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LONGITUDINAL MOTION CONTROL FOR FLARE PHASE OF LANDING

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Abstract

The discussion presented in the paper is focused on selected problems of synthesis of automatic flight control law, designed especially for longitudinal motion control of a landing aircraft, when pursuing a flare manoeuvre, just before a touch-down. Some general aspects of control process, that is executed in such a case, are considered in order to disclose its predictive character. Novel solutions, developed in the Instytut Lotnictwa for on-board sub-systems, designed to measure/estimate a rate-of-climb/descent flight parameter in such a manoeuvre, are also described in order to present their effectiveness and 'new quality' entered in control process. The discussion is illustrated by some results obtained by simulation with the mathematical model of Cessna 402C aircraft used as an example These results are proving the efficiency of proposed solutions and their potential for future research.

INTRODUCTION

Let us consider the classical automatic feedback control system (Fig.1), where the reference signal Y_C (*C - „commanded"*) represents a desired output of controlled object [10], [19],[20].

One of the most specific features of such systems is that at every current moment of time only the history of signal Y_C (past + current moments) is known, while the behaviour of Y_C in future is not. There are however a class of important control tasks of a different nature - in these tasks the knowledge of future values and behaviour of the reference signal is available at every current moment of time. Thus, the problem appears: how to take advantage of this information in control system to improve the quality of control. This task is widely addressed in literature as moving time - horizon control and/or predictive control especially for flight control tasks [1], [6], [9], [11], [12].

For some important control tasks, the accessibility of the knowledge of future values and behaviour of the reference/command signals is a *sine qua non* condition for the realisation of the task. The automatically controlled terrain - following flight at low altitudes (*NoE – Nap of the Earth*) is a good example. Such missions are usually associated with military aviation, because of the essential sense of such flights for some kinds of tasks realised by air forces [1], [4], [6], [9], [11], [12], [21].

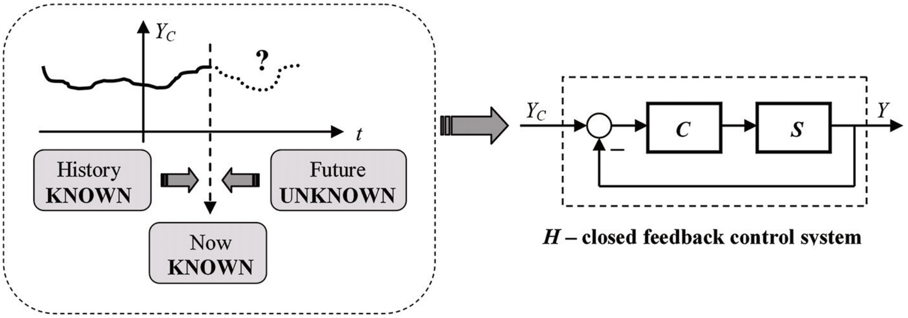

Fig.1. Feedback control – classical structure: S – controlled system (object), C – controller

The most widely known technical solutions have also military character, like LANTIRN system (*Low Altitude Navigation and Targeting Infrared for Night* originated in 1986 [20]) or motion control systems developed for manoeuvring missiles (e.g. *Cruise, Tomahawk* etc.). Principle of operation of such systems consists in forming the set of control signals for the airplane's motion on the basis of the results of on-line scanning of the terrain profile before the aircraft. The process of scanning is realised with the appropriate device being capable to '*see*' the terrain before the aircraft on some distance *L*. It is important to notice that the result of scanning is limited due to the possibly existing 'shadowed areas' (Fig.2). For instance, the LANTIRN system is equipped with the *Terrain Following Radar (TFR)* to complete this task [21]. Then, the more or less complex control law is executed to assure the safe motion of the aircraft [1], [6], [9], [11].

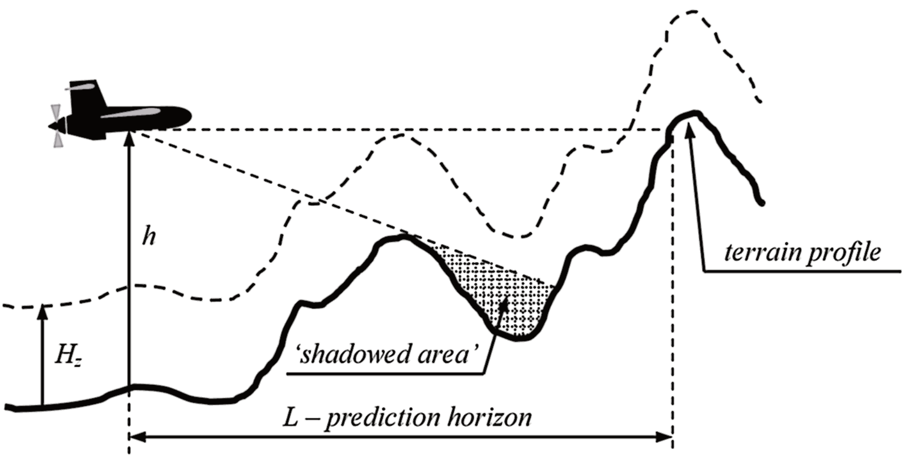

Fig. 2. Terrain-following flight at low altitude [12]

It is worth to notice that NoE flights differs from this illustrated in Fig.1, because of prediction horizon is defined in spatial domain (Fig.1). It is a distance ahead of the aircraft that is scanned by the equipment capable to 'see' a terrain to be covered by future motion of the aircraft. In case from (Fig.1) the prediction horizon is defined in time domain (see Appendix A for more details).

