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EXCITATION OF A LOCALIZED NONLINEAR NORMAL MODE OF A BLADED DISK ASSEMBLY LUMP MASS NONLINEAR MODEL

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Abstract

This article examines a lump mass model of a perfectly symmetric bladed disk assembly of five sectors with nonlinearities in blades. Nonlinear modal analysis and the existence of localized Nonlinear Normal Modes (NNMs) has been shown in previous work [1]. In this article, we demonstrate numerically that it is possible to excite the localized NNMs using travelling waves excitation. Practically this is very important, in case of operation of the assembly in nonlinear regime, then the localized modes must be taken into account, for the proper life-assessment of the assembly. Also this work will be continued with the determination of Nonlinear 'Cambell's' diagram of helicopter blades assemblies, with final aim to control their nonlinear dynamics.

INTRODUCTION

In the literature, of linear bladed disk assemblies, localized modes (whereas the energy is spatially confined) is only characteristic of mistuned assemblies and not of perfectly cyclic [2]. Localization of energy in perfectly symmetric cyclic structures, due to nonlinearities, can be found in nonlinear systems [1,9,14]. Taking into account nonlinearities, we must use the theory of NNMs in order to determine the dynamics. Going back to 60's, a pioneer in this field was Rosenberg who defined Normal Modes (similar and nonsimilar), the *vibrations in unison of admissible systems* [6]. Expansion of this work in forced vibrations has been done by Manevitch [7], Mikhlin [8] and Szemplinska-Stupnicka [9], and in 1990's analytically by Caughey and Vakakis [10], Shaw and Pierre [11], Vakakis et al [12], also by Warminski [13-15], and numerically by Kerschen et al. [16], Georgiades [17], Peeters [18,19], and many others. Since Rosenberg, the definition of Nonlinear Normal Modes is extended nowadays to *the periodic motions* of a dynamical system which include the cases of internal resonances with subharmonic NNMs and travelling waves NNMs [16]. We study a model by taking into account only five blades which correspond to helicopter bladed disk assembly. In design of a helicopter bladed disc with geometric nonlinearities, localized NNMs must be taken into account [5].

SYSTEM DEFINITION-EXCITATION FORCES

In case of inextensional beam, the equation of motion in bending vibration, excluding linear and nonlinear inertia terms (which in many cases are very small – they are higher order terms than the order of equation of motion), the nonlinear part is of cubic geometric nonlinearity, for both cases of isotropic and also for symmetric composite beam with orthotropic lamina [20,21]. We construct our cyclic linear viscous model considering cubic nonlinearities in blades. The system is dissipative and it is presented in Figure 1. Modal analysis of the corresponding conservative system (without dissipation) has been examined in paper [1] which showed the existence of localized Nonlinear Normal Modes. The equations of motion of this model are given by,

(blades) $$m\ddot{x}_i + k\left(x_i - X_i\right) + k_{nl}\left(x_i - X_i\right)^3 + d_s\left(\dot{x}_i - \dot{X}_i\right) = F_i\left(t\right)$$

(disks) $$M\ddot{X}_i + k\left(X_i - x_i\right) + k_{nl}\left(X_i - x_i\right)^3 + d_s\left(\dot{X}_i - \dot{x}_i\right) + K\left(X_i - X_{i-1}\right) + \quad (1)$$

$$+K\left(X_i - X_{i+1}\right) + d_s\left(\dot{X}_i - \dot{X}_{i-1}\right) + d_s\left(\dot{X}_i - \dot{X}_{i+1}\right) + d_m\dot{X}_i = 0$$

whereas, $x, \dot{x}, \ddot{x}$ are the displacements, velocities and accelerations of blades and $X, \dot{X}, \ddot{X}$ are the displacements, velocities and accelerations of disks respectively. $F_i(t)$ is the excitation force which is applied only in blades and also i=1,...,5 and $X_6=X_1$, $X_0=X_5$, $\dot{X}_6 = \dot{X}_1$, $\dot{X}_0 = \dot{X}_5$ (the cyclic conditions).

