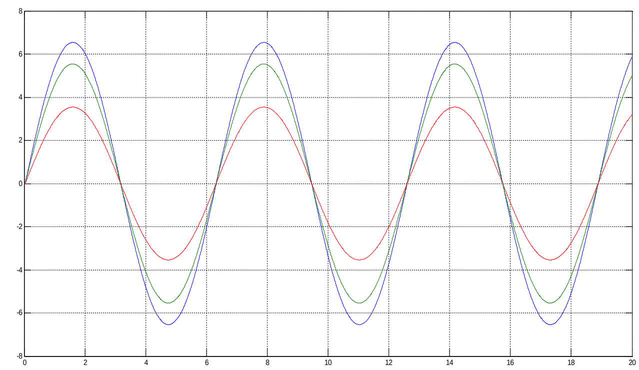

Fig. 1. State vector x for discrete-time system from example 1
Rys. 1. Wektor stanu x układu dyskretnego z przykładu 1

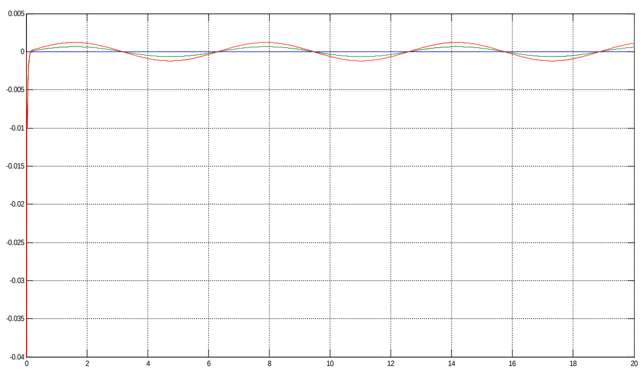

Fig. 2. Observer output signals error for discrete-time system from example 1
Rys. 2. Błąd wyjścia obserwatora dla układu dyskretnego z przykładu 1

Example 2.

Find perfect observer for observable continuous-time linear system with matrices

$$A=\begin{bmatrix}0&1&0\\0&0&1\\-1&-1&-1\end{bmatrix},\ B=\begin{bmatrix}1\\2\\1\end{bmatrix},\ C=\begin{bmatrix}1&0&0\end{bmatrix},\ D=0 \quad (27)$$

Step 1. System (27) is in the canonical form (17) – (18) so matrix $T=I_n$.

Step 2. Using n-p equations from the state equations and output equation we obtain

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\dot{\hat{x}}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\hat{x}+\begin{bmatrix}1\\2\\0\end{bmatrix}u+\begin{bmatrix}0\\0\\-1\end{bmatrix}y \quad (28)$$

Step 3. Perfect observer is equal (28), because of $T=I_n$.

Simulation results are presented below.

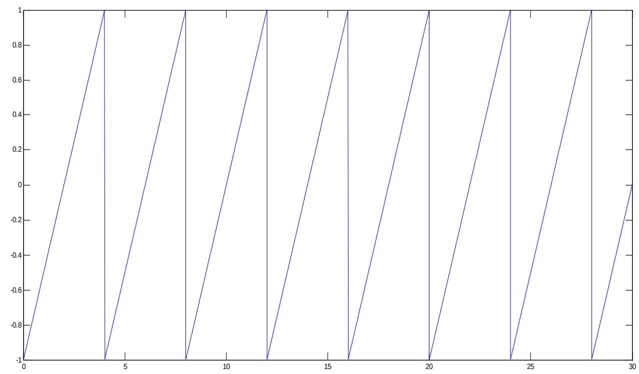

Fig. 3. Input signal u for continuous-time system from example 2
Rys. 3. Sygnał wejścia u układu ciągłego z przykładu 2

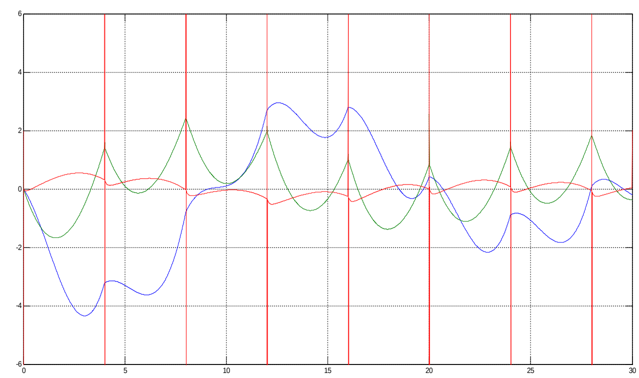

Fig. 4. Perfect observer output signal x for continuous-time system from example 2
Rys. 4. Sygnał wyjściowy obserwatora doskonałego x układu ciągłego z przykładu 2