Approach & landing manoeuvre is one of the most obvious case with the desired trajectory of flight known in advance. To get a deeper insight into predictive nature of motion control process during this manoeuvre came the interesting episode that happened in Air Force Officers

Training Centre in Dęblin (Poland) at late twenties of 20th century is worth discussing. The case was not popular in technical literature, because it was described in a novel belonging to belles lettres kind of literature [15]. The author, Janusz Meissner, those times the Chief Instructor for pilot's training in the Centre, years later described it precisely.

It was the case of young air force pilot, Stanisław Latwis learning the art of pilotage (see [11] for details) and met strong difficulties in approach and landing manoeuvre. Meissner guessed the reason at once - the problem was in the way Latwis was looking at the runway during approach and flare phases of landing. He was fixing the line of sight at fixed point on the airfield, so, when the aircraft flew over this point, he always found himself confused and bewildered - too high above the ground with the speed too low, thus, obvious result happened. However Meissner tried to explain that he had to move the line of sight forward, to keep it ahead of the airplane and look at '*the whole airfield*' the case seemed hopeless. Latwis was unable to stop fixing his line of sight at fixed point on the ground when the airplane was approaching the airfield.

Fortunately, Latwis invented a simple exercise that helped him in overcoming his disability. He placed more than a hundred of stones along a straight line, post - spaced one about three steps after another. Then he started walking along this line, trying to '*move his eyes*' from one stone to the next one, to keep the line of sight about '*twenty stones before him*'.

When succeeded, he did the same running, then riding a bicycle (Fig.3) [12] and finally, driving a motorcycle. After a few days of such training he was able to complete correctly an approach and landing manoeuvre and shortly after that, he passed practical exams and became his career as an air force pilot.

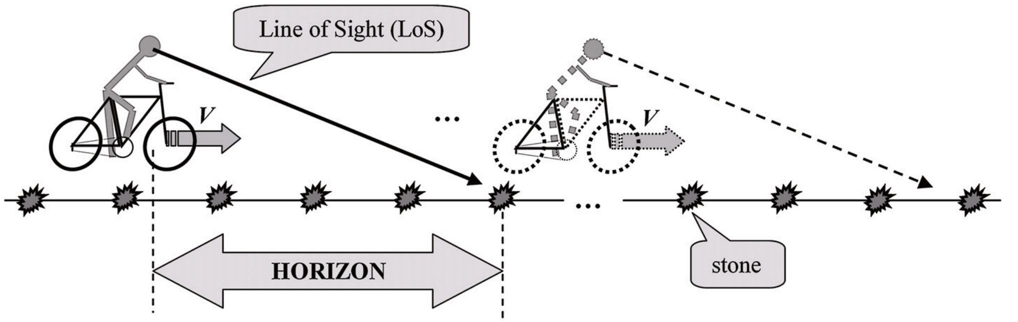

Fig. 3. Latwis´s exercise-'bicycle' phase [12]

It is not easy to explain all aspects of this „*experiment*" and obtained results with clear interpretation. Some of crucial questions belong rather to psychology, *e.g.* what was the nature of Latwis's disability and what about possibilities to overcome it by training.

Obviously, the way that pilot is looking at the runway in landing manoeuvre is crucial and doing it properly allows him/her to assess precisely the current state of the aircraft - height above the ground, velocity, pitch angle, angle of attack, rate of descent, etc. On the other hand, when the airfield is visible, the pilot controlling the aircraft to complete the approach & flare manoeuvre knows „something" in advance about a desired motion of the airplane in the immediate future. Thus, an interesting question appear: is there a possibility to make use of this idea in automatic control system?

However the result of the „experiment" described above is not easy to be applied directly in order to improve known technical solutions, it is interesting as the inspiration for engineering tasks and is treated this way in following paragraphs.

1. FLARE: - AUTOMATIC CONTROL OF AIRCRAFT MOTION

Typical scenario of flight phase ending the approach & landing manoeuvre is pictured in Fig.4. During the approach phase the aircraft is moving along the glide path with typical glide slope (flight trajectory angle) $\gamma_0 \approx 3^\circ$ (0.5236 rd) with the end located about 15 - 20 m above the runway's threshold. Then flare manoeuvre becomes to reduce the vertical speed of the aircraft $\dot{h}$, to fulfil the final requirement: $\dot{h} \leq 1$ m/s at the moment of touch-down (h denotes here the altitude and $\dot{h}$ - vertical speed of the aircraft). According to flight procedures the distance in horizontal direction, covered by the aircraft accomplishing flare manoeuvre is typically about 600 m.