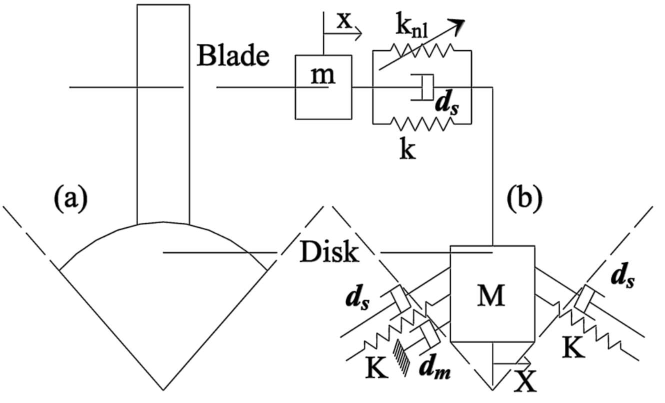


Figure 1. One sector of the model a) continuous model, b) discrete model.

The excitation force is applied only in blades of this system and it is standard travelling waves excitation given by [4],

$$F_i(t) = A\cos(\omega t \pm (i-1)\,\varphi) \quad (2)$$

With i=1,...5, the positive sign defines forward travelling waves and the negative sign backwards travelling waves, and φ is the phase difference or spatial frequency (rad/sector) between adjacent blades and is given by,

$$\varphi = \frac{2\times\pi\times EO}{5} \quad (3)$$

whereas EO is the selected Engine Order excitation. Typical case of travelling waves excitation is depicted in Figure 2.

The values of the parameters of the system are given by,

$$M=1,\ \ m=0.3,\ \ K=1,\ \ k=1,\ \ \mathrm{k}_{nl}=0.1,\ \ d_s=0.015,\ \ d_m=0.015 \tag{4}$$

The natural frequencies of the corresponding conservative system (without damping) are presented in Table 1. There are 8 modes in pairs and 2 single modes. The first mode with 0 frequency is the rigid body mode whereas the addition of grounded viscous dampers in disks affect to extremely high critical damping ratio in this mode, without influence in our studies. Also in Table 1 are indicated the characteristics of the modes in terms of nodal circles and nodal diameters. In the case of 0 (1) Nodal Circles of normal modes means that at each sector the blade mass is in phase with disk mass (the blade mass is out of phase with disk mass). The case of 0, 1 or 2 nodal diameters corresponds to circumferential examination of amplitudes of mode shapes and we count how many circumferential points of adjacent sectors have opposite sign in amplitudes e.g. for 1 nodal diameter 2 points etc. [1]. This is very important because it is indicating which EO of travelling waves excitation should be used for the excitation of the corresponding mode.

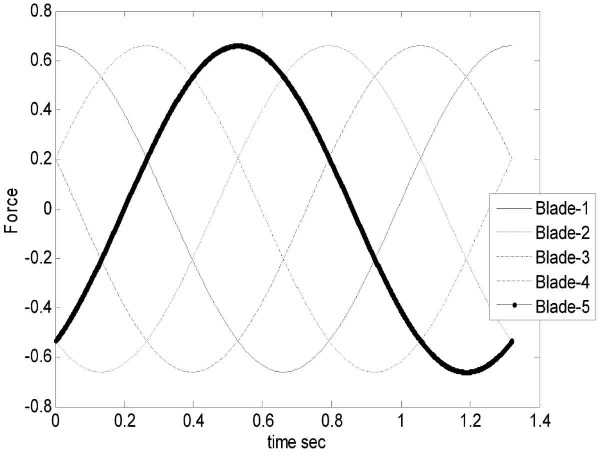


Figure 2. Travelling waves excitation with ω=4.76 rad/sec

Table 1

Mode	Frequency of conservative system (rad/sec)	Critical Damping Ratio (%)	Nodal Circles	Nodal Diameters
1	0	$8.74\ 10^7$	0	0
2,3	0.99	1.22	0	1
4,5	1.43	1.25	0	2
6	2.08	1.64	1	0
7,8	2.18	1.76	1	1
9,10	2.43	2.03	1	2