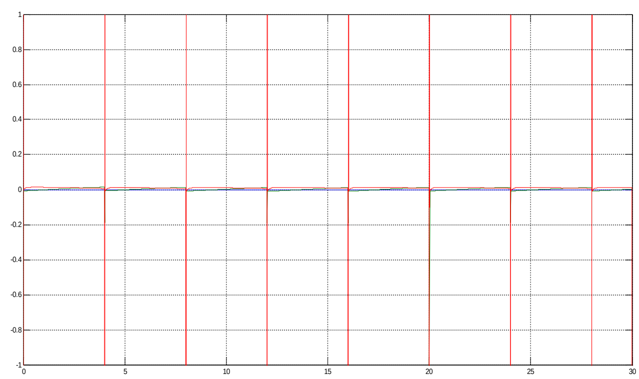

Fig. 5. Error of the perfect observer output signal from example 2
Rys. 5. Błąd sygnału wyjściowego obserwatora z przykładu 2

The input signal contains some discrete points, so its derivative not exists in these points and the error of the observer output signal is very big in these points Fig. 5. Observer output signal is small for the other time values, where it depends on the dynamics of its derivative.

Example 3.

Find perfect observer for observable continuous-time linear system with matrices

$$A=\begin{bmatrix}0&1&0\\0&0&1\\-1&-1&0\end{bmatrix},\quad B=\begin{bmatrix}1\\2\\1\end{bmatrix},\quad C=\begin{bmatrix}1&0&0\end{bmatrix},\quad D=0 \qquad (29)$$

Step 1. System (29) is in the canonical form (17) – (18) so matrix $T=I_n$.

Step 2. Using n-p equations from the state equations and output equation we obtain

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\dot{\hat{x}}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\hat{x}+\begin{bmatrix}1\\2\\0\end{bmatrix}u+\begin{bmatrix}0\\0\\-1\end{bmatrix}y \qquad (30)$$

Step 3. Perfect observer is equal (30), because of $T=I_n$.

Simulation results are presented below.

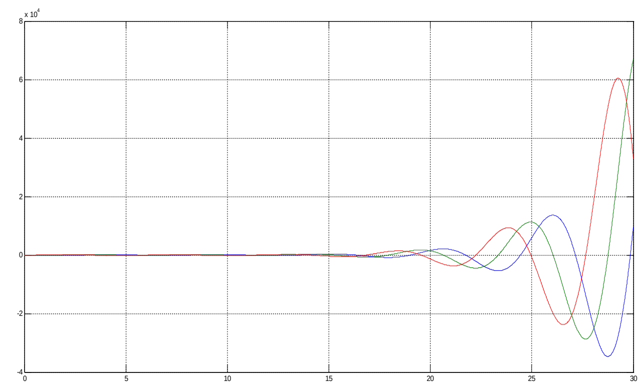

Fig. 6. Perfect observer output signal x for continuous-time system from example 3
Rys. 6. Sygnał wyjściowy obserwatora doskonałego x układu ciągłego z przykładu 3

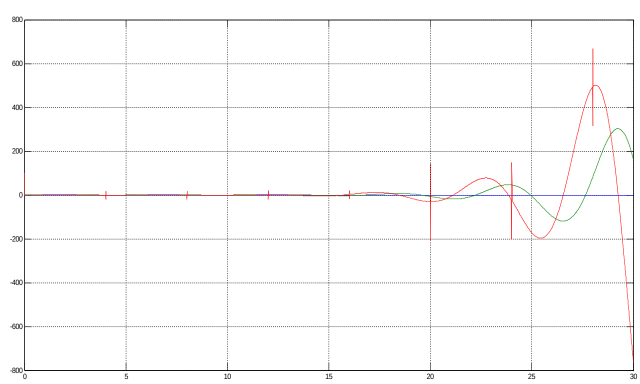

Fig. 7. Error of the perfect observer output signal from example 3
Rys. 7. Błąd sygnału wyjściowego obserwatora z przykładu 3

System presented in the example 3 is unstable but the perfect observer exists and correctly reconstructs the state vector. State vector has been reconstructed with an error depending on the dynamic of the derivative of the observer input signals.

Comparing the results of the example 2 and example 3 we notice an essential feature. Perfect observer is the same for stable system (23) and unstable one (25), too. The perfect observer is independent of the system dynamics. Equation of the perfect observer is not dependent of the system state transmittance denominator.

6. Conclusions

Perfect observers for standard continuous-time and discrete-time systems have been investigated. Necessary and sufficient conditions for the existence of the perfect observers have been established. Procedures for designing of the perfect observers have been derived and illustrated by numerical examples. Some simulations results for the perfect observers have been also presented. An extension of the considerations for nonlinear and fractional continuous-time systems is also possible but it is not trivial and it will be a subject of the future research and publications.

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