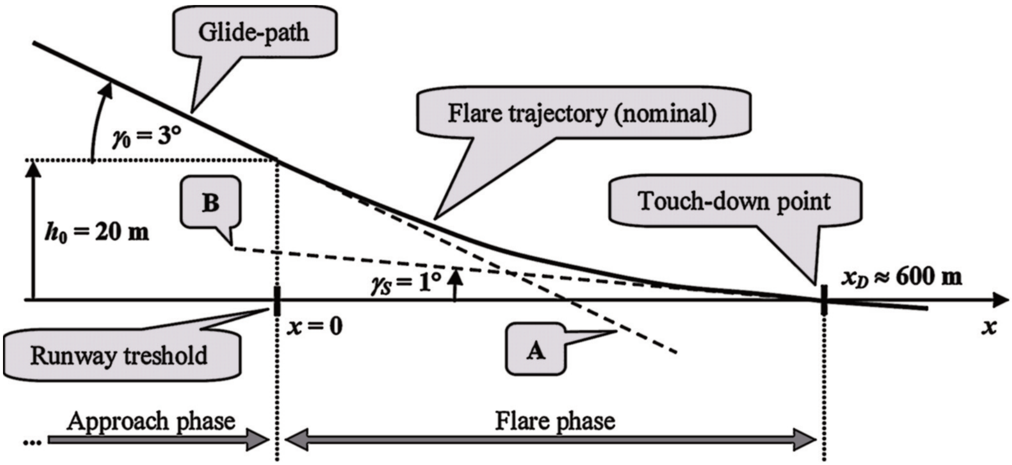

Fig. 4. Flare-out manoeuvre

In [8] the control algorithm is discussed that is capable to guide the aircraft accomplishing this manoeuvre. This algorithm is presented in form of the block diagram in Fig.5, where: PI, PID are typical controllers, K_θ, K_q–constant gains, U-forward velocity of the aircraft, U_{ref}–desired forward velocity, h – current altitude, h_{ref}– desired altitude corresponding with nominal flare trajectory in Fig.5, θ- pitch angle, q-pitch rate, δ_E–deflection of the elevator, δ_{TH}– throttle position. Especially interesting is the additional signal θ_{FLARE}. In the approach phase of flight it is equal to zero, but when flare phase starts it is switched to constant value (4° the aircraft considered by the authors of [8]). It is important to notice that the signal h_{ref} representing the shape of flare nominal trajectory (complex exponential curve) is computed at very beginning of flare manoeuvre (when the aircraft is passing over the threshold of the runway), thus is known *a priori*, before flare manoeuvre. Despite of this, it is used in control algorithm classically- to compute the control error: h_{ref}- h at every current moment of time.

The controller (Fig.5) does not make use of the access to future values of h_{ref} signal, although computation of this signal is the most complicated step of control process.

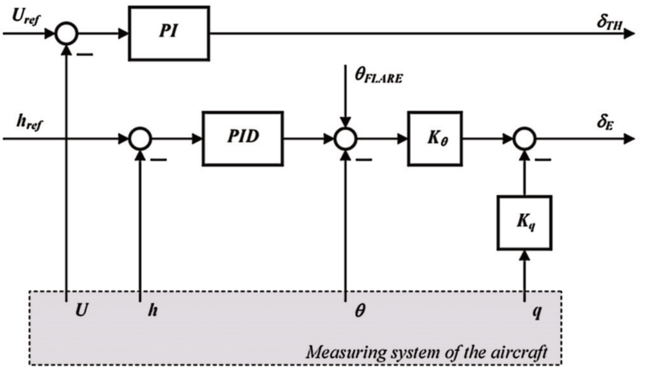

Fig. 5. Control algorithm for approach and flare manoeuvre- classical case

The simple solution is proposed instead of this presented in Fig. 5. We use the algorithm proposed in [13], [17], [18] which is capable to estimate precisely the vertical speed of the aircraft (the rate of climb/descent). Authors of these works proved (theoretically, by simulations and experiments), that using filtering algorithms (Kalman, complementary, etc.) for data fusion, it is possible to obtain precise estimates of rate of descent $\langle \dot{h} \rangle$, enough for control system requirements (see Appendix B for more details).

The control algorithm with $\langle \dot{h} \rangle$ used as feedback signal is presented in Fig.6 in the form of block diagram. The loop for horizontal speed stabilisation is not presented for simplicity– it is assumed to be the same as in Fig.5. Moreover, after numerical experiments variations of horizontal speed, even without stabilisation, turned out to be small (see the next paragraph). The loop with q, θ feedback signals is the classical Stability Augmentation System (SAS) for short period motion.

In fact the system stabilizes the required, constant value of rate of descent $\langle \dot{h} \rangle$. This value corresponds to the line 'B' in Fig. 4–the straight line tangential to the flare trajectory in touch down point. This line is also treated as required trajectory h_{ref}, so computation of h_{ref} and is extremely simplified. There is also the loop for controlling the altitude error h_{ref} - h, but if this error is small the loop opens due to dead zone and the control process focuses on rate of descent stabilisation. Parameters (gains) of the controller were computed by classical rules of control theory [10], [19], [20].

And what about predictive character of the control process? It is important to notice that from the beginning of the control process, the controller, uses $\langle \dot{h} \rangle_{ref}$ parameter, characterising the final part of flare trajectory, trying, to reduce as quickly as possible the rate of descent to reach the value which is desired for touch down and known a priori. The h_{ref} signal is also computed „*in advance*" for final part of manoeuvre. Moreover, this treatment is more flexible than presented before, because of possibility of shifting h_{ref} trajectory along the runway (axis x in Fig.4).

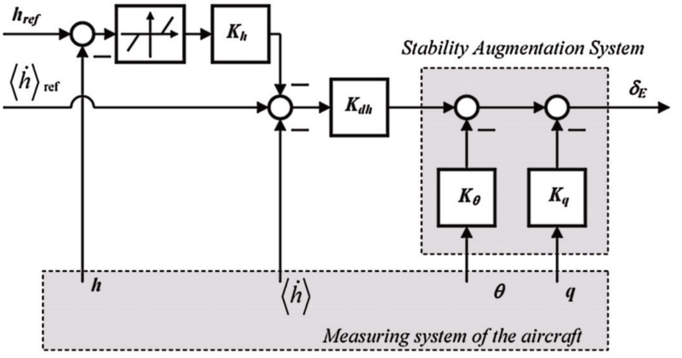

Fig.6. Control algorithm for flare manoeuvre – the estimate of vertical speed (rate of descent) is used as feedback signal

2. RESULTS OF SIMULATIONS

To illustrate the discussion presented in previous paragraphs, some numerical experiments (simulations) were completed. The mathematical model describing a longitudinal motion of Cessna 402C aircraft [14] (see Appendix C for details) was used as an example. This model is the linear one - its' structure and parameters were obtained by linearisation procedure applied to the full set of nonlinear equations of motion in the neighbourhood of a trajectory of typical approach and landing manoeuvre, accomplished by such an aircraft. The scenario of each simulation was typical for flare manoeuvre: the aircraft started to flare from the position 15 m above the runway threshold, being in trimmed flight, with the rest of flight parameters trimmed for the approach phase of flight.