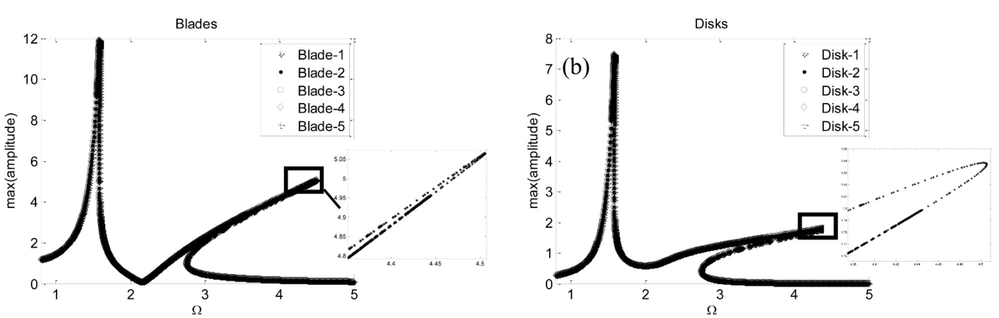
EXCITATION OF NNMS

In case of non-autonomous systems we determine the steady states (which are periodic motions of k – periods which corresponds to j – periods of the excitation frequency e.g. for k=j=1, there is 1 – 1 steady states on the system) and we examine the resonance points (k – j, NNMs) in nonlinear Frequency Response Functions (FRFs- maximum amplitudes of displacements or Frequency-Mechanical Energy plots) which, in relative small damping, are very close to curves of maximum amplitudes of displacements with variation of frequency or frequency-energy plot curves of autonomous systems [6,12,16].

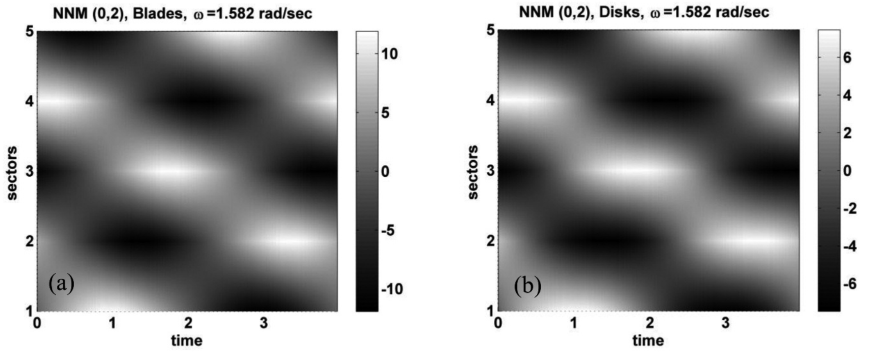
In order to determine numerically the resonance point of the system (NNM) with variation of frequency we use pseudo-arclength and also sequential continuation technique with codes written in Matlab which initially has been developed at University of Liege [15, 16] and has been modified significantly at University of Lublin to meet the requirements of our research.

The travelling waves excitation is applied only in blades. This one used in this article is of EO=2 with amplitude A=0.66 and it is depicted in Figure 2, in case of ω=4.76 rad/sec. By application of this excitation, with variation of frequency at periodic motions, we can find the corresponding Nonlinear FRF which is the variation of maximum amplitudes of Blades and Disks at each excitation frequency.