Flight trajectory is the most representative result of such simulation is the trajectory of landing aircraft that is shown in Fig. 7. This trajectory shows the correct shape and expected behaviour, with touch-down point located about 530 meters behind the runway threshold.

At the altitude of 6 m above the ground the slope of the trajectory reached the value closed to the desired one (reached the desired value of the descent speed) and this slope remind close to constant till touch - down point.

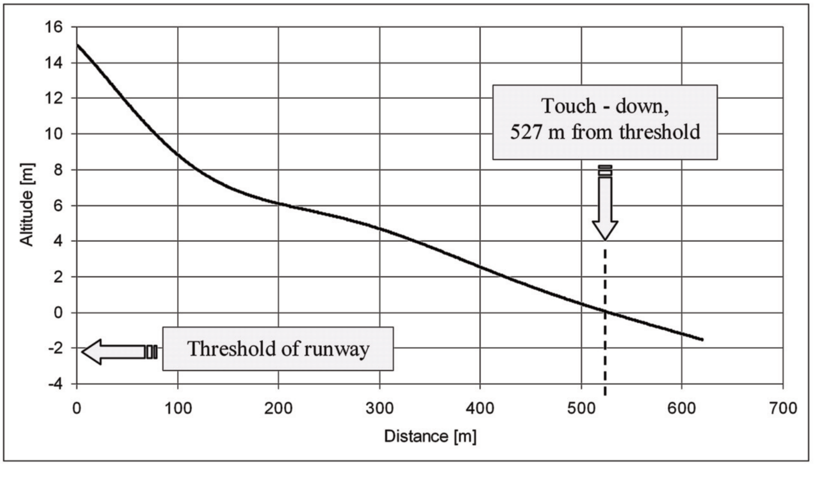

Fig. 7. Flare manoeuvre- aircraft trajectory

Next, two diagrams present the behaviour of the most important variables: angle of attack, pitch angle, pitch angular rate and trajectory angle (increments of these variables related to the values for which the set of motion equations was linearised). To give a good insight into the process these results are presented in two different domains: in spatial domain (Fig. 8) - as the function of altitude, and in time domain (Fig. 9). These results prove again the correctness of transient of the manoeuvre, following preliminary expectations.

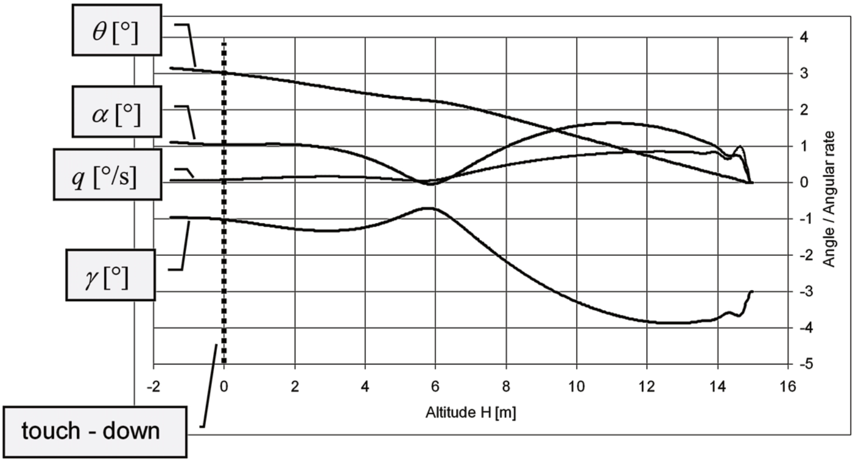

Fig. 8. Flare manoeuvre - „*angular*" parameters of longitudinal motion versus altitude

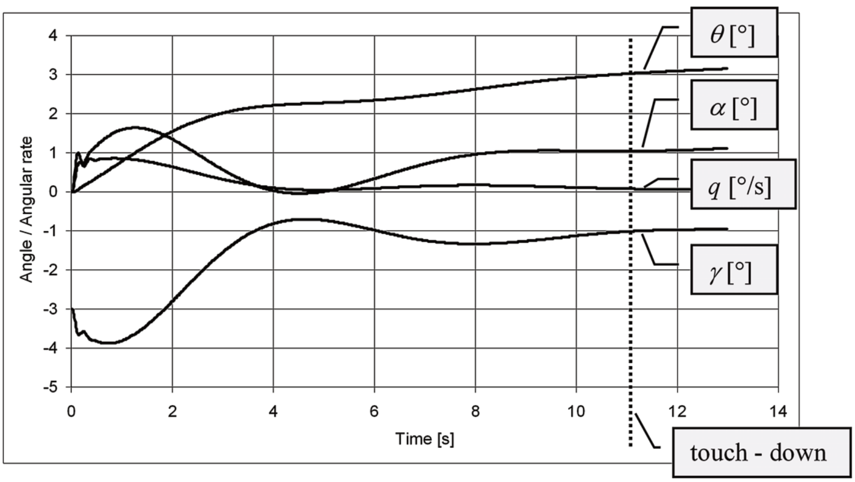

Fig. 9. Flare manoeuvre- angular parameters of longitudinal motion versus the time

Fourth diagram (Fig.10) presents one of the most important relationship: vertical speed of the aircraft (rate of descent) versus altitude. This diagram proves that the value of vertical speed in touch - down point (about -0.8 m/s) got into the range of allowable and desirable values [-1.0, -0.5] m/s. This means that during the flare phase of landing manoeuvre, the proposed automatic control system managed to reduce the value of this parameter more than three times when compared with its' „*trimmed"* value for the approach phase (-2.5 m/s).

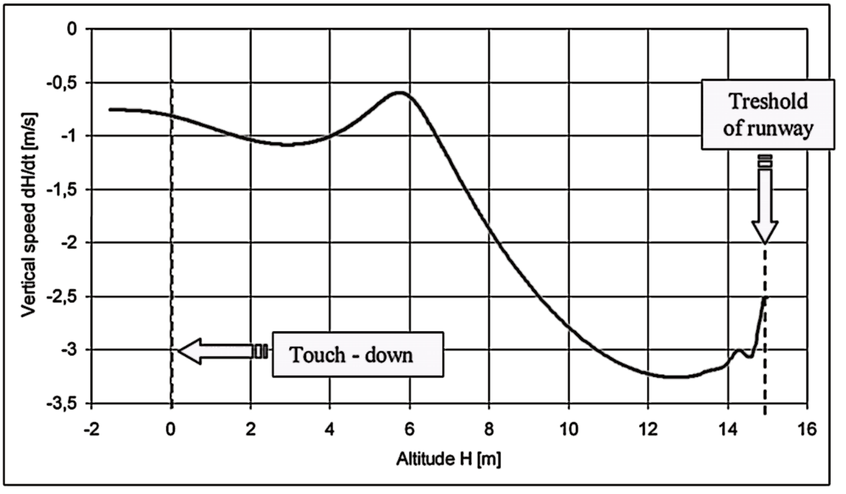

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Fig. 10. Flare manoeuvre – rate of descent motion versus the altitude

Last two diagrams complete the picture of simulation results. One (Fig. 11) shows the time-story of control signal - the deflection of elevator, as well as the time lag imposed by inertial dynamics of the actuator (the difference between commanded and actual values of deflection of the elevator).

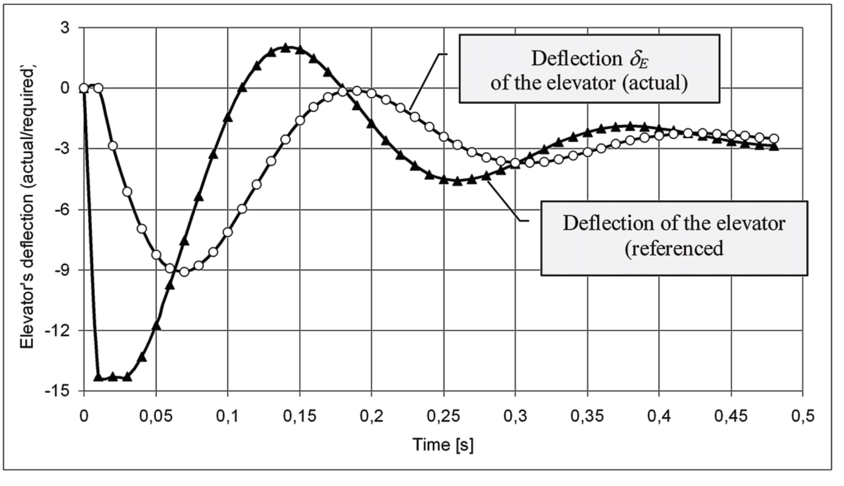

Fig. 11. Flare manoeuvre- motion of the elevator

The last diagram (Fig. 12) proves that the incremental variation of linear velocity in flare manoeuvre (about 1 m/s) compared with the absolute value of this speed in approach phase (48 m/s) is so small, that for presented simulations the control loop for linear speed stabilisation can be neglected. This does not mean of course that this loop can be rejected from the design of real control system !

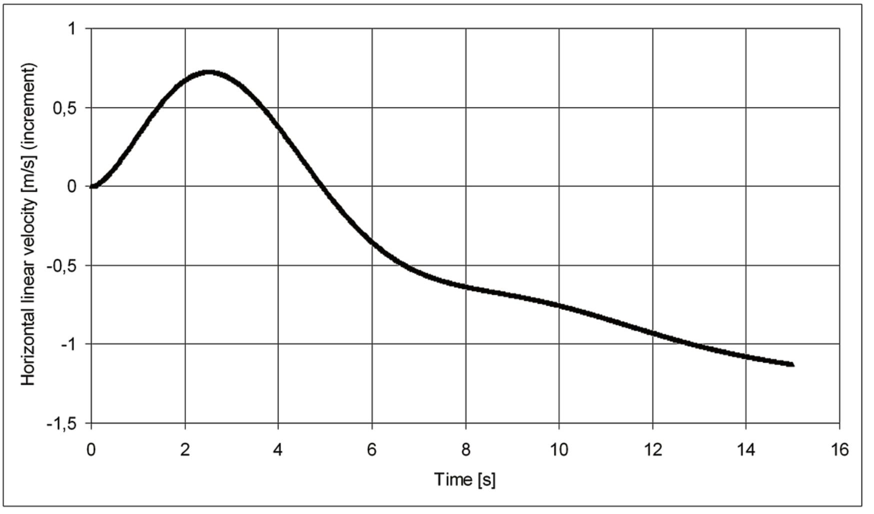

Fig. 12. Flare manoeuvre- fluctuation of the horizontal speed

3. CONCLUSIONS

Presented solution for longitudinal motion control in flare manoeuvre is the simple algorithm based on the advanced measuring/estimating system for rate of descent in approach & olanding manoeuvre. This system is designed in the Institute of Aviation and is expected to have good perspectives in the future. Proposed longitudinal motion control system for flare manoeuvre was verified by simulation with the mathematical model of Cessna 402C aircraft, used as an example. Results of simulations are close to expectations, despite of simplicity of control law. The main advantage of proposed solution is its high potential for further delopment.