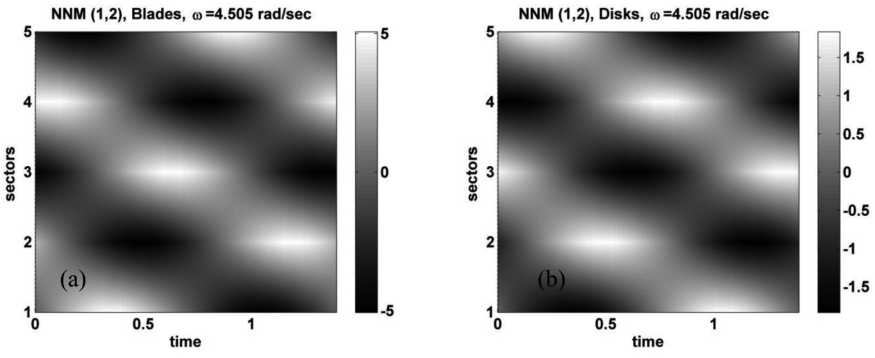
In Figures 3 (a, b), are depicted the Nonlinear Frequency Response Function for blade maximum responses (disks maximum responses). There are 2 resonance points in these Nonlinear FRF's the first one at ω=1.582 rad/sec, and the second at ω=4.5 rad/sec. Since it is used EO=2, it is expected to be the "evolution" with energy of linear modes (4,5) and (9,10), respectively. Indeed, this is true, which confirms detailed examination at resonance points, as depicted in Figures 4a,b and 5a,b. More precisely, Figure 4a(b) depicts the displacements of blades (disks) for each sector, in time, for the resonance point at ω=1.582 rad/sec, whereas one can observe that the motions are no "synchronous". They are in travelling waves form which corresponds to 1:1 internal resonance as shown in [17]. It can be noted that in Figures 4a,b there are 4 inclined 'lines' which follows the zero crossings or the maximum absolute amplitudes of the displacements. They correspond to 4 circumferential nodes in pairs of "anti-diametrical" positions and therefore in 2 nodal diameters. Comparison of Figures 4a with 4b, by means of the displacements of blades, with disks for the same sector, shows that they are in phase therefore correspond to 0 zero nodal circle modes. Therefore this mode corresponds to (4,5) linear modes. In Figure 5a(b) are depicted the displacements of blades (disks), for each sector in time, for resonance point at ω=4.5 rad/sec. Similarly we can see that they correspond to 2 nodal diameters and also 1 nodal circle, therefore to (9,10) linear modes.



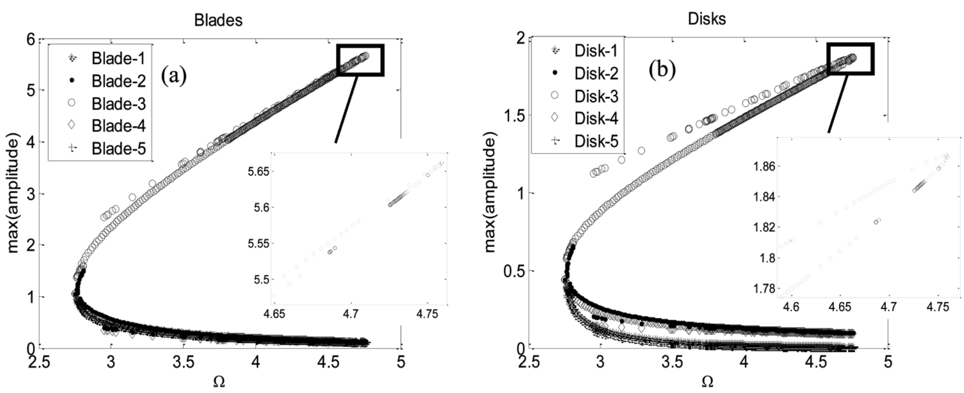
Figures 3. Nonlinear FRF's of a) blades, b) disks



Figures 4. Responses at resonance w=1.582 rad/sec, of a) blades b)disks.