APPENDIX A RATE-OF-CLIMB/DESCENT MEASUREMENT/ESTIMATION [5],[17],[18]

The avionic equipment installed on-board in the aircraft for measurement / estimation of altitude and rate of climb / descent flight parameter affects significantly on flight safety, particularly when flight control in approach & landing manoeuvre is considered. One of relevant results, achieved in the Institute of Aviation (ILot) during last years within this area, is a development of innovative algorithms and appropriate electronic hardware designed to carry on this measurement/estimation operations with improved accuracy. Actually, two such systems are designed, especially in order to be used as the on-board aid for the approach & landing phase of flight.

- the first system is based on the idea of accuracy improvement of estimates achieved by the fusion of measuring signals received from several types of sources (e.g. pressure, inertial and other sensors [17], [18]).
- the other solution is the electronic miniaturised, microwave sensor [5], operating on the principle based on radio – altimeter concept, thus, processing the information from one source.

In [17] the synthesis of data fusion (integrated) system is presented. The design is based on the set of not expensive *MEMS* sensors (*Micro Electro - Mechanical Systems*) supported by *GPS*. All of these elements are integrated by the effective fusion of different methods for altitude measurement and vertical speed estimation in one measuring / estimating system, to achieve required accuracy and reliability. The idea of proposed solution is focused on *„... improvement of relatively weak time - stability of MEMS sensors' scaling factors by sufficiently frequent correction based on GPS measurement, known for good stability in long time intervals"* [17]. The system delivers estimates of:

▪the altitude, absolute and true (this last determined relatively to the height of terrain data stored in memory) within the range: -300 ÷ 6000 m above sea level, with 5 m accuracy up to 3000 m and 10 m accuracy within the rest of operating range

▪the vertical speed, computed repetitively within the operating range of 20 m/s, with ±0.1 m/s accuracy and 50 Hz repetition frequency.

The schematic diagram presented in Fig.A1 [12] illustrates the idea of complementary filtering – the effective method used for data fusion. Transfer functions enclosed by broken line create the complementary filter (see [17], [18] for details of design).

Basic version of the system makes use of three measuring signals:

▪static pressure (*MEMS* sensor)

▪vertical acceleration (*MEMS* sensor)- the component of acceleration along the axis of local vertical with rejected influence of gravitation and curvilinear motion

▪altitude estimate delivered by *GPS* receiver

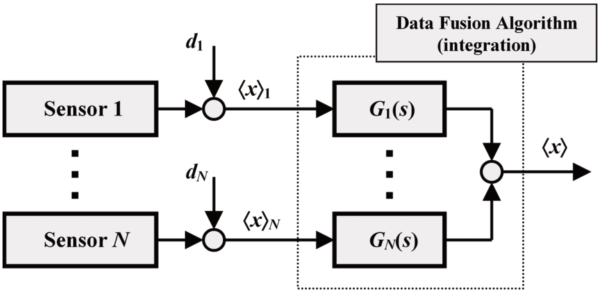

Fig. A1. Complementary filtering: $\langle x\rangle_i$=1,...*n*,-signal from sensors , d_i-disturbance signals, G_i- transfer functions of filter components, ‹x›- final estimate [13]

For altitude measurement/estimation three signals are used as the input of data fusion algorithm:

▪barometric altitude - obtained by the appropriate processing of static pressure

▪*„inertial altitude"* estimate - obtained by double integration of vertical acceleration

▪GPS altitude estimate

For the rate of climb/descent estimation algorithm also makes use of three sources:

▪barometric rate of climb/descent estimate - obtained by differentiation of barometric altitude

▪*„inertial"* estimate of rate of descent/climb – obtained by vertical acceleration integration;

▪the altitude estimate delivered by *GPS* receiver to augment the process of current corrections of scale factors for accelerometer and pressure sensor.

The structure of the measuring/estimating system is open-it is ready to be extended with other sources of measuring signals (sensors)–*e.g*. the radio–altimeter, also developed in the Institute of Aviation (aforementioned in the initial part of the paragraph).

Presented idea was verified in series of in-flight tests completed on *An-28 „Bryza"* aircraft. Experiments are focused especially on approach & landing manoeuvre to get the assessment of the possibility to use this system as a source of feedback signals in automatic control system guiding the aircraft during this manoeuvre. The *An-28 „Bryza"* aircraft was equipped with the radio - altimeter to for verification of measurement results (the accuracy of measurements delivered by this instrument was not worse than 0.9 m within the range of low altitudes). Vertical speed estimates, obtained by data fusion are compared with the result of off – line differentiation of radio – altitude. Maximum value of discrepancy between these two signals was less than 0.1 m/s, thus, taking into account the declared accuracy of radio - altimeter, the accuracy of vertical speed estimate obtained by data fusion was assessed to be not worse than 0.05 m/s. This result proves that the system works correctly.

Results of in-flight tests proved that it is possible to fulfil aforementioned requirements. Measuring system, based on fusion of measuring signals obtained from GPS system, vertical accelerometers and static pressure sensor, turned out to be capable to compute precise estimate of vertical speed (with estimated accuracy about 0.05 m/s) as well as the estimate of altitude (with accuracy better than 1 m in discussed case,). Additionally it is worth a notice that:

- in case of GPS signal fade the system continues to work and both vertical speed and altitude estimates are computed continuously, however measuring accuracy decreases
- satisfying accuracy and reliability of MEMS sensors and GPS receivers makes this system applicable also for small and low cost aircraft of General Aviation category
- it is possible to extend the proposed structure by including other measuring sub-systems (e.g. Doppler radio – altimeter or ultrasonic altimeter) in the process of data fusion, to increase the accuracy and reliability of the whole system
- overall dimensions are small enough for UAV applications.

APPENDIX B: PREDICTIVE CONTROL – THE CLASSICAL APPROACH

It is convenient to consider a discrete time domain for discussing the idea of automatic control with anticipated behaviour of controlled object. So, we assume that time instants (samples) are denoted by natural numbers: 0, 1, ..., *k, k+1*, and all increments of time from one sample to the next one are equal i.e.: $t_{k+1}-t_k=\Delta t=const$ for every k. The most typical idea of predictive control algorithm is then based on two general assumptions:

Assumption 1: Mathematical model (B1) of a controlled system is known, with *y, u* denoting respectively output and control signals (in general, vectors of appropriate dimensions) and *G*- a causal operator. This model is capable can be used for predicting by computation future behaviour of controlled object when the future sequence of control signal samples is known.

$$y_{k+1} = G(y_k, y_{k-1}, \dots, u_k, u_{k-1}, \dots) \tag{B1}$$

Assumption 2: At every current moment of time a reference trajectory y(ref), representing a desired value of output signal, is known over a finite set of future time instants, called the prediction horizon. This means that at the moment k values: $y_{k+1}^{(ref)}, y_{k+1}^{(ref)}, \dots,. y_{k+L}^{(ref)}$ are known and L is the prediction horizon.

Then, control signal applied to the controlled object is computed by optimisation of quadratic quality index:

$$I(k) = \sum_{j=k}^{k+L-1} \left[y_{(j|k)} - y_j^{(ref)} \right]^T \mathbf{Q} \left[y_{(j|k)} - y_j^{(ref)} \right] + u_{(j|k)}^T \mathbf{R} u_{(j|k)} \tag{B2}$$

where **Q, R** are symmetric, positive - definite weighting matrices of appropriate dimensions, $u_{(k|k)}, u_{(k|k+1)}, \dots, u_{(k|k+L-1)}$ is a series of control signal candidate future samples, while $y_{(k|k)}, y_{(k|k+1)}, \dots, y_{(k|k+L-1)}$ is a series of predicted output signal samples.

Functional (B1) can be treated as a function of $u_{(k|k)}, u_{(k|k+1)}, \ldots, u_{(k|k+L-1)}$, because, accor ding to Assumption 1, $y_j^{(ref)}$ samples appearing in this relationship are known and approp riate samples of $y_{(j|k)}$ can be computed using the model (B1).

Indeed: $$y_{(k+i|k)} = G\left(y_{(k+i-1|k)}, \ldots, y_{(k+1|k)}, y_k, \ldots, u_{(k+i-1|k)}, \ldots, u_{(k+1|k)}, \; u_k, u_{k-1}, \ldots\right) \quad \text{(B3)}$$

where i = 1, 2, ... , $k+L$-1. Thus the control algorithm can be presented as follows [6]:

Step 1: Measure the y_k; sample of output signal

Step 2: Obtain the control sequence: $u_{(k|k)}, u_{(k|k+1)}, \ldots, u_{(k|k+L-1)}$ by optimising the quality index (2)

Step 3: Apply $u_{(k|k)}$ as $\boldsymbol{u}_k$

Step 4: Time update: $k := k + 1$

Step 5: **GOTO** Step 1

APPENDIX C: CESSNA 402C–THE MATHEMATICAL MODEL

To illustrate the discussion some numerical experiments have been completed, on the basis of mathematical model of longitudinal motion in approach & flare manoeuvre.

The state space model of Cessna 402C aircraft [14] has been chosen as an example with the state vector $\mathbf{X}=[u,\alpha,q,\theta]^T$,where: u- longitudinal velocity in body axes, α- angle of attack, q-pitch angular rate, θ-pitch angle, and control vector $\mathbf{W}=[\Delta\delta_E,\Delta\delta_{TH}]^T$, where δ_E, δ_{TH}-positions of the elevator and throttle respectively.

The model is a linear one, being a result linearisation in the neighbourhood close to the reference trajectory representing the approach flight along a glide path before flare and touch down. It is assumed that such reference (nominal) motion takes place at sea level and is defined by the following nominal values of state variables: $u=u_0$=48m/s, $q=q_0$=0rd/s and trajectory angle (3°-the typical slope of glide path).

The general form of the model is of the following: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{w}$, with constant matrices **A**, **B** of appropriate dimensions and both, state: $\mathbf{x}=[\Delta u, \Delta\alpha, \Delta q, \Delta\theta]^T$ and control: $\mathbf{w}=[\Delta\delta_E,\Delta\delta_{TH}]^T$ vectors, representing increments (denoted by) differing true values of variables and their nominal values, corresponding to reference trajectory. The structure of **A** and **B** matrices is presented below:

$$\mathbf{A} = \begin{bmatrix} X_u & X_\alpha & 0 & -g \\ Z_u & Z_\alpha & Z_q & Z_\theta \\ M_u(1+Z_u) & M_\alpha + M_\alpha Z_\alpha & M_q + M_\alpha Z_q & M_\alpha Z_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} X_{\delta E} & X_{\delta TH} \\ Z_{\delta E} & 0 \\ M_{\delta E} + M_\alpha Z_{\delta E} & 0 \\ 0 & 0 \end{bmatrix},$$

where numerical values of matrices' components are the following:

A:	X_α = 21.01	X_u = -0.053	X_q = 0	X_θ= -9.806
	Z_α = -1.05	Z_u = -0.002	Z_q = -0.024	Z_θ = 0
	M_α = -12.3	M_u = 0.016	M_q = -6.22	M_θ = 0

B: $X_{\delta E} = -0.046$ $X_{\delta TH} = 0.0005$
$Z_{\delta E} = -0.96$
$M_{\delta E} = -13.55$