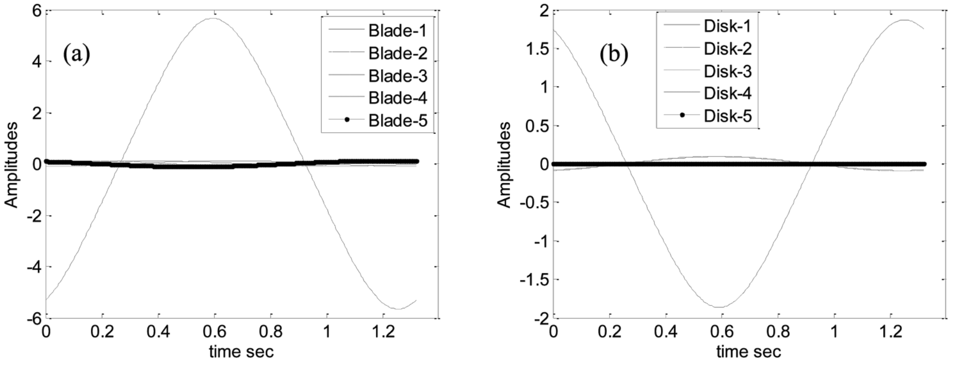


Figures 5. Responses at resonance ω=4.5 rad/sec, of a) blades b)disks.



Figures 6. Nonlinear FRF’s of a) blades, b) disks

We continued the examination of Nonlinear FRF’s and we found also another one Nonlinear FRF curve. In Figures 6a,b we may observe the Nonlinear FRF’s for a curve which corresponds to a bifurcated curve of the autonomous system. This Nonlinear FRF is rather significant for the assembly since it is related with a localized Nonlinear Normal Mode only in one sector (blade-“3”, disk-“3”, the system is cyclic so there is no circumferential origin which means no sense in sector 3). In Figure 7 a,b are depicted the displacement of blades (disks) in time, for this excited mode which corresponds to resonance point with ω=4.76 rad/sec. It is clear that it represents 1-nodal circle, since the blades are in opposite phase with disks. The nodal diameters are no longer clear that are still 2 due to the almost zero amplitudes of all the other sectors apart the 3rd one. The excitation of the assembly of this NNM has been done with the travelling waves applied only in blades (Figure 7).



Figures 7. Responses at resonance of a) blades, b) disks

CONCLUSIONS AND FUTURE WORK

We showed numerically, that using travelling waves excitation we can excite a localized Nonlinear Normal Mode. This phenomenon may play essential role in design of bladed disk assemblies. We will continue our research taking into account rotation of the assembly (centrifugal and Coriolis forces) and construct the Nonlinear 'Campbell' diagram using Nonlinear Normal Modes techniques. The final aim is to control the dynamics of helicopter blades with active elements e.g. avoiding the excitation of localized Nonlinear Normal Modes.

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LOKALIZACJA NIELINIOWYCH POSTACI DRGAŃ DYSKRETNEGO MODELU UKŁADU WIRNIKOWEGO

Streszczenie

W pracy przedstawiono analizę drgań dyskretnego, idealnie symetrycznego, modelu struktury składającej się z dysku wraz z dołączonymi pięcioma nieliniowymi sektorami, reprezentującymi nieliniowe łopaty wirnika. Szczegółową nieliniową analizę modalną, jak również możliwość lokalizacji nieliniowych postaci drgań (NNM) zawarto w artykule [1]. W bieżącej pracy przedstawiono numeryczną analizę nieliniowych postaci drgań wzbudzonych za pomocą poruszającej się fali. W przypadku gdy badany układ jest nieliniowy możliwe jest wystąpienie tzw. lokalizacji postaci drgań. Zjawisko to ma istotne znaczenie praktyczne. Powinno być wzięte pod uwagę w celu prawidłowej eksploatacji struktury wielołopatowej oraz zwiększenia jej czasu "życia". Przedstawione badania będą wykorzystane do wyznaczenia diagramu Cambell'a dla wirnika śmigłowca i w konsekwencji posłużą do opracowania strategii sterowania jego nieliniową dynamiką.