This original model is extended with two additional variables: *S*-representing horizontal position of the aircraft along the runway, and *H*-representing height above the ground. Thus:

$$\dot{S} = u\cos\gamma = \underbrace{u_0\cos\gamma_0}_{\dot{S}_0} + (\cos\gamma_0)\Delta u - (u_0\sin\gamma_0)\underbrace{(\Delta\theta - \Delta\alpha)}_{\Delta\gamma} + \dots(\text{terms of higher order})$$

$$\dot{H} = u\sin\gamma = \underbrace{u_0\sin\gamma_0}_{\dot{H}_0} + (\sin\gamma_0)\Delta u + (u_0\cos\gamma_0)\underbrace{(\Delta\theta - \Delta\alpha)}_{\Delta\gamma} + \dots(\text{terms of higher order}),$$

and the linear matrix vector equation added to the original model takes the following form:

$$\frac{d}{dt}\begin{bmatrix}\Delta S \\ \hline \Delta H\end{bmatrix} = \left[\begin{array}{c|c|c}\cos\gamma_0 & -U_0\sin\gamma_0 & U_0\sin\gamma_0 \\ \hline \sin\gamma_0 & U_0\cos\gamma_0 & -U_0\cos\gamma_0\end{array}\right]\begin{bmatrix}\Delta u \\ \hline \Delta\theta \\ \hline \Delta\alpha\end{bmatrix} .$$

Finally, the extended model with new, extended, state vector $\tilde{\mathbf{x}} = [\ \Delta u, \Delta\alpha, \Delta q, \Delta\theta, \Delta S, \Delta H\ ]^T$ and control vector $\mathbf{w} = [\ \Delta\delta_E, \Delta\delta_{TH}\ ]^T$ not changed, is built on the following matrices:

$$\tilde{\mathbf{A}} = \left[\begin{array}{c|c|c|c|c|c} -0.053 & 21.01 & 0 & -9.806 & 0 & 0 \\ \hline -0.002 & -1.05 & -0.024 & 0 & 0 & 0 \\ \hline 0.0406 & 0.615 & -6.5152 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0.9986 & 2.5121 & 0 & -2.5121 & 0 & 0 \\ \hline 0.0523 & -47.9342 & 0 & 47.9342 & 0 & 0 \end{array}\right] ; \quad \tilde{\mathbf{B}} = \left[\begin{array}{c|c} -0.046 & 0.0005 \\ \hline -0.96 & 0 \\ \hline -1.742 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \end{array}\right]$$

Assuming that during the approach & flare manoeuvre the aircraft's velocity is approximately constant ($\Delta u \approx 0$) this model can be reduced to obtain the simplified one of *SISO* structure (*Single Input – Single Output*), convenient for control law synthesis:

$$\frac{d}{dt}\begin{bmatrix}\Delta\alpha \\ \hline \Delta q \\ \hline \Delta\theta\end{bmatrix} = \left[\begin{array}{c|c|c} -1.05 & -0.024 & 0 \\ \hline 0.615 & -6.5152 & 0 \\ \hline 0 & 1 & 0 \end{array}\right] \cdot \begin{bmatrix}\Delta\alpha \\ \hline \Delta q \\ \hline \Delta\theta\end{bmatrix} + \begin{bmatrix}-0.96 \\ \hline -1.742 \\ \hline 0\end{bmatrix}\Delta\delta_E$$

$$\Delta v_Y = [-47.9342 \mid 0 \mid 47.9342] \cdot \begin{bmatrix}\Delta\alpha \\ \hline \Delta q \\ \hline \Delta\theta\end{bmatrix} = \left\langle \frac{d\Delta H}{dt} \right\rangle ,$$

where the output signal Δv_Y represents the linear increment of vertical velocity, referenced to the nominal vertical velocity $u_0\sin(\gamma_0)$.

This is the estimate of time – derivative of height increment ΔH. The spectral form of this model based on transfer function, defined in frequency domain, is the following:

$$G(s)=\frac{\Delta v_Y(s)}{\Delta\delta_E(s)}=k\frac{(1+\tau_1 s)(1+\tau_2 s)}{s(1+T_1 s)(1+T_2 s)} \quad ,$$

where: k=46.0168, T_1=-1.0528, T_2=-6.5122, τ_1=0.4898, τ_2=-5.1468.

It is easy to notice that both poles of this function are placed within the left half of the plane of complex variable, proving the stability of short–period motion, however one of zeros is placed within the right half, proving the nonminimal–phase feature of this dynamical system, when treated as an object of automatic control.

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PIOTR MASŁOWSKI

STEROWANIE RUCHEM PODŁUŻNYM W FAZIE WYRÓWNANIA PODCZAS LĄDOWANIA

Streszczenie

Dyskusja przedstawiona w artykule koncentruje się wokół wybranych zagadnień syntezy praw sterowania automatycznego ruchem podłużnym lądującego samolotu, podczas manewru wyrównania poprzedzającego moment przyziemienia. Omówiono niektóre ogólne aspekty procesu sterowania w takim przypadku, aby ujawnić jego predykcyjny charakter. Dyskusja nowych rozwiązań, rozwijanych w Instytucie Lotnictwa dla urządzeń pokładowych przeznaczonych do pomiaru/estymacji prędkości opadania/wznoszenia podczas wykonywania takiego manewru, podkreśla ich efektywność i „nową jakość' wprowadzaną przez nie do procesu sterowania. Rozważania ilustruje kilka wyników obliczeń symulacyjnych, wykonanych dla modelu matematycznego samolotu Cessna 402C, który został wykorzystany jako przykład. Wyniki te potwierdzają poprawne działanie proponowanych rozwiązań i ich duży potencjał dla przyszłych badań.